

# Racial Discrimination in Labor Markets with Posted Wage Offers

by Kevin Lang, Michael Manove and William T. Dickens \*

We analyze race discrimination in labor markets in which wage offers are posted. If employers with job vacancies receive multiple applicants, they choose the most qualified but may choose arbitrarily among equally qualified applicants. In the model, firms post wages, workers choose where to apply, and firms decide which workers to hire. Labor-market frictions greatly amplify racial disparities, so mild discriminatory tastes or small productivity differences can produce large wage differentials between the races. Compared with the nondiscriminatory equilibrium, the discriminatory equilibrium features lower net output, lower wages for both white and black workers and greater profits for firms. (JEL: J70)

Economic theory suggests that wage discrimination against groups of workers is unlikely to persist in a competitive economy, because in the presence of such discrimination, profits can be had by hiring members of the discriminated-against groups. Consequently, in trying to account for differences in the treatment of worker groups, economists have tended to rely either on real productivity differences or else on market imperfections that tend to block the antidiscrimination market response.

We offer a model of racial discrimination in the labor market in which market imperfections transform weak discriminatory preferences or small productivity differences into large wage differentials. We analyze labor markets characterized by wage posting, wherein employers attach wage

offers to announced job openings. Wage posting is a commonly observed labor-market phenomenon, perhaps because workers would be less likely to invest in the job-application procedure for an unknown wage. Indeed, when wage posting is the norm, the failure to post a wage could be viewed as a negative signal. Posted wage offers are assumed binding on the employer and cannot be conditioned on the identity or race of the worker to be hired.

We will show that wage posting lends itself to persistent discrimination. The labor market in our model has the flavor of monopsonistic competition, with a large number of firms that post wage offers and a large number of workers that respond to the posted wage offers. The intuition is straightforward. Because a binding wage offer has been posted, the employer cannot pay less to applicants who are subjected to discrimination elsewhere than he pays to other workers. Thus the antidiscrimination market response cannot function.

In our baseline model, employers find black workers to be slightly less desirable employees than white workers. Although perceived differences are small, they are sufficient to ensure that employers will choose a white worker in preference to a black worker if both apply for the same job. Consequently, black workers want to avoid the cost of applying to firms that are likely to receive applications from whites. Blacks can accomplish this by applying to firms with wage offers that are low enough to discourage white applicants. In equilibrium, blacks and whites will be employed by different firms (segregation), blacks will receive lower wages with the wage differential far exceeding the taste or productivity differential (wage discrimination), and firms will retain higher profits.

Our argument shows that the labor-market structure we depict could amplify even modest racist tendencies or small productivity differences to yield highly visible economic outcomes with significant social consequences. This is not to deny that active racism exists in the labor market; we mean to suggest only that even mild racist tendencies can produce segregated workplaces and

wage discrimination against blacks. Indeed, in the limit, our equilibrium holds even when whites and blacks are equally productive and no employer has any racial preferences whatever, that is, even when firms are unwilling to pay anything more in order to hire a white in preference to a black worker.

Our results require that firms be committed to their posted wage offers, and that wage offers cannot be conditioned on the type of worker. Race-contingent posted wage offers would be an egregious and public violation of civil rights legislation that most employers would wish to avoid. Furthermore, in white racist social environments, wage discrimination in favor of blacks would be a gross violation of social norms, and wage discrimination against blacks would inevitably lead to hiring discrimination in their favor, also socially proscribed. Thus we should not expect to see race-contingent wage offers, even in the absence of civil rights legislation. Evidence from the 19th and early 20th century American South supports this view (Robert Higgs, 1989).

Our principal conclusion is that in an economic environment with posted wage offers, segregation and wage discrimination against black workers can arise even when all information is symmetric, information about posted wage offers and about employers' discriminatory behavior is perfect and employers and workers lack substantial racist motives. This discrimination creates economic inefficiency, reduces total output, decreases wages for both black and white workers and increases profits.

Although based entirely on individualistic maximizing behavior without collusion or cooperation, the model has a flavor that is reminiscent of certain Marxian models in which capitalists increase profits by dividing workers against themselves.

In subsequent sections we construct and analyze a wage-posting model in both a nondiscriminatory and a discriminatory setting. Because of space considerations, many technical details and

formal proofs will be omitted without comment, but all can be found in Lang, Manove and Dickens (2004), which is the online working-paper version of this article.

## I. The Wage-Posting Model in a Nondiscriminatory Regime

We analyze the wage-posting game without discrimination, or equivalently, with homogeneous workers. The solution of the game will serve as a benchmark, and additionally, it will yield the solution for white workers in the game with discrimination against blacks. We draw on a model sketched in Lang (1991) and formalized by James D. Montgomery (1991).

Suppose that all workers are equally productive, and firms make no distinctions between them. Each firm has one unfilled position and posts a wage in the hope of attracting at least one applicant. Workers observe the wage offers that have been posted and decide where to apply. Each worker can apply for only one job. Recognizing that a higher wage offer is likely to attract more applicants for the job opening, workers trade off a higher wage offer against a lower probability of employment and probabilistically—without coordination—spread their applications among firms in a way that tends to equalize their expected incomes. The number of applications each firm receives is a random variable, and there is a positive probability that a firm will receive no applications. Firms recognize that raising the wage will increase the expected number of applicants and thus lower the probability of having to bear the cost of a job vacancy and the resulting inability to produce.

Consider a two-stage game with a large and fixed number  $N$  of identical firms and random number  $\tilde{Z}$  of identical workers, where  $\tilde{Z}$  is Poisson distributed with mean  $Z \equiv E(\tilde{Z})$ . This is the distribution that would arise if agents from a large population were to make independent and equally probable decisions to enter the job market. It will be important to our model that the realization of the random variable  $\tilde{Z}$  not be observable, either to firms or to workers. By contrast, the mean  $Z$  of the distribution is assumed to be common knowledge.

In the first or wage-setting stage of the game, firms simultaneously announce their wage offers, which they are committed to pay each worker hired. In the second or worker-application stage, workers observe the profile of wage offers and simultaneously apply to firms for jobs. In general, worker  $j$  will adopt a mixed strategy whose outcome will be an application to only one firm  $i$ . At the end of the game, firms apply the following hiring rules to their applicants: a firm that receives no applications cannot hire or produce; a firm that receives one application hires that applicant; and a firm that receives more than one application hires one applicant at random.

Firm  $i$ 's strategy consists of its choice of a single wage offer  $w_i$ . The vector  $\mathbf{W} \equiv \langle w_i \rangle$  denotes the profile of strategies for all firms. A worker's (mixed) strategy is a vector-valued function  $\mathbf{q}(\mathbf{W}) \equiv \langle q_i(\mathbf{W}) \rangle$ , where each  $q_i(\mathbf{W})$  is the probability that the worker will choose to apply to firm  $i$ . We restrict the worker's strategy choices to those consistent with the anonymity of firms: if  $w_i = w_j$  then  $q_i(\mathbf{W}) = q_j(\mathbf{W})$ . If all workers adopt the same mixed strategy, the number of workers that apply to a given firm  $i$  will have a Poisson distribution, whose mean we denote by  $z_i$  where

$$(1) \quad z_i = q_i(\mathbf{W}) Z.$$

Firm  $i$ 's payoff is its expected operating profits (revenue minus variable cost), given by

$$(2) \quad \pi_i = (1 - e^{-z_i})(v - w_i),$$

where  $v$  is the value of the worker's output, and  $1 - e^{-z_i}$  is the probability that the firm fills its vacancy. A worker's payoff is the firm's wage offer  $w_i$  if he is hired by firm  $i$ , and zero, otherwise.

We proceed to search for an equilibrium  $\{\mathbf{W}^*, \mathbf{q}^*(\cdot)\}$  of the wage-posting game that is symmetric among workers (all workers use the same mixed strategy  $\mathbf{q}^*(\cdot)$ ). In the solution concept as applied to the wage-setting stage of the game, we substitute the common notion of a competitive equilibrium for that of a Nash equilibrium: the only difference being that in competitive equilibrium agents are required to be price-takers in a sense to be described below, whereas in Nash equilibrium agents

are required to take into account even the very small effect that their own behavior may have on market prices. We will use the term “subgame-perfect competitive equilibrium” to describe a solution concept for a multistage game that is parallel to subgame-perfection, but with a competitive equilibrium substituted for a Nash equilibrium in the first stage.

We now find the unique symmetric equilibrium  $\mathbf{q}^*(\mathbf{W})$  of the worker-application subgame for any given wage-offer profile  $\mathbf{W}$ . Workers trade off each firm’s wage offer against the expected number of competing job applicants so as to maximize their expected incomes. In equilibrium, workers obtain the same expected income (to be called the “market expected income”) at every firm to which they apply. Workers will apply with positive probability to any firm that offers a wage above the market expected income, and the expected number of applicants will rise to a level exactly sufficient to reduce expected income at that firm to the market level. Workers will not apply with positive probability to any firm that offers a wage less than or equal to the market expected income, because competition from other applicants (no matter how slight) would force expected income to fall below the market level. The market expected income is increasing in the wage offers of firms and decreasing in the number of workers in the pool of applicants. A firm’s expected number of applicants (and the probability that each worker will apply) is a continuous function of its wage offer. If the wage offer is increased by a small amount, the expected number of applicants will rise until the expected income at the firm falls back to the market level (now very slightly higher). We proceed to model the situation more formally.

Let the wage-offer profile of firms be  $\mathbf{W} = \langle w_i \rangle$  with  $\mathbf{W} \neq \mathbf{0}$ , and consider the worker-application subgame, in which workers apply for jobs. Suppose a firm has a pool of potential job applicants, each with the same non-negative probability of applying to that firm, and suppose  $z$  is the expected number of applicants to the firm from that pool. Imagine that an additional designated worker applies to the firm. The probability  $f(z)$  that the additional applicant will be

hired is given by

$$(3) \quad f(z) \equiv \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{e^{-z} z^n}{n!},$$

where  $e^{-z} z^n/n!$  represents the Poisson probability that  $n$  other applicants would appear,<sup>1</sup> and  $1/(n+1)$  is the probability that the designated worker would be hired in that case. Manipulation of the series yields

$$(4) \quad f(z) \equiv \begin{cases} 1 & \text{for } z = 0 \\ (1 - e^{-z})/z & \text{for } z > 0 \end{cases}.$$

Thus, if  $K_i$  denotes the expected income or payoff that the designated worker can obtain by applying to firm  $i$ , we have

$$(5) \quad K_i \equiv w_i f(z_i).$$

Suppose now that firms have set wage offers  $\mathbf{W} \equiv \langle w_i \rangle$ , and suppose the worker application subgame has an equilibrium in which all workers adopt the same mixed strategy. Let  $K \equiv \max_i \{K_i\}$  denote the maximum expected income available in that equilibrium. Because workers will choose to apply only to firms with  $K_i = K$ , we may think of  $K$  as the market expected income. If a firm  $i$  offers a wage  $w_i$  greater than  $K$ , then the expected number of applications  $z_i$  will be large enough to reduce  $K_i$  to  $K$ . If a firm offers a wage  $w_i$  less than or equal to  $K$ , then  $K_i$  must be less than  $K$ , even when the expected number of applicants is very small. Thus no worker will apply to such a firm in equilibrium. We can conclude that in any symmetric equilibrium of the worker application subgame,  $K_i$  is given by

$$(6) \quad K_i = \begin{cases} K & \text{for } w_i \geq K \\ w_i & \text{for } w_i < K \end{cases},$$

$z_i$  satisfies

$$(7) \quad \begin{aligned} z_i &> 0 & \text{for } w_i > K \\ z_i &= 0 & \text{for } w_i \leq K \end{aligned},$$

and

$$(8) \quad z_i = f^{-1}\left(\frac{K}{w_i}\right) \quad \text{for } w_i \geq K.$$

Equation (8) implies that given  $\mathbf{W}$ , the total expected number of applicants is

$$(9) \quad \sum_{i=1}^N z_i \equiv \sum_{\{i | w_i \geq K\}} f^{-1}\left(\frac{K}{w_i}\right),$$

which depends on only on the value of  $K$ . Therefore, in equilibrium,  $K$  must take a value that satisfies

$$(10) \quad \sum_{\{i | w_i \geq K\}} f^{-1}\left(\frac{K}{w_i}\right) = Z,$$

because  $Z$  is the parametrically fixed expected number of applicants. Since  $f^{-1}$  is strictly decreasing in  $K$  and because the summand can lose but not gain terms as  $K$  increases, the left-hand side of the equation is strictly decreasing in  $K$ . Consequently, the equation has a unique solution for  $K$ , denoted here by  $K^*(\mathbf{W})$ . It follows that equations (6)-(8) and the relation  $q_i Z = z_i$  yields a vector of application probabilities  $\mathbf{q}^*(\mathbf{W})$  that defines a unique symmetric equilibrium of the worker application subgame with offered wages  $\mathbf{W}$ .

Now we search for equilibria of the entire wage-posting game. Our solution concept will be the *subgame-perfect competitive equilibrium (SPCE)*, a simplification of standard subgame-perfection in which aggregate variables are assumed constant with respect to the changes in the strategy of an individual agent. We say that  $\{\mathbf{W}^*, \mathbf{q}^*(\cdot)\}$  is a subgame-perfect competitive equilibrium, symmetric among the workers, if

- i. each firm's  $w_i^*$  is a best response to the other components of  $\mathbf{W}^*$  and to the workers' strategies  $\mathbf{q}^*(\cdot)$  on the assumption that the market expected income  $K^*(\mathbf{W})$  remains fixed at  $K^*(\mathbf{W}^*)$  and is not sensitive to the firm's own wage;<sup>2</sup> and,
- ii.  $\mathbf{q}^*(\mathbf{W})$  is a best response of each worker to any vector of offered wages,  $\mathbf{W}$ , and to the choice of  $\mathbf{q}^*(\mathbf{W})$  by all other workers.

Let  $r \equiv Z/N$  denote the ratio of the expected number of job applicants to the number of firms.



**Proposition 1** *The game between firms and workers has a subgame-perfect competitive equilibrium  $\{\mathbf{W}^*, \mathbf{q}^*(\cdot)\}$  that is unique among those in which all workers adopt the same mixed strategy. In this equilibrium, all workers adopt the strategy  $\mathbf{q}^*(\cdot)$ , as defined above, and all firms adopt the strategy  $w^*$  given by*

$$(11) \quad w^* = \frac{vr}{e^r - 1}.$$

*The expected income of each worker is*

$$(12) \quad K^*(\mathbf{W}^*) = ve^{-r},$$

*and the operating profit of each firm is*

$$(13) \quad \pi^* = [1 - (1 + r)e^{-r}]v.$$

*As  $r$  goes from 0 to  $\infty$ ,  $\pi^*$  goes from 0 to  $v$  and  $w^*$  and  $K^*(\mathbf{W}^*)$  go from  $v$  to 0.*

The basic steps of the derivation are straightforward. We know from (8) that  $z_i$  satisfies  $w_i = K^*(\mathbf{W})/f(z_i)$ , and substitution into (2) yields

$$(14) \quad \pi_i = (1 - e^{-z_i})v - z_i K^*(\mathbf{W}).$$

With  $K^*(\mathbf{W})$  held constant, the first-order condition for profit maximization implies

$$(15) \quad z_i^*(\mathbf{W}) = \log \frac{v}{K^*(\mathbf{W})},$$

and it follows that  $z_i^*(\mathbf{W})$  is the same for all  $i$ . Since each worker applies to exactly one firm, we have  $z_i^* = Z/N = r$ , so that (12) follows from (15). Equations (8), (14) and the definition of  $f$  then yield (11) and (13).

The equilibrium in Proposition 1 is unique among those in which all workers have the same expected income. We believe that it is also unique among those in which any offered wage is offered by a large number of firms. There may be other equilibria in which individual workers and firms are able to circumvent the anonymity of the labor market by coordinating on a unique wage that only one firm offers and for which only one worker applies. However, in the context of a large

impersonal labor market, we find such equilibria implausible.

## II. The Wage-Posting Model in a Discriminatory Regime

Here, we generalize the model developed in the previous section to allow for two types of worker, black and white. The total numbers of white and black workers are Poisson-distributed random variables  $\tilde{Z}$  with mean  $Z$  (whites) and  $\tilde{Y}$  with mean  $Y$  (blacks) where both  $Z$  and  $Y$  are large. The number of firms,  $N$ , is also large. This structure yields compact closed-form solutions, which, in turn, permit straightforward comparative statics.

The productivity of white workers is given by  $v$ . The productivity of black workers is  $v(1 - \delta)$ . The parameter  $\delta$  is small or zero. We will continue to refer to  $\delta$  as a measure of a physical productivity difference, but  $\delta$  could just as well represent the distaste of racist employers for black workers. (In the latter case, of course, firms would be maximizing their expected utility instead of expected profits.) For now we assume that aside from race there are no observable productivity-relevant differences between workers.

The first stage of the game is the wage-setting stage. Firms simultaneously announce their wage offers. We let  $\mathbf{W} \equiv \langle w_i \rangle$  denote the wage profile. Each firm is committed to its posted wage and cannot hold up applicants by reducing the wage offer later. The wage offer cannot be conditioned on worker type.<sup>3</sup> The second stage is the job-application stage. Workers observe  $\mathbf{W}$  and apply to firms. Workers adopt mixed strategies of the form  $\mathbf{q} \equiv \langle q_i \rangle$  that are consistent with the anonymity of firms: if  $w_i = w_j$ , then  $q_i = q_j$ .

The discriminatory wage-posting game has a third stage that we call the hiring stage. In contrast to the nondiscriminatory game, in which employers chose randomly among all applicants, the choice of hiring policy in the discriminatory regime is part of the employer strategy. If the productivity-reduction parameter  $\delta$  is positive, and if employment discrimination is not effectively penalized,

then a discriminatory hiring policy in favor of whites would be the employer's best response. The employer would choose randomly among white applicants if he had white applicants; otherwise he would choose randomly among black applicants. If  $\delta$  is zero, then all hiring policies are best responses, including discrimination in favor of blacks or applying different hiring probabilities to whites and blacks.

If firms do not discriminate at the hiring stage, then the first two stages of the game in this section are equivalent to the nondiscriminatory game of the previous section, and the equilibrium is unchanged aside from minor notational differences. Here we analyze the equilibria of the more interesting case, when the discriminatory hiring strategy is the adopted best response. We search for equilibria in which all workers of a given type adopt the same mixed strategy. As before, our solution concept will be the subgame-perfect competitive equilibrium.

We assume that a firm cannot credibly commit to a hiring policy inconsistent with its best response at the hiring stage. In particular, the structure of our model is based on the presumption that a firm's promise not to discriminate against blacks (or, stronger, to discriminate in their favor) would not be believed by black workers. We return to this point later.

**The Workers' Equilibrium Strategy in the Discriminatory Regime:** We stipulate that firms follow the discriminatory strategy in the last (hiring) stage of the game, and using backwards induction, we analyze the worker-application stage. We search for equilibria in which all workers of the same type (either black or white) adopt the same mixed strategy.

First let us consider the situation of white workers. Given that wage offers have been set and that all firms will use the discriminatory strategy, white workers can consider black workers to be invisible, because blacks can have no effect on the probability that a white will be hired. Therefore, the equilibrium response of whites to  $\mathbf{W}$  is identical to that of workers in the nondiscriminatory

regime. In an equilibrium of the subgame, the expected income of white workers, here denoted by  $H^*(\mathbf{W})$ , is the same at all firms to which they apply with positive probability, and no greater at firms to which they do not apply. The expected number of white applicants  $z_i$  to firm  $i$  is the continuous function defined by

$$(16) \quad z_i = \begin{cases} 0 & \text{for } w_i \leq H^*(\mathbf{W}) \\ \text{the solution of:} & \\ w_i f(z) = H^*(\mathbf{W}) & \text{for } w_i > H^*(\mathbf{W}) \end{cases}$$

The function  $z_i$  determines a unique symmetric equilibrium strategy  $\mathbf{q}^*(\mathbf{W})$  for white workers.

The situation for black workers is more complicated. As with whites, they will not apply to firms that offer wages that are too low. Moreover, given the discriminatory hiring strategy of firms, blacks will not apply to a firm that sets its wage offer too high, because high wages induce whites to apply with a high probability. Let  $y$  denote the expected number of black applicants to a designated firm. Black applicants will be hired only if no white applicants apply, an event that occurs with probability  $e^{-z}$ . However, given that no white applicants are present, the situation of blacks is parallel to that of the whites. Therefore, the probability that an additional black applicant would be hired is given by

$$(17) \quad g(y, z) \equiv e^{-z} f(y),$$

so that his expected income would be  $wg(y, z)$ .

Suppose now that the worker-application subgame has an equilibrium strategy profile in which all black workers adopt the same mixed strategy (we already know that whites adopt  $\mathbf{q}^*(\mathbf{W})$ ). Set  $J^*(\mathbf{W}) \equiv \max_i \{w_i g(y_i, z_i)\}$ , the maximum expected income available to blacks in that equilibrium. Note that  $J^*(\mathbf{W})$  must be less than  $H^*(\mathbf{W})$ , the maximum expected income available to whites, because an additional white applicant always has a better chance of being hired than an additional black applicant does. Blacks will apply to firms with positive probability if and only if they

can attain the maximum expected income  $J^*(\mathbf{W})$  there. This is not possible for  $w_i \leq J^*(\mathbf{W})$ . Furthermore, for wage offers beyond a certain threshold, denoted here by  $\hat{w}(\mathbf{W})$ , the expected number of white applicants will be sufficiently high to force the expected wage for blacks below  $J^*(\mathbf{W})$  again. The expected number of black applicants will be positive for wage offers between these two limits and exactly sufficient to equalize expected incomes at  $J^*(\mathbf{W})$ . More formally, we can write that  $y_i$  is the continuous function of  $w_i$  defined by

$$(18) \quad y_i = \begin{cases} 0 & \text{for } w_i \leq J^*(\mathbf{W}) \text{ or } w_i \geq \hat{w}(\mathbf{W}) \\ \text{the solution of:} & \text{for } J^*(\mathbf{W}) < w_i < \hat{w}(\mathbf{W}) \\ w_i g(y, z_i) = J^*(\mathbf{W}) & \end{cases} .$$

For any wage-profile  $\mathbf{W}$ , there are mixed strategies  $q^*(\mathbf{W})$  and  $s^*(\mathbf{W})$  for white and black workers that form a unique symmetric equilibrium of the job-application subgame. (The argument is akin to that made in the previous section.) Consequently, the equilibrium expected incomes,  $H^*(\mathbf{W})$  for white workers and  $J^*(\mathbf{W})$  for black workers, depend only on  $\mathbf{W}$ , are the same across all firms that receive applications from the respective types, and satisfy  $J^*(\mathbf{W}) < H^*(\mathbf{W})$ .

As in the case of the nondiscriminatory game, the equilibrium of the job-application game is an extension of the Harris-Todaro model. Workers of each type distribute themselves so that the expected income is the same at all jobs to which they apply. Low-wage jobs receive no applicants. Jobs that are very likely to attract white applicants do not attract black applicants.

**The Firms' Equilibrium Strategy in a Discriminatory Setting:** We now search for a subgame-perfect competitive equilibrium of the three-stage game. As before, in equilibrium, all firms will offer wages that have a positive probability of attracting applicants. Therefore, in the relevant range, a firm's expected operating profits are given by

$$(19) \quad \pi_i = (1 - e^{-z_i})(v - w_i) + e^{-z_i}(1 - e^{-y_i})((1 - \delta)v - w_i),$$

where  $z_i$  now represents the expected number of white applicants and  $y_i$ , the expected number of

black applicants.

We proceed to eliminate several categories of possible equilibria. First, we point out that in equilibrium, there are no firms that attract both white and black applicants. If a firm offered a wage that attracted both white and black workers, and gradually lowered that wage, the expected number of white applicants would fall, but the expected number of black applicants would rise at an even faster rate. The proof of that statement begins with the parts of (16) and (18) that define positive values of  $z_i$  and  $y_i$ ; namely,

$$(20) \quad w_i f(z_i) = H^*(\mathbf{W})$$

and

$$(21) \quad w_i g(y_i, z_i) = J^*(\mathbf{W}).$$

Then, holding  $H^*(\mathbf{W})$  and  $J^*(\mathbf{W})$  constant, as the equilibrium concept requires, the equations are differentiated implicitly with respect to  $w_i$ , and the sum  $dz_i/dw_i + dy_i/dw_i$  is shown to be negative. Because blacks and white have almost the same productivity, firms that are attracting both black and white applicants gain in two ways by lowering wages: their probability of having a job vacancy falls and their expected labor costs fall as well. This leads to the following proposition:

**Proposition 2** *In any subgame-perfect competitive equilibrium, some firms will offer wages that attract only white applicants and the remaining firms will offer wages that attract only black applicants.*

Consequently, like earlier taste-based discrimination models, our model implies complete racial segregation. This is true even for  $\delta = 0$ , provided only that when productivities are the same, employers choose to hire whites in preference to blacks.

In what follows, we shall refer to firms that attract only white applicants as “white firms” and firms that attract only black applicants as “black firms.” Note however that aside from their choice of wage offers, these firms are identical in every way.

We now derive the characteristics of the labor-market equilibrium. Let  $N_z$  and  $N_y$  be the numbers of white and black firms, with  $N_z + N_y = N$ , the total number of firms. Let  $r_z \equiv Z/N_z$  and  $r_y \equiv Y/N_y$  denote the mean number of applicants to firms in each category. The following propositions present closed-form solutions for equilibrium wage-offers, expected incomes and profits at white and black firms as functions of  $r_z$  and  $r_y$ , which are themselves endogenous variables.

**Proposition 3** *Let  $\mathbf{W}^*$  be an equilibrium wage-offer profile, and suppose that  $w_k^*$  is an element of  $\mathbf{W}^*$  that attracts only white applicants. Then, in equilibrium we have*

$$(22) \quad w_k^* = \frac{vr_z}{e^{r_z} - 1}.$$

*The expected income  $H^*(\mathbf{W}^*)$  of white workers is*

$$(23) \quad H^*(\mathbf{W}^*) = ve^{-r_z},$$

*and the operating profits  $\pi_k^*$  for white firms are*

$$(24) \quad \pi_k^* = [1 - (1 + r_z)e^{-r_z}]v.$$

**Proposition 4** *Let  $\mathbf{W}^*$  be an equilibrium wage-offer profile, and suppose that  $w_j^*$  is an element of  $\mathbf{W}^*$  that attracts only black applicants. Then, for  $\delta$  sufficiently small, we have in equilibrium*

$$(25) \quad w_j^* = H^*(\mathbf{W}^*).$$

*The expected income  $J^*(\mathbf{W}^*)$  of black workers is*

$$(26) \quad J^*(\mathbf{W}^*) = \frac{1 - e^{-r_y}}{r_y} H^*(\mathbf{W}^*),$$

*and the operating profits  $\pi_j^*$  of black firms are*

$$(27) \quad \pi_j^* = (1 - e^{-r_y})[(1 - \delta)v - H^*(\mathbf{W}^*)].$$

*Black workers are strictly worse off than white workers.*

The proof of Proposition 3 (white workers) is identical to that of Proposition 1, which describes the equilibrium for the nondiscriminatory case. The proof of Proposition 4 requires added steps. Equation (25) follows from the fact that whites will not apply to a firm that offers a wage of

$H^*(\mathbf{W}^*)$  or less, but will apply, with positive probability, to a firm that offers anything more than  $H^*(\mathbf{W}^*)$ . If  $w_j^*$  were strictly less than  $H^*(\mathbf{W}^*)$ , then the equilibrium wage offer to blacks would be unconstrained by the existence of whites. This means that for a small enough  $\delta$ ,  $w_j^* < H^*(\mathbf{W}^*)$  would have to be sufficiently close to the white wage  $w_i^* > H^*(\mathbf{W}^*)$  so as to belong to the interval  $[H^*(\mathbf{W}^*), w_i^*]$ , a contradiction. But if  $w_j^*$  were greater than  $H^*(\mathbf{W}^*)$ , whites would apply to black firms, a violation of Proposition (2). Equation (25) follows. Because all firms with black applicants offer the same wage, the expected number of black applicants must be  $r_y \equiv Y/N_y$ , the ratio of black workers to black firms. The probability that a black firm will have an applicant is  $1 - e^{-r_y}$ , and (25) implies (27).

Having established Propositions 3 and 4, it remains to characterize the equilibrium values of  $r_z$  and  $r_y$ . Let  $\bar{r} \equiv \frac{Z+Y}{N}$  denote the ratio of the expected total number workers to firms, and let  $\alpha \equiv Y/(Z + Y)$  denote the ratio of the expected number of black workers to the expected total number of workers. Unlike  $r_z$  and  $r_y$ , both  $\bar{r}$  and  $\alpha$  are parameters of the model. We have:

**Proposition 5** *Exactly one pair of values of  $r_z$  and  $r_y$  is consistent with a subgame-perfect competitive equilibrium of the discriminatory game. Those values, to be denoted by  $r_z^*$  and  $r_y^*$ , are defined by solution for  $r_z$  and  $r_y$  of the equations*

$$(28) \quad r_y = \frac{\alpha \bar{r} r_z}{(1 + \alpha)r_z - \bar{r}},$$

and

$$(29) \quad r_y = \ln \frac{1 - \delta - e^{-r_z}}{e^{-r_z} r_z - \delta}.$$

Both  $r_z^*$  and  $r_y^*$  are increasing in  $\bar{r}$  and  $\alpha$ . In addition,  $r_y^* < \bar{r} < r_z^*$ .<sup>4</sup>

Equation (28) is derived directly from the definitions  $r_z, r_y$  and  $\bar{r}$ . Equation (29) is derived from the equilibrium requirement that black and white firms have equal profits. Demonstrations of the remaining assertions are straightforward technical exercises.



We have thus established that an equilibrium must take the following form: Some firms offer high wages and attract only white applicants. Other firms offer low wages and attract only black applicants. In equilibrium, firms offering the low wage must make the same expected profit as firms offering the high wage, so that the vacancy rate must be higher at low-wage firms ( $r_y < r_z$ ). Conversely, blacks must have a lower unemployment rate than whites, a counterfactual implication to which we return in the extensions section. Despite their lower rate of unemployment, blacks are worse off than whites in this model. The proposition also demonstrates the seemingly paradoxical fact that as the proportion  $\alpha$  of blacks is parametrically increased, the ratio of workers to firms within each worker group increases, even though the overall ratio of workers to firms is held constant. This will have interesting consequences, which are discussed below.

We sum up the results of this section as follows:

**Proposition 6** *Among strategy profiles in which all workers of the same type adopt the same strategy,  $r_z^*$  and  $r_y^*$ ,  $w_k^*$  and  $w_j^*$  and  $\mathbf{q}^*(\cdot)$  and  $s^*(\cdot)$  describe a unique subgame-perfect competitive equilibrium of the discriminatory wage-posting game.*

**The Effect of Discrimination on Wages, Profits and Output:** Although we have established that in the discriminatory equilibrium black workers are worse off than white workers, we have not determined who, if anyone, benefits from the discrimination and who suffers from it. With the nondiscriminatory equilibrium as a benchmark, it can now be seen that discrimination lowers the wages of all workers, reduces national income and increases profits.<sup>5</sup>

For a fixed number of firms and small  $\delta$ , output is decreasing in the total number of job vacancies. But as the number  $r$  of workers per firm increases, the probability  $1 - e^{-r}$  that a firm has a job vacancy decreases at a decreasing rate. Because of this, expected total vacancies are minimized when the proportion of workers to firms is equated across worker types. Inasmuch as

$r_z^* \neq r_y^*$  in the discriminatory equilibrium, this implies:

**Proposition 7** *With  $\delta$  sufficiently small, output in the nondiscriminatory equilibrium is strictly greater than output in the discriminatory equilibrium.*

Furthermore, in contrast with most discrimination models, we have

**Proposition 8** *With  $\delta$  sufficiently small, the wages and expected incomes of both white and black workers are less, and operating profits are more, in the discriminatory equilibrium than in the nondiscriminatory equilibrium.*

This proposition follows from the definition of these quantities for white workers in equations (11), (12), and (13) and in (22), (23), and (24) and from the fact that in the discriminatory equilibrium black workers have a lower wage and expected income than do whites.

It is not surprising that discrimination hurts black workers, but it may be less clear why it also affects white workers adversely. The reason is that by lowering wages in the black sector, discrimination increases the profitability of hiring blacks. This, in turn, induces more firms to set wages that attract only black workers, which reduces the demand for white workers and thus their wages. These results are similar to a recurring theme in Marxist labor economics—that capitalists use various devices to create false distinctions and disunity among workers, which reduces their power and lowers all their wages.

### III. Extensions and Empirical Relevance

Of necessity the model is highly stylized. Some of our assumptions may seem very strong, and the reader may appropriately question whether our results are robust to changes in model characteristics. In this section we briefly address extensions that generalize and add realism to the

model. We then build on these extensions to show that it is possible both to fit the model to a set of stylized facts regarding exit hazards from unemployment and to generate significant wage differentials numerically.

**Free-Entry Equilibrium:** We add to our model a preliminary stage in which firms decide whether or not to enter the market. If firm  $i$  enters, it must pay an entry cost (or fixed cost) in the amount  $c_i$ . The  $\bar{n}$  potential entrants are ordered by their entry costs, so that  $c_1 < c_2 < \dots < c_{\bar{n}}$  with some but not all of these entry costs less than productivity  $v$ . Expected profits for a firm in business are defined as the operating profits less entry costs; expected profits for firms not in business are zero.

As we demonstrated in the previous section, there is a unique equilibrium associated with any fixed number of entrants  $n$ , so that entry will continue until a marginal entrant cannot earn positive expected profits. The free-entry game has a unique equilibrium with symmetric strategies among workers of a given type. Relative to the nondiscriminatory free-entry equilibrium, the discriminatory free-entry equilibrium has the following properties: aside from the marginal entrant, every firm makes greater profit; all white workers have lower wages and lower expected incomes; all black workers have lower wages and expected incomes, and net output (output less entry costs) is lower. In the discriminatory equilibrium, blacks have lower expected wages than do whites while no such difference can arise in the equilibrium without discrimination.

Our analysis, with appropriate modifications, would also apply if entering firms were required to purchase capital and capital were sold in a market with an upward-sloping supply. Switching from a nondiscriminatory regime to a discriminatory one would yield a capital gain to the holders of capital. Only if marginal firms were identical in their costs and there were an infinitely elastic supply of capital would profits be unaffected by moving from a nondiscriminatory regime.

**Match-Specific Productivity:** In keeping with the model so far, we assume that white and black workers may differ but that within each race workers are *ex ante* homogeneous. However, after workers apply for a job but before the firm decides which worker to hire, an observable match-specific component of productivity is revealed, independently drawn from the random variable  $\varepsilon_H$  for white workers and  $\varepsilon_J$  for black workers, where  $E[\varepsilon_H] = E[\varepsilon_J] = 0$ . Thus net worker productivity is given by

$$(30) \quad \tilde{v} = \begin{cases} v + \varepsilon_H & \text{for white workers} \\ v(1 - \delta) + \varepsilon_J & \text{for black workers} \end{cases} .$$

Because of the match-specific component of productivity, some blacks may be seen to be more productive than some whites at a given job. Therefore, even if some whites apply for a job, a black worker might turn out to be the most productive applicant and be hired.

Without match-specific productivity, our model yields a discriminatory equilibrium even when productivity for blacks and whites is the same. But if match-specific productivity is incorporated into the model, a discriminatory equilibrium cannot be supported when the productivity distributions satisfy all of the following conditions:<sup>6</sup>

- i. whites and blacks have the same average level of productivity ( $\delta = 0$ );
- ii. the distributions of match-specific productivity are without mass points; and
- iii. match-specific productivity is distributed identically for the two groups.

However, when any of these conditions is dropped, the discriminatory equilibrium re-emerges for some distributions of match-specific productivity.

Suppose we relax the first condition, so that  $\delta > 0$ . The necessary conditions for a firm to have a positive number of both white and black applicants in the equilibrium of the worker-application subgame are similar to (20) and (21), but now  $f$  is decreasing rather than constant in  $y_i$ , because

the probability that any given black will be hired in preference to a white is positive. For the same reason,  $g$  must now decrease more slowly in  $z_i$ . Because the match-specific productivity of workers varies, the expected productivity of the firm's best applicant will be increasing in both  $z_i$  and  $y_i$  and the firm's profit function must be generalized to take this into account.

If the variation of  $\varepsilon_H$  and  $\varepsilon_J$  is not too large compared with  $\delta v$ , the probability that a black will be hired in preference to a white, if positive, will be small. When it is small enough, the derivatives of the modified functions  $f$  and  $g$  will be sufficiently close to those of the original that the the proof of Proposition 2 remains valid. This leads to the conclusion that a discriminatory equilibrium can be supported under these circumstances, even when the probability that a black will be hired in preference to a white is positive.

If  $\delta = 0$  and blacks and white have the same distributions of match-specific productivities, one would expect blacks to be hired in preference to whites a substantial fraction of the time. But even then, if we drop the second condition and allow the productivity distributions of blacks and whites to have mass points at the same productivity levels, a discriminatory equilibrium may be supported. The mass points create a nonzero probability of a productivity tie between a black and a white applicant. If firms choose whites over blacks of equal productivity, then the probability of being hired will be lower in the presence of white applicants than of black applicants and this adverse effect will be greater for blacks. Therefore blacks will avoid firms that attract whites and the segregation result will go through, although the wage differential will be smaller than in the absence of match-specific productivity.

The obvious example is a case with just two points in the distribution each with probability .5. If two blacks or two whites apply, each has a probability of .5 of being selected. If a white and a black apply, the white is chosen with probability .75. Replacing a white co-applicant with a

black co-applicant raises a white's probability of employment by fifty percent but doubles a black's probability of employment. Therefore a black would be willing to give up fifty percent of the wage in order to move from a firm with one white applicant to one with one black applicant (besides himself), but a white would give up only one-third of his wage.

To see what happens when the third condition is relaxed so that  $\varepsilon_H$  and  $\varepsilon_J$  may have different distributions, consider the extreme case where employers think "all blacks look alike" but can distinguish between whites who are good and bad matches (each with probability .5). If there are three black applicants, each will have a one-third chance of being chosen. If there are two white applicants and one black applicant, the black will be chosen only if both whites are bad matches which happens with probability one-fourth. Thus blacks prefer competing with other blacks. The authors have constructed an example of a segregated equilibrium along these lines in which blacks have lower wages even though, on average, blacks are more productive than are whites.

Espen R. Moen (2003) also generates segregation in a directed search model with match-specific productivity. However, in contrast with our model, the presence of the low types drives up the wages of the high types, making them inefficiently high. The critical difference between our two models does not lie in the effect of match-specific productivity. Unlike us, Moen considers large firms that are committed to hire all applicants at their posted wage, provided only that the applicants surpass a specified productivity threshold. This means that at the time they apply for jobs, Moen's black and white workers are not directly competing with one another. It is the direct competition, which, we think reflects a real-world phenomenon, that drives the model in this paper. Moen's segregation result is striking, but it derives from other considerations.

**Valuable Unemployment:** If unemployment is valued at  $u$ , then labor market equilibrium requires workers to set expected income plus expected unemployment value to its maximum ( $H$  for

whites and  $J$  for blacks) at every firm to which they apply. Profits in this case are given by the expression

$$(31) \quad \pi = (1 - e^{-z})(v - w) + e^{-z}(1 - e^{-y})(v(1 - \delta) - w),$$

which is maximized with respect to  $w, z$ , and  $y$ , subject to the workers' equilibrium conditions:

$$(32) \quad (w - u) \frac{1 - e^{-z}}{z} = H - u$$

when  $z > 0$  and

$$(33) \quad (w - u) e^{-z} \frac{1 - e^{-y}}{y} = J - u$$

when  $y > 0$ . With an appropriate change of variables this problem becomes mathematically isomorphic to (14) and our proofs go through.

**Multiple Periods:** If we extend the model to multiple periods but prohibit on-the-job search, very little of substance changes. The model becomes similar (but not identical) to the model in which unemployment is valuable. It is relatively straightforward to show that the equilibrium must involve separation. The more important question is whether sizable wage differentials can persist when unsuccessful searchers can apply elsewhere in the next period. We believe the answer to this question to be “yes.” However, the counterfactual prediction that blacks should experience less unemployment would persist. Therefore, we turn to a somewhat more realistic example that allows simultaneously for multiple periods and heterogeneity in workers' discount rates within both black and white groups.

**A Numerical Exercise:**<sup>7</sup> If workers are homogeneous within race, the model implies that blacks exit unemployment more rapidly than do comparable whites. This is inconsistent with the data. Van den Berg and van Ours (1996) estimate separate unemployment duration models for black men, black women, white men and white women. Their approach allows them to estimate an underlying baseline hazard for escaping unemployment for each group and to measure the extent of heterogeneity around that average. Unfortunately, it does not allow us to ascertain to what

extent either the mean differences or the differences in heterogeneity are attributable to known characteristics. They find that blacks exhibit considerably more heterogeneity in their probability of exit from unemployment than do whites. Combining the differences in heterogeneity and exit hazards suggests that relative to whites there is a substantial group of blacks with high exit hazards and another with low exit hazards. We will show that such an outcome is consistent with our model provided that we introduce some heterogeneity into the personal characteristics of workers.

The high exit rate of blacks from unemployment in our model reflects the fact that they apply to low-wage jobs and never compete directly with whites. If, for some reason, some blacks chose to apply to jobs for which whites also apply, then blacks applying to these jobs would tend to have a relatively low exit hazard and the whites a relatively high one. We generate this type of behavior in the framework of our model by introducing heterogeneity in the rate at which workers discount future income (but keeping the distribution of discount rates the same for both types). Whites with high discount rates ought to accept relatively low wages in return for relative certainty of employment, while blacks with low discount rates ought to accept a relatively high risk of continued unemployment in return for the possibility of a high future wage. Therefore, we postulated that examples could be constructed in which low-discount-rate blacks and high-discount-rate whites would apply for the same jobs. The following numerical example confirms this intuition.

Suppose that there are patient workers with  $\rho = .015$  and impatient workers with  $\rho = .09$ . The proportion of patient workers is the same for blacks and whites. We normalize the lifetime value of output ( $v$ ) to 1 and set the flow cost of a vacancy to .25. We assume that the number of whites with  $\rho = .09$  is sufficiently high relative to the number of blacks with  $\rho = .015$  to be consistent with the requirements of our equilibrium. With these parameters, the equilibrium has four wages: a high wage to which only patient whites apply; a somewhat lower wage to which only some impatient whites apply; a yet lower wage to which some impatient whites and all patient blacks apply and



a low wage equal to the expected wage of impatient white workers to which only impatient black workers apply.

With these parameters, impatient black workers have the fastest exit rate from unemployment and patient black workers have the slowest. If 10 percent of workers are black and 20 percent of each group is patient, then, on average, the black-white wage differential is approximately 8 percent, a reasonable value for the skill-adjusted wage differential. Moreover, assuming that all groups enter unemployment at the same rate, the steady-state unemployment rate among blacks is approximately 20 percent higher than among whites, a plausible number for a skill-adjusted unemployment differential.

This exercise was not designed to calibrate the model, which is too stylized to justify such an attempt. However, the numerical example suggests that the model is capable of generating empirically significant wage and unemployment differentials that are consistent with empirical regularities.

## **IV. Discussion**

The most general game we modeled in this paper has four stages: firm entry, wage posting, worker applications, and hiring. Once the hiring stage is reached, firms will be almost indifferent to the race of the workers they hire—their hiring choices have at most a negligible effect on profits. Yet the hiring choice of firms, however capriciously it may be made, is the tail that wags the dog. If firms discriminate against blacks in the hiring stage (or are expected to do so), the discriminatory equilibrium described above will prevail. If firms choose workers without regard to race (or are expected to do so), then the nondiscriminatory equilibrium will prevail. We believe that the first is the more natural equilibrium for a number of reasons.

First, if firms have even a very slight preference for white over black workers, they will use the discriminatory strategy. In particular, if firms maximize profits and prefer white to black workers

given equal profits, then in the case of equal productivities only the discriminatory equilibrium remains. Although we do not wish to suggest that most employers are racists, anecdotal evidence suggests that many employers have at least very mild discriminatory preferences.

Second, the discriminatory equilibrium yields higher profits to every firm than does the nondiscriminatory equilibrium. If firms as a group create the expectation that they will discriminate in the hiring stage, they stand to make more money. Moreover, they would lose nothing by fulfilling such expectations. An ethos of discrimination among firm owners would be consistent with their economic interests.

Finally, the discriminatory equilibrium outcome does not require that firms would actually use discriminatory strategies, but only that black workers believe that they do. A belief in discrimination is sufficient to induce blacks to apply to low-wage firms where they would not be in competition with whites. Some firms would then choose a low-wage strategy designed to attract blacks just as in the equilibrium in which the strategies of firms entail discrimination. Since blacks do not apply to high-wage firms, their beliefs are not contradicted in equilibrium. This constitutes what Drew Fudenberg and David K. Levine (1993) term a self-confirming equilibrium.

Widespread and ongoing litigation over employment discrimination suggests that a substantial number of people believe that some firms discriminate. Even public enforcement of antidiscrimination laws has proved insufficient to dispel the belief that many so-called Equal Opportunity Employers do not live up to their announced policy. In the context of our model, we have assumed that it is impossible for firms to make credible promises not to discriminate. However, at the cost of some simplicity, we could substitute the much weaker assumption that in the eyes of potential employees, a firm's announced policy of nondiscrimination reduces the probability of discrimination but does not drive it to zero. This would be sufficient to obtain labor market segregation, and the

broad outlines of our results would continue to hold: jobs would be segregated and firms attracting black workers would offer lower wages. All firms seeking to attract blacks would announce a policy of nondiscrimination.

Consequently, we believe that the equilibrium in which firms discriminate, or at least blacks believe that they do, is the natural equilibrium.<sup>8</sup> That being so, our model provides useful insights on the role of antidiscrimination policy. Such policies can be justified on pure efficiency grounds, but the distributional impacts are also significant. Although effective antidiscrimination measures would have the greatest impact on black workers, they would increase the incomes of white workers as well. Only owners of capital would be affected adversely.

The model also highlights the importance of targeting employment discrimination rather than wage discrimination alone. No firm practices wage discrimination in our model, but employment discrimination results in significantly lower wages for black workers anyway. Moreover, a significant discriminatory outcome can result from even very mild preferences for white workers. Employers do not have to be overt racists to create a situation collectively in which blacks are dramatically disadvantaged. The required preference for discrimination is sufficiently minimal that employers may not even be aware of it. This constitutes a justification for affirmative action designed to offset even mild discriminatory tendencies. The costs of combating such tendencies need not be high.

## REFERENCES

- Fudenberg, Drew, and Levine, David K., "Self-Confirming Equilibrium," *Econometrica*, May 1993, 61(3), pp. 523-45.
- Higgs, Robert, "Black Progress and the Persistence of Racial Economic Inequalities, 1865-1940," in Steven Shulman and William Darity, Jr., eds., *The Question of Discrimination: Racial Inequality in the U.S. Labor Market*, Middletown, CT: Wesleyan University Press, 1989, pp. 9-31.
- Lang, Kevin, "Persistent Wage Dispersion and Involuntary Unemployment." *Quarterly Journal of Economics*, February 1991, 106(1), pp. 181-202.
- Lang, Kevin, Manove, Michael and Dickens, William T., "Racial Discrimination in Labor Markets with Posted Wage Offers," manuscript, Boston University, 2003, <http://www.bu.edu/econ/faculty/manove/DiscrimOnlineVersion.pdf>
- Montgomery, James D., "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics*, February 1991, 106(1), pp. 163-79.
- Moen, Espen R., "Competitive Search Equilibrium," *Journal of Political Economy*, April 1997, 105(2), pp. 385-411.
- \_\_\_\_\_, "Do Good Workers Hurt the Bad Workers or Is It the Other Way Around," *International Economic Review*, May 2003, 44(2), pp. 779-800.
- Shi, Shouyong, "Wage Differentials, Discrimination and Efficiency," University of Toronto, manuscript, 2004.
- Shimer, Robert, "The Assignment of Workers to Jobs In an Economy with Coordination Frictions," National Bureau of Economic Research (Cambridge, MA) Working Paper No. 8501, October 2001.

## Footnotes

\*Lang: Department of Economics, Boston University, 270 Bay State Rd, Boston MA 02215 (email: lang@bu.edu); Manove: Department of Economics, Boston University, 270 Bay State Rd, Boston MA 02215 (email: manove@bu.edu); Dickens: The Brookings Institution, 1775 Massachusetts Ave NW, Washington DC 20036 (email: wdickens@brook.edu). The authors acknowledge and appreciate funding under NSF grants SES-0339149 (Lang and Manove), SBR-9709250 (Dickens) and SBR-9515052 (Lang). We are grateful to Olivier Blanchard, Sandy Darity, Lawrence Katz, Albert Ma, Douglas Orr, Michael Peters, James Rebitzer, Rafael Repullo, Robert Rosenthal, Andrew Weiss and two anonymous referees for helpful comments on this paper.

<sup>1</sup>From the point of view of the designated worker,  $z$  would represent the expected number of applicants to the firm *aside from himself*. The intuition is that there are very large number of potential workers who may apply to firm  $i$ , each with a very small probability. The *ex ante* probability that a designated worker will apply is so small that, *ex post*, his decision to apply boosts the expected value of the total number of applications by an amount close to 1. Thus, in the limit, the expected number of applicants to a firm from the point of view of a worker who has decided to apply is greater by one than the same expectation formed by others. The same logic applies to entry of workers into the labor market.

<sup>2</sup>This is a reasonable assumption for firms to make if  $N$ , the number of firms, and  $Z$ , the expected number of workers in the job-market, are both large. Just as competitive suppliers are assumed to ignore the effect of their own actions on market price, so our firms are assumed to ignore the effect of their own wage movements on the market-wide expected income of workers. In formal games, competitive equilibrium is usually modeled with a continuum of agents, but for this case, our refinement yields a far simpler model.

<sup>3</sup>Robert Shimer (2001) and Shouyong Shi (2004) consider the case of race-contingent wage, ruled out in this model. Surprisingly, with race-contingent wages, if the productivity difference between the two groups is not too large, the equilibrium involves both groups applying for the same jobs but the *less* productive workers being offered *higher* wages conditional on being hired. Of course, since they are both less costly and more productive, the more productive workers are always hired in preference to the other workers. The counterfactual prediction that blacks would receive higher wage offers than whites seems to us to be very problematic, and is an additional reason that we do not pursue this route.

<sup>4</sup>As is common in the literature, we are ignoring the integer constraint on  $N_z$  and  $N_y$ , which are assumed to be large numbers. Constraining  $N_z$  and  $N_y$  to be integers would change the equilibrium in only minor ways.

<sup>5</sup>In a wage-posting model with a different structure from our own, Espen R. Moen (1997) proves that in his context wage-posting leads to an efficient outcome when all workers of the same productivity apply to jobs that offer the same wages. Moen's efficiency condition is violated in our model, and we obtain the results his model would suggest.

<sup>6</sup>The authors would like to thank an anonymous referee for pointing this out.

<sup>7</sup>Details of the calculations are available from the authors upon request.

<sup>8</sup>Note that separating equilibria with segregation can be derived for directed search models in a wide range of settings. See Moen (2003) for a model rather different from our own that nevertheless yields a separating equilibrium.