Chapter 9: Options in banking and financial markets

As we have seen, many traditional bank assets and liabilities are very much like bonds. In this chapter you will learn how many bank assets and liabilities combine bond-like elements with important option-like components. In recent years, the banking business has increasingly focused on option-like aspects of its assets and liabilities. This shift in focus has occurred for two reasons. First, because many traditional banking products contain option components that are valuable to the customer and costly to the bank, increasing competition in banking has forced banks to price their products more carefully. Second, there has been an explosion of new bank products that contain option-like components. Options are everywhere in modern banking, and they will become even more important in banking in the years ahead as banks seek to construct attractive new products in an increasingly competitive financial marketplace.

9-1 Options are everywhere

An **option** is simply a right to undertake a specific activity--usually, to buy or sell a financial asset. An important feature shared by all options is that their payouts depend on the value of an other asset, called the underlying security. Because the value of an option depends on the value of an underlying security, options are derivative securities. Options are unique among financial instruments in that they confer rights on their owner without any corresponding obligation. Ownership of an option does not guarantee that the option holder will receive any cash flows --this makes options different from assets with specified payout schedules, such as bonds. Another important feature of options is that they typically have **expiration dates**: there is a limit on the amount of time an option-owner has to use, or **exercise**, her option.

One well-known example of an option is a stock option: this option confers on its owner the right to buy or sell the underlying stock at a specified price on or before the expiration date. In this chapter we focus mainly on stock options, but the concepts we develop can be applied directly to options on other securities. We will learn how options are priced and how the value of the option depends on the characteristics of the underlying security. However, before going on it is useful to think about the various kinds of options encountered in everyday life.

Options in everyday life

What do the following things have in common: a pair of tickets to a concert; frequent flier miles; a bottle of Old Rotgut whiskey? The answer is: they are all options--options that occur in everyday life. Let's see why. Concert tickets confer on their owner the right to attend the concert. However, a ticketholder is not *obligated* to attend the concert. If a more interesting opportunity arises, the ticketholder can either sell the tickets or simply choose not to attend the concert. This is the essential characteristic of an option--a right without a corresponding obligation. Of course a right without any obligation is a valuable thing--this is why we usually must pay for concert tickets, and this is why financial options are not usually given away.

Frequent flier miles are options too. After the frequent flier has flown a specified number of miles on a particular airline she receives the right to fly free on the airline in question (subject to many restrictions, however). There are restrictions on the frequent flier's ability to transfer this right--the airlines generally allow one to transfer the free ticket to a family member, but prohibit transferring it to anyone else. The frequent flier miles are nevertheless options--the frequent flier can use the free ticket if she wants, or she can decide not to use it.

What about the bottle of Old Rotgut? Suppose your Uncle Frank really likes to drink Old Rotgut whiskey, but you cannot stand the taste of the stuff. Uncle Frank is planning to visit you on the weekend if it's too rainy for him to go golfing. Knowing this, you purchase a bottle of Old Rotgut: you will serve the whiskey to your uncle if he visits. If he goes golfing instead, you will either give the Old Rotgut away or just let it sit around in the back of a cupboard. In any case, you have no intention of actually *drinking* it! Once again, you have purchased an option--the right to do something without the obligation to do anything at all.

Options in banking

Options are everywhere in banking too. They occur frequently in loan contracts between banks and their customers. Because options are valuable, it's important for banks to recognize the options embedded in their loan contracts and to make sure that they charge appropriate prices to their customers for these embedded options.

One type of option that is very common in loan contracts is the **prepayment option**.

Just as the name indicates, this option allows the borrower to repay the loan in advance of its stated maturity date. The right to prepay a loan is not a necessary part of the standard loan contract--the bank *can* write a loan contract prohibits prepayment. If the bank allows the customer to prepay the loan, then the bank has issued the customer a valuable option. A prepayment option is valuable to a borrower because it allows the him the flexibility to end the borrowing arrangement early if he wishes. For example, the owner of a small firm may have unexpectedly high sales in a particular period, generating unexpectedly high cash flow to his firm. If this happens, the firm's owner may wish to repay all or part of a loan. A good reason for doing this is that the interest rate that he can earn if he saves the unexpected cash (say, in a savings account) is probably lower than the interest rate on the loan.

When a house is sold, the mortgage-holder typically repays the mortgage. That is: the mortgage-holder exercises the prepayment option that is included with standard mortgage contracts. Another example concerning the mortgage market is refinancing: when interest rates fall, individuals can benefit from repaying their high-interest-rate mortgages (exercising the prepayment option) and obtaining new mortgages with the prevailing, lower interest rate.

Because the prepayment option is valuable, a bank will not give it away. The bank will charge the customer for the option, either as a higher initial interest rate, or as a direct fee, or with a specified penalty that will be assessed should the customer choose to prepay.

Another option that is important in banking is the **default option**. An individual or firm may take out a loan in good faith, with every intention of repaying the loan, but economic circumstances may make it impossible to repay the loan. If this occurs, the individual or firm is said to **default** on the loan. New businesses are at high risk of default--cash flows to new firms are highly variable, and more than half of the new businesses incorporated each year fail within five years. Limited liability laws make default an attractive option for owners of these businesses.

An individual who takes out a loan to purchase real estate may find that the value of the real estate falls to the point that the market value of the property is substantially less than the amount owed to the bank. In such a situation the individual may choose to default on the loan and allow the bank assume ownership of the property. This individual has exercised the default option implicit in the loan contract. This is what is currently happening in Japan--on a nationwide basis!

These examples show that bank loans--the largest component of bank assets--commonly have options built into them. This has been true as long as there have been banks; the very nature of the bank-customer relationship means that an important economic function of banks is to sell options to their customers.

On the liability side of the bank's balance sheet, which is dominated by deposits, the most important options are those involving early withdrawal of those deposits. Demand deposits are, of course, available "on demand." Other types of deposit accounts--CDS, for example--have specified minimum periods of time that the funds must remain with the bank in order for the depositor to receive the stated interest rate for that type of deposit. The depositor may withdraw her funds in advance of this minimum period of time--she may exercise an early-withdrawal option--but the bank typically charges her for this by charging an early withdrawal penalty.

Recently, some unusual options have begun to appear on the liability side of the bank balance sheet. In 1985, Chase Manhattan Bank offered a new type of deposit account for which the interest rate on the account was linked to the stock market index. The idea was that the depositor would receive a specified, minimum level of interest no matter what happened to the stock market, but that the interest rate on the account would rise if the stock market rose. This account implicitly incorporated an option on the stock market--the depositor could participate in the stock market if the stock market performed well, and could stay out of the stock market if it did poorly.

These are just a few of the ways in which options have become a central part of modern banking and financial markets. To understand more fully how options are used in these markets, we must learn the basic concepts and terminology of the options markets and learn how to determine the value of an option. Then, in Chapter 10, we will put these tools to use in understanding how options are used--and priced--in banking and in the fast-paced world of financial engineering.

9-2 Option basics

As we discussed earlier, *an option is a right*. An option does not obligate its owner to do anything. The owner of an option may exercise the option if he feels it is worthwhile to do so, or he may simply tear it up if the option is not useful to him.[1] Options can be written on many underlying assets: we'll focus initially on options on stocks, but there are also options on gold, other commodities, stock market indexes, foreign currencies, and government bonds.

There are two types of options: call options and put options. The most common type of option, a **call option**, confers on its owner the right to purchase, or "call away," the underlying asset from its owner. The second type of option is a **put option**, which gives its owner the right to sell, or "put," the underlying asset to another individual. An option contract must specify the following:

(1) whether the option is a call option or a put option;

(2) the underlying security upon which the option's value is based;

(3) the **exercise price** of the option (also called the **strike price**): the price at which the holder of a call option can purchase the underlying security, or the price at which the holder of a put option can sell the underlying security

(4) the expiration date of the option

(5) the number of units of the underlying security that may be purchased (a call option) or sold (a put option)

Now that we know the defining characteristics of options, let's look at how stock options are bought and sold.

How options are traded

Stock options are traded in two ways. There are organized options exchanges in Chicago, New York, London, and other major financial centers where the stocks of major corporations are traded--these are called "listed options" because they are listed on these exchanges. The largest options exchange--the Chicago Board Options Exchange (CBOE)--

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[1]While the option holder *may* tear up an option, it would be a much better idea to sell it! So long as an option has not expired, it still has value--this will become clearer as we go on.

has an annual dollar volume of business greater than that of the American Stock Exchange. The CBOE was created in 1973. This is a very new market, compared with the other major financial markets: the New York Stock Exchange was created in 1792, and the first organized futures exchange in the U.S. was incorporated about 1870.

The options exchanges standardize the terms of the option contract to guarantee a ready supply of buyers and sellers. That is: the options exchange specify the number of shares of stock covered by one option contract (i.e., the number of shares that can be purchased with a call or sold with a put); the exercise price of the option; and its exercise date. This leaves just the price of the contract, which is determined by the market.

The stocks of smaller corporations are traded in the "over-the-counter" market. This market has no centralized, physical location--it is a network of specialists and investment banks in the U.S. who buy and sell options among themselves and for their clients. The over-the-counter market also provides custom-made options for clients who want to purchase or sell options with characteristics different from those of the listed options--different exercise prices or expiration dates, for example. Before the CBOE opened in 1973, all options trading was carried out in the over-the-counter market.

There are two parties to an option contract. One individual is the **option writer**--he issues the option contract and receives a payment called the **option premium** for doing so. The other individual, the option purchaser or **option holder**, buys the option and thereby obtains the rights associated with the option contract. The option purchaser is said to have a **long position** in the option, while the option writer is said to have a **short position** in the option. An individual may establish either a short position or a long position in either a call option or a put option. For example, a long position in a call option means that the individual has purchased the right to purchase stock at the exercise price on or before the option's expiration date. A short position in a put option is the probably the most difficult position to understand. If person A establishes a short position in a put, person A sells to person B the right to sell person A the underlying stock at the option's exercise price.

Box 9-1 is a glossary of the terms and concepts used in discussing options markets. Not only are there many terms to use--there are often many different terms used to mean the

Box 9-1: THE LANGUAGE OF OPTIONS MARKETS

American option: an option that can be exercised any time before its expiration date

at the money: an option is "at the money" if its exercise price is equal to the price of the underlying security

call option: the right to purchase the underlying security

European option: an option that can only be exercised on its expiration date

exercise price (strike price)**:** the price at which the option can be exercised

expiration date: the last date that the option can be exercised

in-the-money: a call option is "in the money" if the exercise price is less than the price of the underlying security; a put option is "in the money" if the exercise price of the option is greater than the price of the underlying security

intrinsic value: the amount of money you could make by exercising the option immediately. It's never negative because you can choose not to exercise an out-of-the-money option.

listed option: an option traded on one of the organized options exchanges, such as the Chicago Board Options Exchange (CBOE)

long position: an individual with a long position in an option has purchased the option. A long position in a call entails the right to purchase the underlying security; a long position in a put entails the right to sell the underlying security.

option holder (option purchaser)**:** the individual who has purchased the option.

option premium: the price paid by the option purchaser (to the option writer) in exchange for the option

option writer (option seller)**:** the individual who has written, or sold, the option.

out-of-the-money option: a call option is out of-the-money if the exercise price is greater than the price of the underlying security; a put option is out-of-the-money if the exercise price is less than the price of the underlying security

over the counter market: a network of brokerage firms and other dealers who can create custom options for their clients. The premium, exercise price, and expiration date are negotiated between the two parties.

put option: the right to sell the underlying security

short position: An individual with a short position in an option has sold, or written, the option. A short position in a call entails the obligation to sell the underlying security to the option holder (option purchaser), at the exercise price, if requested to do so. A short position in a put option entails the obligation to purchase the underlying security, at the exercise price, from the option holder if requested to do so.

time value: the part of the option premium not accounted for by the option's intrinsic value.

same thing. In our discussion of options markets we will stick to one term for each concept--the one given in boldface in Box 9-1. However, you should be aware that all of these terms are in common use, and you may encounter any of them in discussing options with financial market practitioners.

Reading option quotes in the *Wall Street Journal*

Information on listed options is reported each business day in the *Wall Street Journal* and most other major newspapers. Figure 9-1 shows *Wall Street Journal* quotes for AT&T stock

FIGURE 9-1: *Wall Street Journal* option quotes

Friday, October 21, 1994							Monday, October 24, 1994						
Stock/ NY Close	Strike price	Exp. date	Calls vol.	Calls close	Puts vol.	Puts close	Stock/ NY Close	Strike price	Exp. date	Calls vol.	Calls close	Puts vol.	Puts close
AT&T	50	Oct	267	4¼	...	...	**AT&T**						
54½	50	Jan	227	5	314	⅝	54	50	Nov	60	4¼	24	3/16
54½	50	Apr	175	5⅞	467	1	54	50	Jan	49	4⅞	179	⅝
54½	55	Oct	2	1/16	422	⅝	54	55	Nov	730	9/16	90	1¼
54½	55	Nov	293	¾	279	1 3/16	54	55	Dec	227	1 1/16	103	2
54½	55	Jan	537	1¾	188	2	54	55	Jan	393	1½	...	...
54½	55	Apr	97	2⅝	42	3	54	55	Apr	69	2½	15	2⅝
54½	60	Jan	89	⅜	20	5½	54	60	Jan	178	¼	10	5¾
54½	60	Apr	94	15/16	...	...	54	60	Apr	240	13/16	...	...

options listed on the CBOE for two adjacent trading days: Friday, October 21, 1994, and Monday, October 24, 1994. Below the stock name, AT&T, is the price at which the stock closed on that day. On Friday, the stock closed at 54-1/2, or $54.50 per share. The columns to the right contain the option prices (option premiums) for the listed options on AT&T stock. The call options are listed first, followed by the put options. The exercise prices range from 50 to 60--the prices are quoted in dollars per share covered. For example, a call option with exercise price of 55 entails the right to purchase AT&T stock at a price of $55.00 per share at any time before the expiration date. Similarly, a call option with an exercise price of 60 gives its owner the right to buy AT&T stock at $60.00 per share. The exercise prices for listed put options are typically the same as the exercise prices for the call options, however, the put option involves the right to sell the underlying stock. Thus, a put option with an exercise price of 60 gives its owner the right to sell AT&T stock for $60.00 per share. Stock options are typically contracts for 100 shares of the underlying stock, but we will discuss the value of option positions on a per-share-covered basis.

The column to the right of the exercise prices gives the options' expiration dates followed by trading volume (in thousands of contracts) and option prices. For the options listed in Figure 9-1, the expiration dates range from October 1994 until April 1995. CBOE listed options expire on the Saturday following the third Friday of the month. When an option expires, a new option is listed that expires nine months later. The exercise price of the new option is set by the exchange and can be different from the exercise price of the expiring option. The options exchanges choose an exercise price that they believe will be attractive to participants in the options market--typically, this exercise price will be in the range of recent prices for the underlying stock.

A call option is said to be **in-the-money** if its exercise price is less than the stock's price: in our example, the call options with exercise price of 50 are in the money. If you found an in-the-money option lying on the street, or buried in your box of breakfast cereal, it could provide you with a quick windfall. You could exercise the option, buying the stock using the option for less than the stock's market price, and then immediately turn around and sell the stock in the open market for a higher price, earning a quick return for little work. For example, if you found the AT&T call option with an exercise price of 50 that had not yet expired, you could purchase the stock for $50 and sell it for the market price of $54.50. Your gain would be $54.50-$50.00 = $4.50, ignoring brokerage fees incurred in carrying out this transaction. The difference between the stock price and the exercise price for an in-the-money option has a special name--it's called the **intrinsic value** of the option. The preceding example shows why: it's the amount of money that could be gained by exercising the option immediately. We want the intrinsic value for an in-the-money option to be positive, so it's defined differently for calls and puts:

intrinsic value of an in-the-money call = stock price - exercise price
intrinsic value of an in-the-money put = exercise price - stock price

An in-the-money option must sell for at least its intrinsic value; otherwise, smart investors would purchase the option and exercise it immediately, generating arbitrage profits.[2]

An **out-of-the-money** call option is a call option for which the exercise price exceeds the stock price; an out-of-the-money put option is one for which the stock price exceeds the exercise price. Suppose you found a call option on AT&T stock with an exercise price of $60 when the stock price was $54.50. This option is out-of-the-money, and you would not want to exercise it. If you wanted to own the stock, it would be cheaper to purchase it in the market for $54.50 than to purchase it using the call option for $60. For an out-of-the-money option, the intrinsic value is zero. This reflects the fact that an option is a right, not an obligation--no one can force you to exercise an out-of-the-money option.

Should you throw away the call option with the $60 exercise price? No, you shouldn't. As long as the option has not expired, it will still be possible for the price of AT&T stock to move up above $60 per share. That is: an unexpired option leaves open the possibility that the stock price will move in a favorable direction (up for a call, down for a put) making the option even more valuable. The part of the option premium not accounted for by the option's intrinsic value is called the **time value** of the option. You may have noticed that the option prices are higher for expiration dates further in the future, holding fixed the exercise price. The further away is the expiration date, the better the chance that the underlying asset will make a favorable move before expiration. So if there are two options that are identical except for the expiration date, the option with the most time to expiration is the more valuable option.

[2]Sometimes you will observe that the newspaper columns report an in-the-money option selling for less than its apparent intrinsic value, as is the case for the October 50 call option which is selling for 4-1/4, while its intrinsic value appears to be 4-1/2. This can happen when the last trades in each of the option and stock markets do not occur at approximately the same time--note that the newspaper reports last trades in each market.

Let's compute the time value for some of the options shown in Figure 9-1--we'll focus on the Friday, 10/21/94 quotes. The January 50 call option has intrinsic value equal to \$54-1/2-\$50.00 = \$4-1/2, or \$4.50. The price of the option--the option premium--was \$5.00. The time value of the option is the difference between the option premium and its intrinsic value: \$5.00 - \$4.50 = \$0.50. The April 50 call option expires three months after the January 50 option: let's see how much the extra time was worth. The intrinsic value of the April 50 call is the same as for the January 50 call: \$4.50. The premium for the April 50 call was 5-7/8, or \$5.875. So the time value for the April 50 call was \$5.875 - \$4.50 = \$1.375. The additional three months between January and April added \$1.375-\$0.50 = \$0.875 of time value.

The time value for out-of-the-money options is equal to the option premium since intrinsic value is zero for these options. The time value of the January 60 calls was equal to the option price of 3/8 or \$0.375, and the time value of the April 50 puts was 1, or \$1.00.

We have learned that the option premium has two components: intrinsic value and time value. Intrinsic value cannot be negative, while time value is always positive for an unexpired option. To summarize:[3]

- **intrinsic value for a call option = max(0, stock price - exercise price)**
- **intrinsic value for a put option = max(0, exercise price - stock price)**
- **option premium = intrinsic value + time value**

With the basic terminology of options markets in hand, we turn now to the problem of describing the value of simple option positions at the options' expiration dates.

[3]max(a,b) means the larger, or maximum, of a or b. In the expression in the text, this implies that the intrinsic value of an option cannot be less than zero.

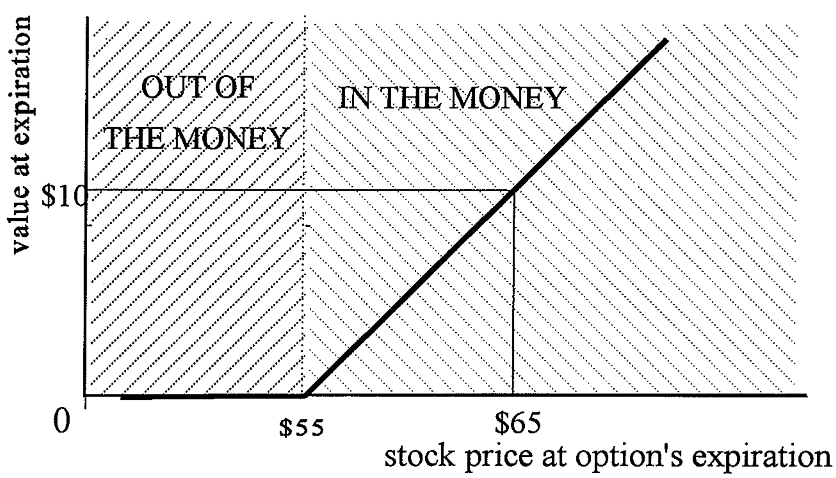
Box 9-2: EUROPEAN VS. AMERICAN OPTIONS

There is an important difference between a European option and an American option. Most so-called European options are traded on the European exchanges and most American options are traded on American exchanges, although there are exceptions. However, the term "European" or "American" does not refer to the location where the option is traded. The terminology actually refers to the exercise provisions of the option. A European option can be exercised only on its expiration date, whereas an American option can be exercised at any point up to and including the expiration date. The additional flexibility allowed by an American option means that an American-style option cannot be worth less than an equivalent European-style option. However, in practice it is usually the case that an option holder would prefer to sell an option rather than exercise it, because if he exercises the option he loses the time value remaining on the option. This explains the well-known adage: "Never kill a live option."

9-3 The value of a call option at expiration

A call option is the right to purchase the underlying security at the exercise. It is customary to show the value of an option position at the option's expiration using a graph like the one in Figure 9-2. It's sometimes called a "hockey-stick" diagram because the line showing

Figure 9-2: The value at expiration of a long position in a call option



the value of an option at its expiration date always looks like a hockey stick. The reason why will become clear as we proceed. Figure 9-2 shows the value at expiration of a long position in the AT&T call option with exercise price equal to $55 per share. If the stock price at expiration is $55 per share or less the option is out-of-the-money: it has zero intrinsic value and thus is worth nothing. If, however, the stock price is higher than the exercise price at expiration, the option is in-the-money at expiration and will be exercised. For example, if the stock price is $65 at expiration the call holder can purchase the stock for $55 per share from the call writer and then immediately sell it in the open market for $65 per share, earning $10 in the process. Notice that the $10 earned in this process is equal to the option's intrinsic value. To sum up: the value of a long position in a call option at the option's expiration is simply the option's intrinsic value, which is equal to the stock price minus the exercise price if the option is in-the-money, and is zero otherwise.

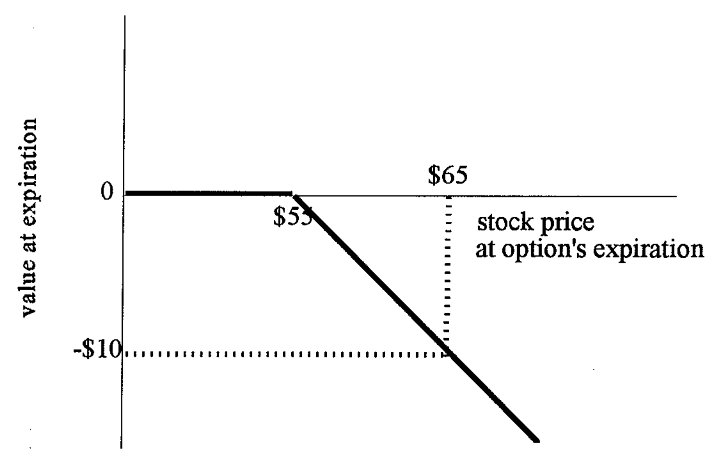
The value of an option position at expiration can also be described algebraically--this will be essential later on when we will want to describe the value of positions that combine many assets at the same time. We will use the notation **P'** to denote the value of the underlying security at the option's expiration date, and will use the notation **E** to stand for the exercise price of the option. Using this notation, we can describe the value of a call option at expiration as:

$$\textbf{value of long position in call option at expiration} = \max(0, P' - E).$$

Now we can see why a diagram of the value of this position looks like a hockey stick--that is, we can see why there is a kink in the diagram. Further, we can see why the kink occurs at the option's exercise price. If the price of the underlying security is below the exercise price the call is worthless at expiration. If the price of the underlying security exceeds the exercise price, the long position in the call option is valuable. There is a fundamental asymmetry in the valuation of options that is reflected in the kink in the diagram. This asymmetry stems from the fact that one can choose to exercise a valuable option, but can always throw away a worthless one.

Now, let's think about how to value a short position in a call option. If an individual who'll we'll call person A has a short position in a call option, he has sold person B (the option holder or option purchaser) the right to purchase the underlying security from person A at the exercise price on or before the option's expiration date. What is the value to person A of this short position, evaluated on the option's expiration date? The value is shown in Figure 9-3. We've used the same option as before--an AT&T call option with an exercise price of $55. If the price of the underlying security is below the exercise price of $55 the option will not be exercised and, as before, the value is zero. Suppose, however, that the price of the underlying security is equal to $65 on the option's expiration date. In this case the option will be

Figure 9-3: The value at expiration of a short position in a call option



exercised by person B against person A, the option writer. In this event, the option writer must go out and purchase the underlying security for $65 and then sell it to the option holder for $55. This entails a loss of $10 for the option writer--the individual with the short position in the call option. The short position in the call option thus loses $1 for every dollar that the price of the underlying security exceeds the exercise price.[4] Using the notation introduced

[4]It doesn't matter whether the call writer already owns the stock or must purchase it in the market. If he already owns it, he is selling a security worth $65 for $55. He still loses $10.

earlier, we can write the value of a short position in a call as

$$\textbf{value of short position in call option at expiration} = -\max(0, (P' - E)).$$

Why does the call writer get involved in a situation where he can only lose at the expiration date? The reason is that he is paid to do so--when he wrote the call option the option purchaser paid him the option premium. Once the call is written, the call writer hopes that the option will expire out-of-the money so that he can just keep the option premium and never hear from the option holder again! On the other side, however, the option holder is hoping that the call is in-the-money at expiration.

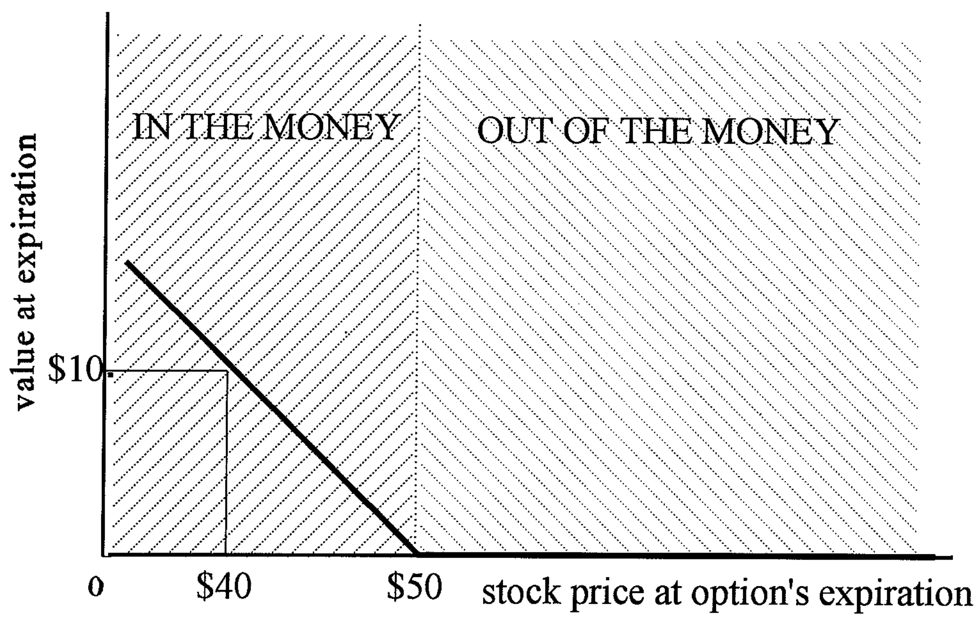
This suggests that an option can also be thought of as a wager, or bet, between the call holder and the call writer. If, at expiration, the price of the stock is higher than the exercise price, the call writer loses and the call holder wins. The call writer loses because he must sell a security (to the call holder) for a price lower than its market price. The call holder wins because she can purchase the underlying security for a price lower than its market price. What if the price of the underlying security is lower than the call's exercise price at expiration? The call will not be exercised--its intrinsic value is zero. In this case the call writer wins because he gets to keep the option premium initially paid by the call holder. The call holder loses because she purchased a call that turned out to be worthless at expiration. In short--the call writer wins when the call holder loses, and vice versa --this is a zero-sum game between these two individuals. This can be seen clearly if we add together the values of the short and long position in the call at the option's expiration: the sum of these two values is zero. Another way to see this is to notice that Figure 9-3 is the mirror image of Figure 9-2.

9-4 The value of a put option at expiration

We can show the value of positions in put options at expiration in the same way as we did for calls. As before, let's first look at the long position and then consider the short position. Since a put entails the right to sell the underlying stock, an individual with a long position in a put (a put holder) will gain if the stock price falls below the put's exercise price. If this happens, the put-holder can purchase the stock in the open market for a low price, and sell it to--or *put it to*--the put-writer for a high price (the put's exercise price).

Let's consider the example of a long position in the April 50 put on AT&T stock for Friday, October 21, 1994, as shown in Figure 9-4. The put will expire worthless if the stock price at expiration exceeds the exercise price of $50. But if the stock price is below $50 per

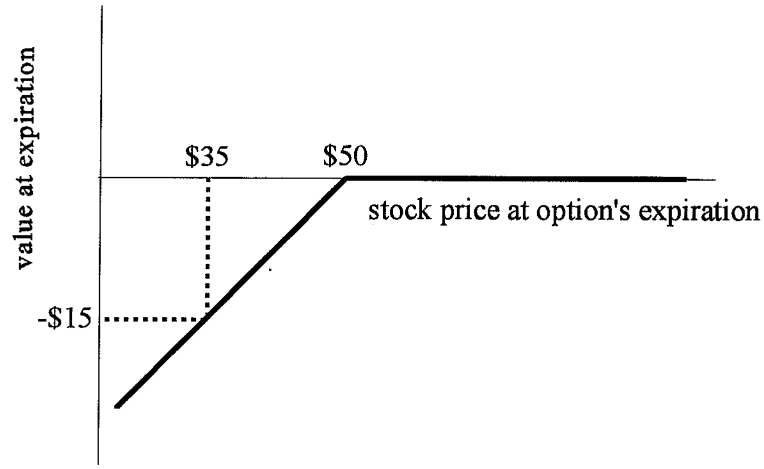
Figure 9-4: The value at expiration of a long position in a put option



share at the put option's expiration date the put owner can buy the stock at this low price and sell it to the put writer at the exercise price of $50. As with the call option positions, the graph of the value of the long position in a put option resembles a hockey stick, with the kink occurring at the option's exercise price. Unlike a long position in a call, however, a long position in a put is valuable when the stock price is *low* at the option's expiration.

Finally, let's look at the value of a short position in a put. An individual, who we'll call person A, who establishes such a position has sold to the put holder, person B, the right to sell stock to person A at the exercise price before the option expires. The graph of the value of a short position in a put is shown in Figure 9-5.

Figure 9-5: The value at expiration of a short position in a put option



Since a put writer loses what the put holder gains, Figure 9-5 is just a mirror image (through the horizontal axis) of Figure 9-4. For example, if AT&T stock is $35 per share when the put option expires the put holder gains $15 per share by purchasing the stock in the market for $35 and then selling it to the put writer for $50. The put writer loses $15 because he must purchase stock for $50 that is worth only $35 on the open market.

9-5 Index options

In March 1983, the Chicago Board Options Exchange introduced a new type of option for which the underlying instrument was a stock index--the S&P 100 Index (the original name of this option contract was the CBOE 100). Shortly afterward, the American Stock Exchange introduced an option on an index created by the Exchange, called the Major Market Index (MMI). Today, there are index options traded on almost every important securities exchange in the world, including the New York and American stock exchanges. Index options are the single most popular type of option traded. In 1989, the volume of trade in index options represented one-third of total trading volume in options on all exchanges.

What is a stock index?

The underlying security for an index option is a stock index, so let's start there. A **stock index** is defined as the value of a group of stocks. For example, the S&P100 index is the value of a particular group of 100 stocks. The index is constructed by adding up the dollar value of one share of each stock--this type of index is a value-weighted index, since each stock's weight in the index is equal to the ratio of the stock's price to the value of the index. Stock indexes can also be share-weighted indexes, where each stock in the index has an equal weight in the index--this means that the same dollar amount is allocated to each stock in the index.

Contract specifications for index options

As with other option contracts, the contract for an index option must specify the underlying security, the exercise price, the expiration date, and whether the option is European or American. It is also necessary to specify the multiple of the contract, which is similar to specifying the size of an ordinary option contract. **Size** refers to the number underlying securities that are controlled by the option. The **multiple** of an index option is number by which the value of the index is multiplied to get the value of the option contract. A typical multiple for an option contract is 100. For such a contract, a price of $3.50 for an index option with a multiple of 100 would mean that the value of the contract was 100 x $3.50 = $350.00.

An unusual feature of index options is that both European and American options are available, even if we just look at exchanges in the U.S. For example, the CBOE has two fairly similar index options: the S&P 100 index option, and the S&P 500 index option. Yet the S&P100 is an American option, while the S&P 500 is a European option. The index options also differ in their expiration dates: some cover just the next three or four months (the S&P 100, the NYSE Index, and the Major Market Index), while others have expiration dates every three months from March to December.[5]

Besides the original stock indexes, which were broad-based indexes made up of stocks from a wide range of industries, there are also index options now traded where the underlying

[5]Need to update this--expiration dates have recently changed.

Figure 9-6: Index option quotations in the Wall Street Journal

INDEX OPTIONS TRADING

Tuesday, July 25, 1995

Volume, last, net change and open interest for all contracts. Volume figures are unofficial. Open interest reflects previous trading day. p-Put c–Call

CHICAGO

CB MEXICO INDEX(MEX)

	Strike	Vol.	Last	Net Chg.	Open Int.
Sep	60p	20	1/4	− 2 3/4	146
Aug	75p	25	1 5/16	+ 3/16	23
Sep	75p	100	2 15/16	− 1/16	765
Aug	80c	26	6 1/4	− 7/8	2,803
Aug	85p	2	5	− 1/8	99
Sep	85c	13	5	− 1 3/8	420
Aug	90c	40	1 7/8	− 1/2	260
Sep	90c	2	3 1/8	− 1 1/8	233
Aug	95c	25	13/16	− 3/16	363
Call Vol.		106	Open Int.		12,768
Put Vol.		147	Open Int.		9,476

S & P 100 INDEX(OEX)

	Strike	Vol.	Last	Net Chg.	Open Int.
Aug	430p	1	1/16	...	1,714
Sep	440p	16	1/4	...	2,134
Aug	445p	430	1/16	...	1,112
Sep	445p	200	5/16	...	220
Aug	450p	80	1/16	− 1/16	2,143
Sep	450p	10	3/8	+ 1/16	1,601
Aug	460c	1	72 7/8	− 1/2	80
Aug	460p	501	1/8	...	3,503
Sep	460p	5	7/16	− 1/16	1,110
Oct	460p	38	1	− 3/16	430
Aug	465p	99	1/8	− 1/16	1,479
Sep	465p	10	5/8	− 1/8	231
Aug	470p	1,841	1/8	− 1/16	8,480
Oct	470p	127	1 5/16	− 7/16	1,522
Aug	475p	270	3/16	− 1/16	2,764
Sep	475p	55	3/4	− 1/8	368
Oct	475p	18	1 3/4	...	...
Aug	480c	4	53 1/8	− 3/8	1,184
Aug	480p	2,380	1/4	− 1/16	11,559
Sep	480p	108	13/16	− 3/16	5,108
Oct	480c	5	58	+ 1/2	143
Oct	480p	50	1 15/16	− 1/16	3,465
Nov	480c	1	59	− 1/2	75
Nov	480p	19	2 7/8	− 1/8	2
Aug	485c	12	51 3/4	+ 4 7/8	130
Aug	485p	466	5/16	− 1/16	5,051
Sep	485p	3,365	1 1/16	− 3/16	1,235
Oct	485p	2	2 1/4	...	...
Aug	490p	1,231	3/8	− 1/8	10,744
Sep	490c	4	46 1/2	+ 1 1/2	4,179
Sep	490p	888	1 5/16	− 1/8	5,149
Oct	490p	22	2 5/8	− 1/8	1,736
Nov	490c	2	50 1/2	+ 1/2	15
Nov	490p	35	3 7/8	...	...
Aug	495c	209	41	+ 3 3/4	3,087
Aug	495p	1,721	1/2	− 1/16	14,908
Sep	495p	315	1 1/2	− 9/16	952
Oct	495p	305	3 1/8	− 1/4	5
Aug	500c	601	36 3/4	+ 4 1/8	4,304
Aug	500p	5,797	5/8	− 1/8	17,520
Sep	500c	500	36 1/2	+ 1 3/8	1,605
Sep	500p	550	1 13/16	− 1/4	8,071
Oct	500p	219	3 1/2	− 1/2	5,461
Nov	500c	100	44 1/2	...	...
Nov	500p	18	5	− 1/2	87
Aug	505c	380	31 1/4	+ 3	3,717
Aug	505p	2,823	3/4	− 1/4	21,851
Sep	505p	160	2 1/4	− 3/8	1,936
Oct	505p	250	4 1/4	− 1/2	220
Aug	510c	591	27 1/4	+ 4	6,520
Aug	510p	7,121	13/16	− 1/8	32,847
Sep	510c	28	29 7/8	+ 2 5/8	5,785
Sep	510p	617	2 13/16	− 9/16	9,042
Oct	510c	152	34	+ 4 7/8	3,097
Oct	510p	261	5	− 1/2	6,498
Nov	510c	2	33 1/2	...	...
Nov	510p	30	6 3/8	− 1	26
Aug	515c	258	22 1/4	+ 2 1/8	10,799
Aug	515p	4,437	1 9/16	− 5/16	24,547
Sep	515c	4	24	+ 1	3,989
Sep	515p	2,230	3 3/8	− 3/8	8,050
Oct	515p	71	6	− 5/8	51
Aug	520c	1,664	18	+ 2 1/2	17,800
Aug	520p	9,919	2 1/4	− 1/2	31,680

RANGES FOR UNDERLYING INDEXES

Tuesday, July 25, 1995

	High	Low	Close	Net Chg.	From Dec. 31	% Chg.
S&P 100 (OEX)	535.66	530.84	534.74	+ 3.63	+ 106.11	+ 24.8
S&P 500 -A.M.(SPX)	561.75	556.34	561.10	+ 4.47	+ 101.83	+ 22.2
CB-Mexico (MEX)	84.84	82.99	83.26	− 1.10	− 19.76	− 19.2
CB-Lps Mex (VEX)	8.48	8.30	8.33	− 0.11	− 1.97	− 19.1
Nasdaq 100 (NDX)	575.12	562.64	572.71	+ 10.07	+ 168.44	+ 41.7
Russell 2000 (RUT)	294.72	293.39	294.70	+ 1.31	+ 44.34	+ 17.7
Lps S&P 100 (OEX)	53.57	53.08	53.47	+ 0.36	+ 10.61	+ 24.8
Lps S&P 500 (SPX)	56.18	55.63	56.11	+ 0.45	+ 10.18	+ 22.2
S&P Midcap (MID)	204.52	202.81	204.36	+ 1.55	+ 34.92	+ 20.6
Major Mkt (XMI)	488.26	484.11	488.11	+ 2.67	+ 88.16	+ 22.0
Leaps MMkt (XLT)	48.83	48.41	48.81	+ 0.27	+ 8.82	+ 22.1
Hong Kong (HKO)	....	....	189.32	+ 1.50	+ 24.24	+ 14.7
Leaps HK (HKL)	....	....	18.93	+ 0.15	+ 2.42	+ 14.7
AM-Mexico (MXY)	103.77	101.66	101.81	− 1.61	− 24.45	− 19.4
Institut'l -A.M.(XII)	572.16	566.41	571.57	+ 4.76	+ 107.92	+ 23.3
Japan (JPN)	....	....	163.85	− 4.35	− 36.21	− 18.1
MS Cyclical (CYC)	351.55	347.44	350.42	+ 2.49	+ 59.44	+ 20.4
MS Consumr (CMR)	247.73	245.29	247.72	+ 1.94	+ 39.19	+ 18.8
Pharma (DRG)	234.45	232.66	234.45	+ 0.95	+ 43.13	+ 22.5
Biotech (BTK)	91.46	89.46	90.93	+ 0.12	+ 8.87	+ 10.8
NYSE (NYA)	300.13	297.72	299.92	+ 1.93	+ 48.98	+ 19.5
Gold/Silver (XAU)	124.74	122.42	122.66	− 2.56	+ 13.33	+ 12.2
OTC (XOC)	822.39	805.16	819.08	+ 13.92	+ 238.82	+ 41.2
Utility (UTY)	251.55	248.72	251.07	+ 1.71	+ 23.81	+ 10.5
Value Line (VLE)	538.74	536.04	538.34	+ 2.18	+ 85.81	+ 19.0
Bank (BKX)	336.62	331.77	336.05	+ 3.60	+ 81.30	+ 31.9
Semicond (SOX)	285.56	278.53	283.25	+ 7.13	+ 143.16	+ 102.2

AMERICAN

AM MEXICO INDEX(MXY)

	Strike	Vol.	Last	Net Chg.	Open Int.
Aug	95c	10	9 1/2	− 1 1/2	10
Aug	95p	20	1 11/16	− 1/16	225
Aug	100c	10	5 3/4	+ 1 3/8	100
Aug	100p	10	3 1/4	− 1 1/4	61
Call Vol.		20	Open Int.		1,836
Put Vol.		30	Open Int.		2,826

BIOTECH(BTK)

	Strike	Vol.	Last	Net Chg.	Open Int.
Aug	90c	1	3 1/4	+ 9/16	28
Call Vol.		1	Open Int.		175
Put Vol.		0	Open Int.		195

HONG KONG INDEX(HKO)

	Strike	Vol.	Last	Net Chg.	Open Int.
Aug	175p	30	5/8	− 1/8	20
Aug	180p	33	1 1/8	− 3/8	18
Sep	185p	5	4 5/8	− 2 1/2	160
Aug	190p	15	3 5/8	− 1 1/4	48
Sep	190p	10	6 1/4	+ 5/8	13
Aug	195c	12	3 3/8	+ 1 1/8	20
Aug	200c	35	1 9/16	+ 7/16	15
Aug	205c	30	1/2	...	...
Call Vol.		77	Open Int.		1,020
Put Vol.		96	Open Int.		2,254

INSTITUTIONAL-AM(XII)

	Strike	Vol.	Last	Net Chg.	Open Int.
Sep	510p	2	7/8	− 11/16	211
Oct	520p	1	2 1/2	− 1/16	15
Aug	525p	6	1/2	− 5/8	8
Aug	535p	3	11/16	− 15/16	2
Sep	535p	2	2 1/4	− 1 1/2	2
Aug	545p	10	1 3/8	− 1 7/16	55
Sep	545p	2	3 1/2	− 1 3/8	25
Aug	555p	10	2 1/8	− 2 3/8	75
Aug	560c	1	12 1/4	+ 2 1/2	201
Aug	565c	26	9 3/8	+ 1/2	277
Aug	565p	5	4 5/8	− 1/4	300

NEW YORK

NYSE INDEX new(NYA)

	Strike	Vol.	Last	Net Chg.	Open Int.
Aug	285p	15	3/8	− 1/2	21
Call Vol.		0	Open Int.		1,847
Put Vol.		16	Open Int.		871

LEAPS-LONG TERM

S & P 100 INDEX – CB

	Strike	Vol.	Last	Net Chg.	Open Int.
Dec 95	40p	2165	1/16	...	47301
Dec 95	42 1/2p	527	1/8	...	45626
Dec 95	45p	40	1/4	...	40671
Dec 95	50p	204	5/8	− 1/16	12693
Dec 95	52 1/2p	49	1 1/4	− 1/8	1333
Dec 97	52 1/2p	20	3 1/2	− 1/8	257
Dec 95	55c	15	1 1/4	...	163
Call vol.		15	Open Int.		49,162
Put vol.		3,049	Open Int.		257,538

S & P 500 INDEX – CB

	Strike	Vol.	Last	Net Chg.	Open Int.
Dec 95	42 1/2p	150	1/8	+ 1/16	9364
Dec 95	47 1/2p	50	1/8	− 1/8	10573
Dec 97	47 1/2p	5	1 5/16	...	636
Dec 95	50p	30	5/16	− 1/8	22965
Dec 95	55p	64	1 1/4	− 3/16	3775
Dec 97	55p	20	2 3/4	− 1/4	2905
Dec 97	60p	1	4 3/4	...	1465
Call vol.		5	Open Int.		10,018
Put vol.		884	Open Int.		200,358

index is made up of stocks from a single industry. For example, there are index options on bank stocks; biotechnology stocks; semiconductor stocks; Mexican stocks; and utility stocks. There are also index options with expiration dates more than one year in the future. These options have a colorful and descriptive name: they're called **leaps**.

Figure 9-6: Index option quotations in the Wall Street Journal
<need full page>

Reading index option quotes in the *Wall Street Journal*

Figure 9-6 shows index option quotes as reported in the *Wall Street Journal*--these quotes are for Tuesday, July 25, 1995. The heading **CHICAGO** indicates that the options listed below the heading are traded on the Chicago Board Options Exchange (CBOE). Let's look at two particular quotes: the October 510 call and the October 510 put on the S&P 100 index. This is reported in the paper as the following two lines:

Expiration month	Exercise price	Volume	Last	Net change	Open interest
Oct	510 c	152	34	+4⅞	3,097
Oct	510 p	261	5	-½	6,498

Both of these options expire in October, and both have exercise price of 510. The first option is a call, as indicated by the small "c" following the exercise price of 510, and the second option is a put option, as indicated by the small "p." The next column, "Volume," reports the number of contracts traded on that day: there were 152 contracts traded for the call option and 261 contracts traded for the put. The "Last" column indicates the price of the last trade in the option: this was 34 for the call option and 5 for the put option. Since the multiple for this index option is 100, the price paid for one call option contract would have been 100 x $34 = $3,400. The column headed "net change" indicates the change in the closing price from the previous trading day: the call option rose by 4⅞. This means that the price of the call option contract rose by 100 x $4⅞ = $478.50. Finally, "open interest" tells how many contracts exist for this particular option.

What do you get if you exercise an index option?

If you exercise a call option on a stock, you purchase the underlying stock at the option's exercise price. But a stock index represents s group of securities, not a single stock. This would make things difficult for, say, a call option writer who had the option exercised against him--he would have to purchase each of the stocks in the index (in the right amounts) for delivery to the call holder. This would be extremely inconvenient. Instead, stock index options are settled in cash, not through delivery of the stocks that make up the index. For example, the holder of a call option on a stock index who finds that the option is in-the-money at expiration will receive the difference between the value of the stock index on the option's expiration date and the exercise price of the option. Suppose that an individual has purchased the Oct 510 call on July 25, 1995--she would have paid 100 x $34 = $3,400 for one contract (recall that the multiple is 100). Now, suppose the value of the S&P 100 index is 526 when the option is about to expire the third Friday in October. This call option is in-the-money since the exercise price is less than the price of the underlying security. When the option holder exercises the option she receives, in cash, the difference between the value of the index and the exercise price of the option, multiplied by the contract's "multiple" of 100. Thus she will receive ($526-$510) x 100 = $1,600 when she exercises the option.

Why have stock index options become so popular?

In simple terms, stock index options allow individuals to place bets on the direction of the stock market. One could do this by purchasing a diversified portfolio of stocks, such as the stocks in the S&P 100 index, but this would be cumbersome and would require a large dollar investment. Options can be purchased much more cheaply, and the returns to options will depend on the return to the underlying stock index.

However, options are popular mainly because of their usefulness in combination with other assets. In Chapter 12 we'll learn how options can be used in creative ways to hedge risks and to produce new derivative securities. To develop our ability to determine the value of combinations of securities, let's look at combinations of stocks, bonds, and options.

9-6 Combinations

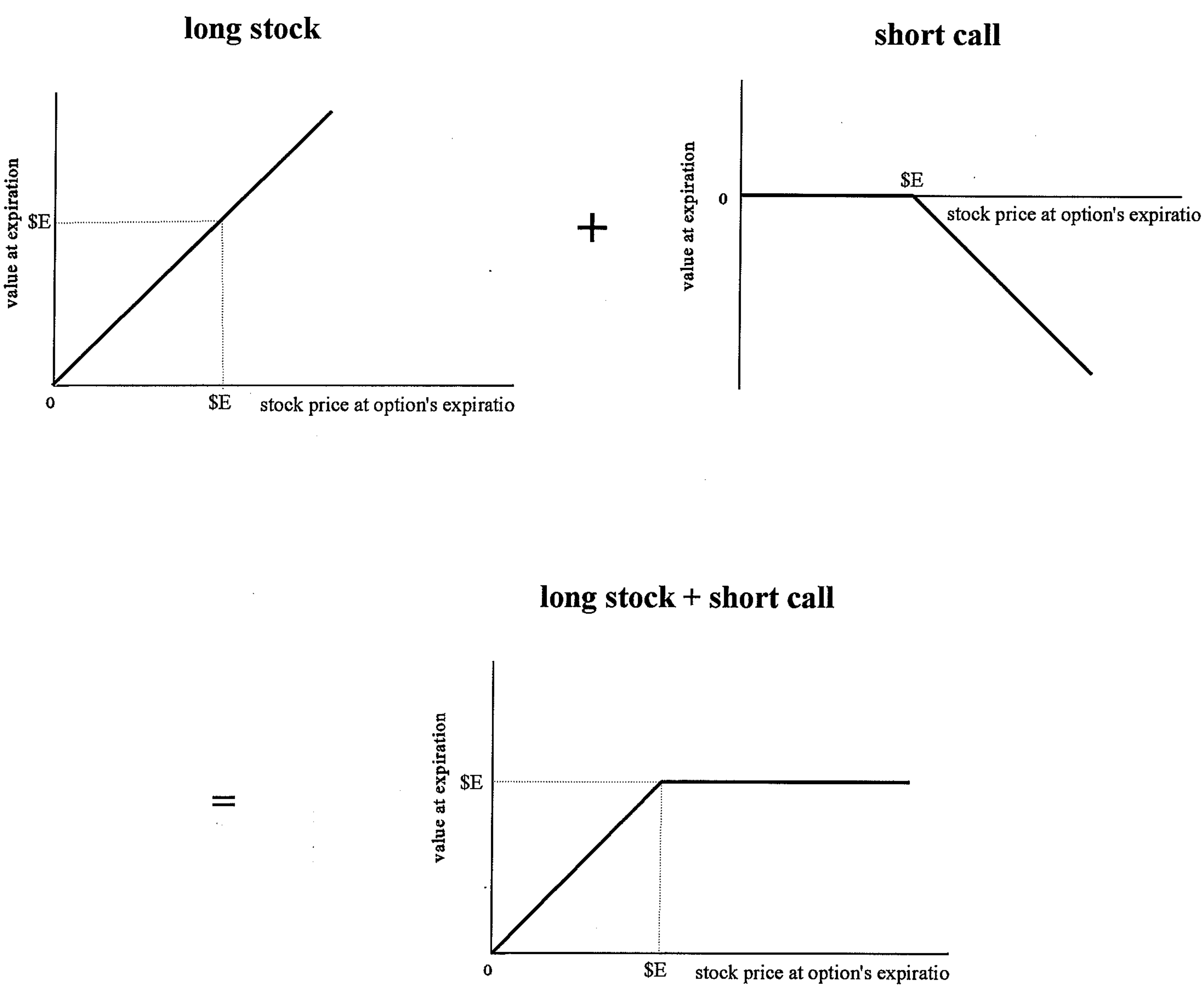
Options are frequently used in combination with other assets such as stocks and bonds, or even just with other options, to create new securities with an especially desirable pattern of payouts. In this section we'll learn how to determine the value of combinations and we'll also learn that there is usually more than one way that securities can be combined to produce a specific pattern of cash flows.

Writing covered calls

An individual is said to have written a **covered call** if she owns the underlying stock and sells a call against that stock (i.e., establishes a short position in the call). The terminology "covered call" refers to the fact that the call writer already owns the stock, so that if the call is exercised against her she is "covered"--she will not need to purchase the stock in the open market at an unknown future price. Let's look at the value of this strategy at the option's expiration date. Figure 9-7 shows that the value at the option's expiration of the stock alone is just a 45° line from the origin since the stock value at the option's expiration is plotted on both the horizontal and vertical axes. The value at the option's expiration of the short call has the shape shown earlier in Figure 9-3, where the kink occurs at the exercise price, **E**. When we combine the long stock position with the short position in the call, we have a position whose value is shown at the bottom of Figure 9-7. If the stock price at the option's expiration date is less than the exercise price, **E**, the option is worthless at expiration and the value of the combined position is equal to the value of the stock. If the stock price at the option's expiration date is greater than the stock price, the option will be exercised against the option writer. That is: the option writer must sell her stock to the option holder for the exercise price, **E**. So for any stock price at expiration that is greater than or equal to **E**, the option writer just receives **E** dollars and transfers her stock to the option holder.

Figure 9-7

Combination of long stock and short call



"Writing a covered call" involves combining a short position in a call with a long position in the stock. The shape of the resulting graph has the same shape as a short position in a put option--see Figure 9-5--except that the flat section occurs at $E rather than $0.

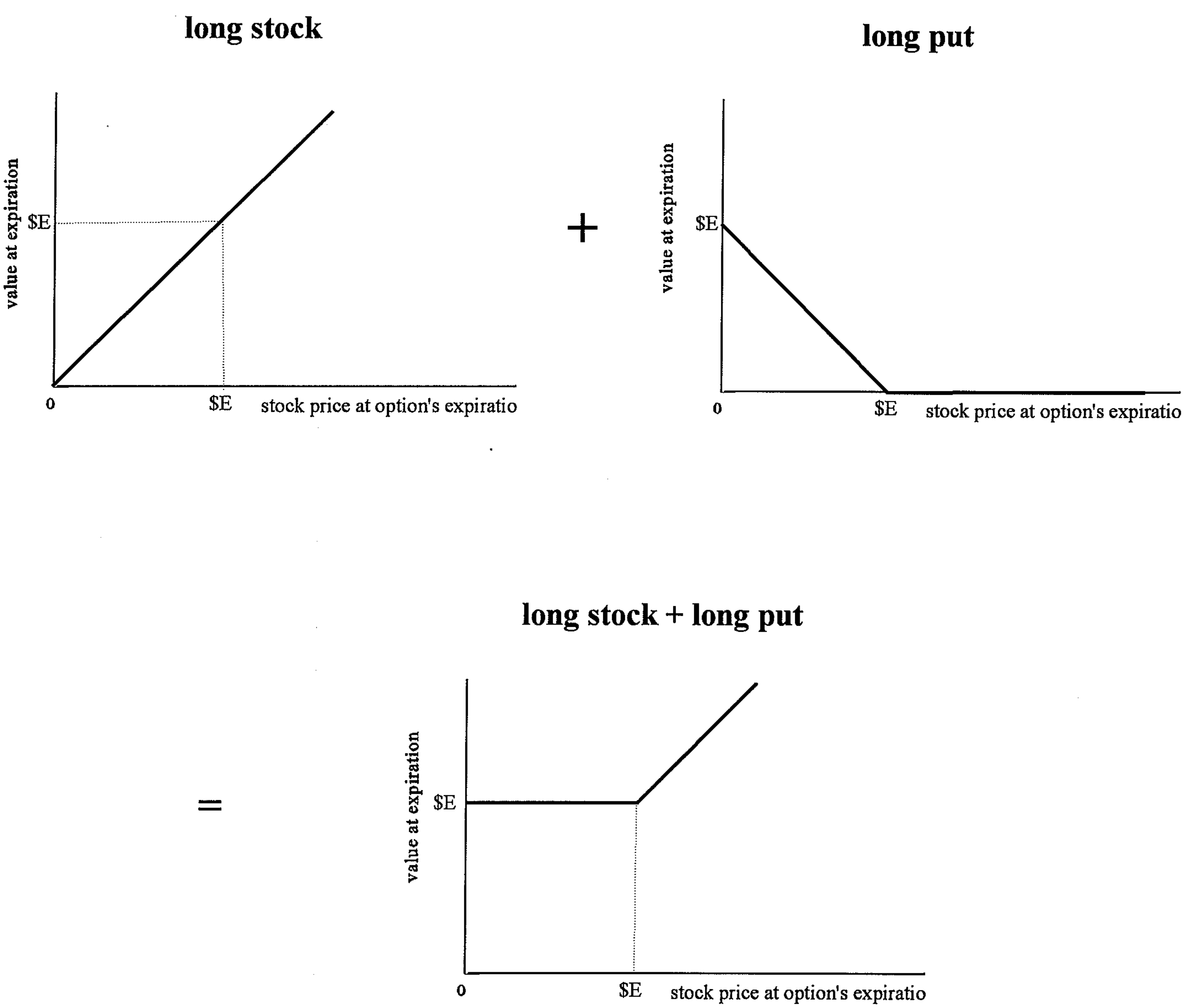
Why would anyone want to write a covered call? Although a detailed analysis of the hows and whys of option strategies is beyond the scope of this book, we can give a partial answer here. One reason to write a covered call is that one is paid to do so--if the call is not exercised against the call writer the call writer gets to keep her stock and she also gets to keep the option premium. This is why writing covered calls is sometimes advocated as a way of "generating extra income" from one's stock. However, the call writer gives up the opportunity to gain from large upward movements in the stock price, as the most she will receive from the stock is the option's exercise price, **E**.

Buying stock and buying a put

Another combination that is frequently used is combining the purchase (or ownership) of stock with the purchase of a put on the stock. Since owning a put means that the stock can be sold for the put's exercise price, this strategy is useful for protecting the investor against downward movements in the stock price. If the stock price falls below the put's exercise price, the stock can be sold using the put for a price higher than in the open market. The value of this combination is shown in Figure 9-8. The first panel of this figure shows the value of the long position in the stock; the next panel shows the value of the long position in the put. The value of the combination is shown at the bottom of Figure 9-8. We see that the least that the combination can be worth at the put's expiration is the exercise price, **E**. This is the value of the combination for stock prices at expiration smaller than the exercise price. If this happens the individual will exercise the put and will receive $**E**. If the stock price at expiration is above the exercise price the put expires worthless, and the value of the combination is just the value of the stock alone. This looks like a win-win situation--the value is always positive. But remember that establishing the position in the first place involved purchasing both the stock and purchasing the put. As we will see in the next section, the prices of the components of the combination will be set in the market to remove any opportunities for profit without risk.

Figure 9-8

Combination of long stock and long put



Combining a long position in the stock with a long position in a put results in a graph that has the same shape as a long position in a call option, except that the flat section is at $E instead of $0.

A risk-free combination

By now you have probably noticed that combinations of stock and options can produce patterns of payouts that mimic simple option positions. For example, writing a covered call produces a graph that has the same shape as a short position in a put--the value rises with the stock price initially, and then is flat for higher stock prices. The long stock plus long put combination produces a value graph that has the same shape as a long position in a call--the value is flat for low stock prices, and rises for stock prices above the exercise price.

Let's put together a combination that has the same shape as a risk-free return--that is: the value of the combination will be the same for all stock prices at option's expiration. A combination that will do this is:

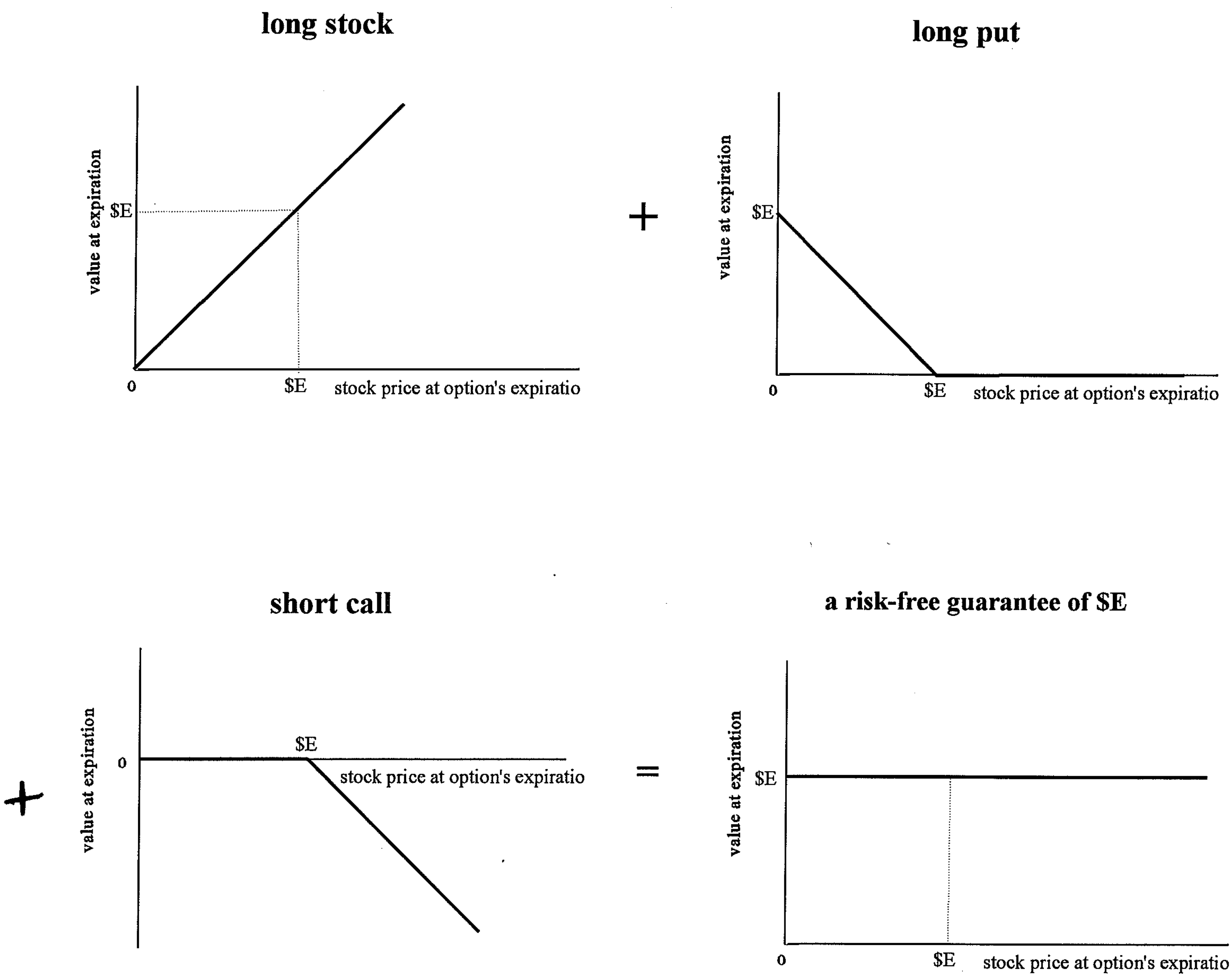
- long position in the stock
- long position in a put on the stock with exercise price **E**
- short position in a call on the stock with exercise price **E**

Notice that the two options involved have the same exercise price--they must also have the same expiration date.

The value of this combination at the options' expiration date is shown in Figure 9-9. Notice that the short call offsets gains from increases in the stock price for stock prices above **E**, while the long put offsets losses from declines in the stock price for stock prices below **E**. The three components together guarantee a payout of **E** no matter what the stock's price. This means that we have found a way to construct a combination of assets that has the same pattern of cash flows as a risk-free bond: we purchase the portfolio now and receive a known return at the options' expiration date. In the terminology of Chapter 4, we have found a *replicating portfolio* for a risk-free bond. Absence of arbitrage opportunities means that a combination that replicates a risk-free bond must yield the same return as an actual risk-free bond. The next section shows how this relationship can be used to determine the price of a put from knowledge of the stock price, the price of a call, and the risk-free interest rate.

Figure 9-9

A risk-free combination



The long put + short call offset the long stock position to yield a guaranteed payout of $E no matter what the stock price at the options' expiration.

9-7 Put-call parity

The preceding section showed how to construct a portfolio with a risk-free return by combining a long position in the stock with a short position in a call and a long position in a put, as summarized in Figure 9-8. In words, this figure says:

buy now:	**long stock + long put + short call**
get later:	**payout of \$E at the options' expiration date**

The payout of \$E can also be viewed as the payout from discount bonds with face value of \$E with maturity date equal to the options' expiration date. So there are two ways to purchase a portfolio that pays \$E at the options' expiration date: the first way is to buy the portfolio shown in Figure 9-8; the second way is to purchase risk-free discount bonds (T-bills) with face value \$E that mature on the options' expiration date. Since these two portfolios are equivalent, they should have the same cost (we are ignoring transactions costs, for the moment). If we purchase the T-bills the current cost, or PDV of the bonds' face value, is

cost of T-bills with face value of \$E = \$E/(1+i)

where i denotes the risk-free interest rate for the period from now until the bonds' maturity date--it is not an annual rate unless the T-bills happen to have exactly one year to maturity. If we purchase the portfolio that replicates the risk-free payout of \$E the cost is

cost of risk-free combination that pays \$E = $P + V_p - V_c$

where P denotes the current price of a share of stock, V_p is the price of the put with exercise price \$E, and V_c is the price of the call with exercise price \$E. Notice that V_c enters with a negative sign--this is because the short position in the call generates cash, while purchasing the stock and the put require current outlays of cash.

The assumed absence of arbitrage opportunities means that these two ways of generating \$E at the options' expiration dates must have the same cost:

cost of risk-free combination that pays $E on options' expiration date	=	cost of T-bills with face value of $E that mature on options' expiration date

Expressed algebraically, this means that

$$P + V_p - V_c = \frac{E}{1+i} \, . \qquad (9\text{-}1)$$

Equation (9-1) is known as the **put-call parity** relationship because it spells out the relationship between the price of a call and a price of a put with the same expiration date and exercise price. The put-call parity relationship can be used to determine the value of a put option on a stock, V_p, given the stock price, the risk-free interest rate, and the value of a call option with the same expiration date and exercise price as the put option. To do this, we just rearrange equation (9-1) to obtain:

$$V_p = \frac{E}{1+i} - P + V_c \, . \qquad (9\text{-}2)$$

The put-call parity relationship is useful because it permits us to focus primarily on call options. If we can value a call option, we can determine the value of a put option with the same exercise price and same expiration date through put-call parity. For example, consider a stock is selling for P = $60 per share and a call option with exercise price equal to $55 that expires in six months and currently sells for V_c = $6.00. The risk-free interest rate earned on a six-month T-bill is i = 0.016 (1.6% for six months). The put-call parity relationship tells us that a put option with exercise price equal to $55 that expires in six months would be worth

$$V_p = \$\frac{55}{1.016} - 60 + 6 = \$0.125 .$$

Financial economists and option market participants have tested whether put-call parity holds in practice. A very large number of studies has verified that the put-call parity is a good description of the relationship between put and call option prices.

Creating a synthetic put option

Suppose that you work for a financial intermediary such as a commercial or investment bank. A client wants to purchase a put option on a stock that has listed call options but no listed puts for the exercise price he wants. Specifically, your client wants to purchase a put option with exercise price **E**=\$70 with an expiration date six months from today. The current price of the stock is **P** = \$85, the interest rate on six-month T-bills is 2% for six months (4% per year); and the price of a call option with **E**=\$70 and six months to expiration is V_c = 17-1/4 = \$17.25 per share. Using the put-call parity relationship, we can find the value of the put as follows:

$$V_p = \frac{E}{1+i} + V_c - P \qquad (9\text{-}2)$$

$$= \frac{\$75.00}{1.02} + \$17.25 - \$85.00 = \$0.875.$$

This calculation shows that the price you should charge for the put is \$0.875 per share of stock that can be purchased with the put (plus your commission, of course).

What if your firm does not want to carry this short position in the put in its portfolio This is a situation that bankers encounter frequently, since many bank products involve selling puts to customers, either explicitly or implicitly. For example, the default option gives the borrower the option to sell, or "put," the borrower's firm to the bank if the firm turns out to have low value. If there were puts traded on the organized exchanges you could just go buy a put--establishing a long position in a put with the same exercise price and expiration date as your short put would leave you with a zero-value position no matter what the stock's price at the options' expiration. However, there are no puts traded on the exchanges for this exercise price--this is why your brokerage firm got involved in the first place. Instead, you will have to create a portfolio that replicates the pattern of payouts of a long position in the put--you will create a synthetic put. We can see how to do this by viewing equation (9-2) as a set of instructions--a blueprint, or a recipe--for constructing a synthetic put option:

synthetic long put = NPV of $E borrowed at the risk-free rate+ long call + short stock

This expression is both a recipe for constructing a synthetic long position in a put, and also a way of computing its value as we have seen above. Let's see what this would cost you in the example under consideration:

cost of synthetic long put = $70.00/1.02 + 17.25 - 85.00 = $0.875.

These calculations show that the cost to you of constructing a synthetic long position in the put is $0.875 per share. You've probably noticed this is exactly what we determined you should charge your customer, not including your commission. So another way to look at this is: you can take the premium you charged your customer for the puts he wanted to buy and then use them to purchase assets that will exactly offset the risk associated with your short position in the put.

Creating a synthetic call option

In the preceding example we showed how to construct a synthetic put option by combining shares of the underlying stock, a call option, and risk-free bonds. A similar approach can be used to create a synthetic call option, as described in Table 9-3.

Table 9-3
Creating a synthetic call option

	COSTS TODAY ($)	PAYOFFS LATER ($)	
		IF P'>E	IF P'<E
A. Long call (Buy call)	V_c	P' - E	0
B. Replicating portfolio for long call			
Buy stock	P	P'	P'
Buy put	V_p	0	E-P'
Borrow E/(1+i) dollars: sell T-bills with face value E	-E/(1+i)	-E	-E
Payoff to replicating portfolio:	**$P + V_p$ -E/(1+i)**	**P'-E**	**0**

An investor who wishes to purchase a call option can either purchase the call option directly, paying V_c, or he can purchase the replicating portfolio, paying $P + V_p - E/(1+i)$. As before, we will ignore transactions costs associated with carrying out these purchases. Since the call option and the replicating portfolio have exactly the same pattern of payoffs, they must sell for the same price. That is: it must be the case that $V_c = P + V_p - E/(1+i)$. Table 9-3 also provides a recipe for constructing a synthetic call option: purchase a share of stock, buy a put option, and sell T-bills.

We have seen that a synthetic put can be created if it is possible to trade calls and to short-sell stock. We have also seen that a synthetic call can be created if it is possible to trade puts and short-sell T-bills. Can calls or puts be produced if neither call options nor put options already exist? This is an important question for bankers since they frequently create custom options for customers. The answer, surprisingly, is "yes." In order to see how this can work, we must take a closer look at theories of option pricing. Before turning to this, however, it is useful to think about the determinants of option value that should be present in any option pricing model.

9-8 The determinants of option value

The preceding section showed how the values of call and put options are linked through the put-call parity relationship. But what determines the value of each option individually? This section reviews the six factors that determine the premiums of both puts and calls. We will focus primarily on the determinants of call value since we can always determine the value of a put through put-call parity.

The price of the underlying asset

A call option must sell for a price at least as large as its intrinsic value. The intrinsic value of an in-the-money call is the price of the underlying asset minus the exercise price of the call, while the intrinsic value of an out-of-the-money call is zero. If the call sold for less than the option's intrinsic value, we know that there would be an opportunity for arbitrage profits. So we have

$$V_c \geq \textit{intrinsic value} = \max(0\ ,\ P - E)\ .$$

Since the time value of an unexpired option is always positive, and since the value of an option equals the intrinsic value plus the time value, we can make the stronger statement that the price of an unexpired option will be strictly greater than the intrinsic value:

$$V_c > \max(0\ ,\ P - E)\ . \qquad (9\text{-}4)$$

For a call option, the value of the option increases with the price of the underlying security, holding fixed all other determinants of option value. This is because increases in the price of the underlying security increase the intrinsic value of the option.

For puts, the intrinsic value of an in-the-money put is the exercise price minus the price of the underlying security. By the same argument, then, we know that the value of an unexpired put option is given by

$$V_p > \max(\ 0\ ,\ E\ -\ P)\ . \qquad (11\text{-}5)$$

For a put, then, the value *decreases* with increases in the price of the underlying security, holding fixed all other determinants of option value, because increases in P mean a decline in the intrinsic value of the put.

Equations (9-4) and (9-5) show that the value of an option can't be less than zero. A value or price less than zero would mean that someone would have to pay to accept the option. But, since an option is a right and not an obligation, it can always be thrown away if it's not valuable. So it makes sense that an option's price cannot be negative.

The exercise price

The exercise price directly affects the value of options through intrinsic value. Looking first at call options, we can see from equation (9-4) that increases in the exercise price, other factors held fixed, will lead to a decline in the intrinsic value of the option and a consequent decline in the value of the option. For puts, increases in the exercise price mean an increase in intrinsic value and an increase in the value of the option, as shown in equation (9-5).

Time to expiration

Option value increases with time to expiration. This is because a longer time to expiration means that there's more time for the price of the underlying asset to move in a favorable direction, and so a greater chance that it will do so. We can see this effect in the *Wall Street Journal* option quotes in Figure 9-1. Holding fixed the exercise price, the prices of call and put options increase with time to expiration.

The risk-free interest rate

It is easy to see how option values depend on exercise price, time to expiration, and the price of the underlying security. It is more subtle to understand why the level of the risk-free interest rate would matter for option prices. Recall that the risk-free rate was important for pricing options using the put-call parity condition, since risk-free bonds formed part of a replicating portfolio for a put option. As a related matter, we will see in section 9-10, an alternative to purchasing an option is to hold a replicating portfolio consisting of risk-free bonds and the underlying stock. Through this channel, a high value of the risk-free interest rate increases the value of an option.

The volatility of the price of the underlying security

We have discussed the idea that taking a position in options represents a "bet" on the future price of the underlying security. A call holder "wins" if the price of the underlying security moves up, and a put holder "wins" if the price of the underlying security moves down. Either option will be more valuable the more likely it is that the price of the underlying security will move significantly before the option expires. That is: an option is more valuable the more volatile is the price of the underlying security.

It may seem strange that volatility increases option value--we learned in Chapter 2 that individuals must be compensated for volatility in returns with a higher average return. That is: volatility *decreases* value for most securities. Why does volatility increase value for options? The answer lies in the unique feature of options--they allow the purchaser to participate in favorable movements of the price of the underlying asset while avoiding unpleasant consequences of unfavorable price movements. Since options let the investor keep only favorable outcomes and discard unfavorable ones, it makes sense that option values should increase with the volatility of the underlying security.

Dividend payments

The payment of dividends on common stock affects the value of the stock's options. When a stock pays dividends, the price of the stock will fall by the amount of the dividend holding fixed all other determinants of the stock price. Since the stock price falls when a dividend is paid, the intrinsic value of an in-the-money call option drops by the amount of the dividend, and the price of the call also falls. Even an out-of-the money call option will fall in value when the stock pays dividends because the fall in the price of the stock makes the call even deeper out-of-the-money. That is: it becomes more unlikely that the stock price will move in a favorable direction before the option expires. The effect of dividend payments on put values is exactly the opposite: since the stock price falls when dividends are paid, dividend payments increase the value of put prices.

9-9 Risk in option contracts

Options are typically considered risky investments--fledgling investors are routinely steered away from them. Yet on first inspection, options may not look very risky at all, in fact, they may seem less risky than holding the underlying asset. After all, the graph for the long positions in both call and put options (Figures 9-2 and 9-4) show clearly that the losses associated with this position are limited while the potential profit is unlimited. The short positions in both calls and puts correspondingly have limited gains and potential for unlimited losses, but short positions in *all* securities are commonly agreed to be risky. Where is the additional risk associated with options?

The risk has two main components. First, an option will expire, and if the favorable movement in the price of the underlying asset does not happen in time, the investor will lose his entire investment. One alternative to holding an option is to establish a position in the underlying stock. Since the stock does not have an expiration date but an option does, options are clearly riskier along this dimension. The second component of additional risk associated with options concerns the volatility of the returns to the option position compared with a position in the underlying stock. Figure 9-1 showed the prices of AT&T stock and its options for two adjacent trading days: Friday, October 21, 1994 and Monday, October 24, 1994. Over the two days shown the price of AT&T stock dropped from $54.50 per share to $54.00 per share--a decline of 0.90% of the stock's initial value. Now, a long position in a call is in some ways similar to a long position in the underlying stock, since both investments have positive return if the stock's price rises. Over the period that the stock declined by $0.50 per share, the price of all the call options declined as well, as shown in Table 9-4. For example, the January 50 call declined in price from $5.00 to $4.875. This 12.5-cent decline in the price of the option is smaller in dollar terms than the 50-cent decline in the stock's price. However, it represents a loss of just over 2.5% of the value of the option--a greater *percentage* loss on the option compared with the 0.9% loss on the stock. The January 55 call declined in price from 1-3/4 to 1-1/2-- a loss of just over 14% of the initial investment. The Jan 60 call lost

33% of its value over the same period. For call options, the percentage change in the value of the option is larger, the more out-of-the money is the option. The same is also true of put options: for a given change in the price of the underlying stock, the percentage change in the value of in-the-money options is smaller than the percentage change in the value of out-of-the money options.

Table 9-4

Sensitivity of call option prices to changes in the stock price

	Price on 10/21/94	Price on 10/24/94	% change in price
AT&T stock	$54.50	$54.00	- 0.9%
AT&T Jan 50 call	$5.00	$4.875	- 2.5%
AT&T Jan 55 call	$1.75	$1.50	-14.3%
AT&T Jan 60 call	$0.375	$0.25	-33.3%

Despite the risk associated with option contracts, options do provide an important aspect of insurance that is not provided by holding the underlying asset. For example, a call option lets an investor participate in favorable upward movements in the price of the underlying stock, without having to participate in unfavorable declines in the stock's price. This insurance is not free, of course--the option's price measures the value of this insurance. Since so many different kinds of bank assets and liabilities contain option-like components, it is important to know what value to assign to these options. In the next section, we describe a way to determine the value of an option by using the prices of other marketable assets.

9-10 A simple model of option pricing

This section presents the two-period **binomial option pricing model**. This is a very simple model that nevertheless manages to incorporate most of the important determinants of option value discussed in section 9-8. Further, it does so in a way that is simple and intuitive. This model will also will serve as a stepping-stone to the more realistic Black-Scholes option pricing model presented in section 9-11.

To simplify matters we assume that there are only two periods: the first period is the present, which we'll call "now," and the second period is "later" which for concreteness we take to be a date three months from today. We will continue let P denote today's stock price (the price "now") and let P' denote the stock price "later" (in three months' time). We will assume that there are only two possible outcomes for the stock price "later." One possibility is that the stock price will go up to $P'=P^u$: the super script "u" stands for *up*. The other possibility is that the stock price goes down to $P' = P^d$: the superscript "d" stands for *down.* The assumption that there are only two possible outcomes for P' explains why this model is called the "binomial" option pricing model: the binomial distribution is one in which, at each point in time, there are only two possible outcomes. These simplifications are highly unrealistic, of course. Later in this chapter we will show how to price options in multi-period settings and we'll allow stock prices to take on a wide range of possible values in each period.

Now, consider a call option on this stock. The call option will expire three months from today and has exercise price **E** which we'll assume lies between P^d and P^u:[6]

$$P^d \leq E \leq P^u \ .$$

The option will be in the money when it expires if the stock price rises to P^u: in this case its value at expiration would be $P^u - E > 0$. The option will expire out-of-the-money if the stock price falls to P^d. In this case the option will not be exercised so its value at expiration will be zero.

Our goal is to determine the value of a call option by constructing a replicating portfolio for the call option. The value of the call will be equal to the value of this replicating portfolio. The replicating portfolio for a call option will consist of the underlying asset, which we'll assume is a share of common stock, together with a risk-free bond.

Let's take a specific example. Suppose that the current stock price is P=$50.00, and the two values it may take on at the option's expiration, three months from today, are P^u =

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[6] This assumption focuses our attention on the most interesting case. If the exercise price is outside these bounds then it is either always in-the-money at expiration or out-of-the-money. The model can be adapted to handle these cases, but they are not of much economic interest so we will ignore them for now.

\$60.00 or P^d = \$40.00. The exercise price of the call option will be **E** = \$50.00: this is an at-the-money option, although this feature is not important for our analysis. We will also need to know the risk-free interest rate: we'll assume that a dollar paid three months from now is worth \$0.99 today, so that 1/(1+i) = \$0.99 where the interest rate i refers to the three-month interest rate, not an annual interest rate. This implies i = 0.0101 (1.01%) for three months.

Table 9-5

The value of a call option at expiration

In the Money at Expiration: Stock price rises to $P^u > E$	**Out of the Money at Expiration:** Stock price falls to $P^d < E$
value of option is P^u-E = \$60.00 - 50.00 = \$10.00	value of option is \$0

The two possibilities for the call option at expiration are shown in Table 9-5. If the stock price rises to P^u = \$50.00 the call option is worth P^u-**E** = \$60.00 - 50.00 = \$10.00. If the stock price falls to \$40.00 the option will not be exercised and will therefore have zero value.

Valuing a call option using a replicating portfolio

The job of an option pricing model is to determine the dollar price of an option with specified characteristics. In the two-period binomial option pricing model it is possible to combine stocks and bonds to perfectly replicate the payouts of a call option. We'll construct this replicating portfolio for the call option in two steps. First, we will construct a portfolio which combines a risk-free bond together with shares of the option's underlying stock. We will design this portfolio to resemble a call option in the sense that it will have zero value if the stock price is below the exercise price, and is positive if the stock price rises beyond the exercise price. After constructing this portfolio we'll determine how much of this portfolio must be held to exactly mimic the payouts of the call option.

Our first task is to construct a portfolio that pays us something if the stock price goes up and nothing if the stock price goes down. To do this, we

- buy one share of the underlying stock
- sell risk-free discount bonds (Treasury bills) with face value equal to P^d that mature on the option's expiration date

Since the cost of a share of stock is P and we'd receive $P^d/(1+i)$ from selling the T-bills, the net cost of the portfolio is $P - P^d/(1+i)$. The cash flows from this portfolio depend on whether the stock price rises to P^u or falls to P^d as summarized in Table 9-6.

Table 9-6

The value at expiration of an option-like portfolio

Stock price rises to P^u		**Stock price falls to P^d**	
receive value of stock:	P^u = \$60.00	receive value of stock:	P^d = \$40.00
pay out face value of bond:	P^d = \$40.00	pay out face value of bond:	$-P^d$ = -\$40.00
Value of portfolio:	$P^u - P^d$ = \$20.00	**Value of portfolio:**	**0**

From Table 9-6 we see that this portfolio bears an important resemblance to a call option, since it pays zero if the stock price falls, but has a positive payout if the stock price rises. But we are not quite through. Notice that the call option pays P^u-E if the stock price goes up, which is \$10 in our example. The portfolio we have constructed pays $P^u - P^d$, which is \$20 in our example. That is: the portfolio pays the same amount as the option if the stock price goes down, but pays more than the option if the stock price goes up. We can fix this problem by adjusting the amount we purchase of the replicating portfolio. Intuitively, in the example it seems that we'd want to purchase half of the portfolio since we only want a payout of \$10.00 if the stock price goes up--exactly half of the payout of \$20.00 yielded by this portfolio. More generally, we would want to purchase a fraction ***h*** of the option-like portfolio, where

$$h = \frac{P^u - E}{P^u - P^d} . \qquad (9\text{-}7)$$

There is a special term for ***h***: it is called the **hedge ratio**--in the next section, we'll learn why it has this name.

Let's return to our example. If we set $\boldsymbol{h} = (P^u - E)/(P^u - P^d) = 10/20 = 0.5$, then we get 0.5 x \$20.00 = \$10.00 from the replicating portfolio when the stock price goes up, which exactly matches the payout from the option if the stock price goes up.[7]

Now let's look at how this works more generally. First, suppose the stock price goes up. Then a fraction $\boldsymbol{h}$ of the option-like portfolio will be worth $\boldsymbol{h} \times (P^u - P^d) = P^u - E$. This is exactly what the call option is worth if the stock price goes up. What if the stock price goes down? Now, a fraction $\boldsymbol{h}$ of the option-like portfolio is worth $\boldsymbol{h} \times 0 = 0$, which is also what the call option is worth if the stock price goes down.

We have seen how to construct a replicating portfolio for the call option. Now, to determine the price that one should pay for this option, we only need to determine the cost of the replicating portfolio. We've done two things: bought $\boldsymbol{h}$ shares of stock at the current price, P, and sold bonds with face value equal to $\boldsymbol{h} \times P^d$. Since the bonds are discount bonds, we receive today $\boldsymbol{h} \times P^d/(1+i)$ for selling the bonds. The total cost of the replicating portfolio is:

$$\textit{cost of replicating portfolio} = h \times \left(P - \frac{P^d}{1+i} \right) \tag{9-8a}$$

$$= \left(\frac{P^u - E}{P^u - P^d} \right) \times \left(P - \frac{P^d}{1+i} \right) . \tag{9-8b}$$

In our example the hedge ratio is $\boldsymbol{h} = 0.50$. Thus, using equation (9-8a), we find that the cost of the replicating portfolio is 0.50 x [50 - (.99x40)] = \$5.20. Since the replicating portfolio exactly reproduces the pattern of payouts of the call option, we have also determined that the price of the call option must be \$5.20. That is: *the assumed absence of arbitrage opportunities means that the cost of the replicating portfolio equals the price of the option.*

Table 9-7 summarizes the steps to constructing a portfolio that replicates the pattern of

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[7] The computation in the text instructs us to purchase half a share of stock for each call option on a single share of stock. What does it mean to buy half a share of stock? Of course this is impossible! However, the typical option transaction involves very large amounts of money, so the problem of fractional shares is not important.

payouts for a call option.

Table 9-7

A. The replicating portfolio for a call option: The general recipe

	COSTS TODAY ($)	PAYOFFS LATER ($)	
		If $P'=P^u>E$	If $P'=P^d<E$
A. Buy call	V_c	$P^u - E$	0
B. Replicating portfolio			
Buy stock	h P	$h\times P^u$	$h\times P^d$
Borrow $h\times P^d/(1+i)$ dollars (sell T-bills with face value $h\times P^d$)	$-h \times P^d/(1+i)$	$-h\times P^d$	$-h\times P^d$
Cash flows from replicating portfolio with hedge ratio $h = (P^u - E)/(P^u - P^d)$	$h\times (P-P^d/(1+i))$ $= (P^u - E)/(P^u - P^d) \times (P-P^d/(1+i))$	$h\times (P^u-P^d)$ $= P^u - E$	0

B. The replicating portfolio for a call option: Our example

	COSTS TODAY ($)	PAYOFFS LATER ($)	
		If $P'=P^u> E$	If $P'=P^d<E$
A. Buy call	$V_c = 5.20$	60- 50	0
B. Replicating portfolio			
Buy stock	$.5\times50 = 25.00$	$.5\times 60=30$	$.5\times 40=20$
Borrow: sell T-bills with face value $h\times P^d$	$-.5\times .99\times 40 = -19.80$	$-.5\times 40=-20$	$-.5\times 40=-20$
Cash flows to portfolio with $h=(P^u - E)/(P^u - P^d) =0.5$	25.00 - 19.80 = 5.20	$.5\times (60-40)=10$	0

The next subsection will give us another chance to see how the hedge ratio is used in practice and will also give us some insight into why it carries this unusual name.

Hedging a call writer's risk

A call writer takes on risk when he sells a call option: if the stock price, P', exceeds the exercise price, **E**, at the option's expiration, the call writer must pay out P'-**E** (see Figure 9-3). One way to protect against this undesirable outcome is to write a "covered call" as discussed in Section 9-6. This means that call writer buys some stock today, at a known price, in case the call is exercised against him at expiration. How much stock should he buy to fully neutralize the risk--to construct a perfect hedge against the risk of a price increase? It turns out that he must buy stock in the amount ***h***: the hedge ratio. To see how this works, suppose that the call writer buys ***h*** shares of stock and sells T-bills with a face value of $h \times P^d$. [8] His cash flows are as shown in Table 9-8.

[8] The sale of the T-bills generates cash. Together with the cash generated from writing the call, there is just enough cash to purchase the required ***h*** shares of stock.

Table 9-8

A. Cash flows from a hedged, covered call: The general recipe

	COSTS TODAY ($)	PAYOFFS LATER ($)	
		If $P'=P^u>E$	If $P'=P^d<E$
Write (Sell) call	$-V_c$	$-(P^u - E)$	0
Buy h shares stock	$h\times P$	$h\times P^u$	$h\times P^d$
Sell T Bills	$-h\times P^d/(1+i)$	$-h\times P^d$	$-h\times P^d$
PORTFOLIO	$-V_c + h\times P - h\times P^d/(1+i) = 0$	$-(P^u - E) + h\times(P^u - P^d) = 0$	0

B. Cash flows from a hedged, covered call: Our example

	COSTS TODAY ($)	PAYOFFS LATER ($)	
		If $P'=P^u>E$	If $P'=P^d<E$
Write (Sell) call	6	50-60	0
Buy h shares stock	$-0.5\times 50 = -25$	0.5×60	0.5×40
Sell T Bills	$0.5\times 40\times 0.95 = 19$	-0.5×40	-0.5×40
PORTFOLIO	$6 - 25 + 19 = 0$	$(50-60) + 0.5\times(60-40) = 0$	0

Using the value of $\boldsymbol{h}$ determined in the previous section, $\boldsymbol{h} = (P^u - E)/(P^u - P^d)$, we find that the call writer is fully hedged: he won't have to pay anything if the stock price goes up or down. He stands to make a certain profit if the selling price of the call, V_c, exceeds the cost of creating the replicating portfolio, which in this case was $5.20. This profit is certain because he has constructed a hedged portfolio, which protects him against increases in the price of the stock. However, in a competitive market, we'd expect that arbitrage profits won't exist (at least, not for very long) so that the call will be priced at $5.20. Thus, the call writer has zero profit.

We have seen that a call writer can produce an effectively riskless position if he simultaneously writes a call, buys the stock, and sells T-bills. The key ingredient of this riskless position is the hedge ratio, $\boldsymbol{h}$, that specifies how many shares of stock to buy per call option that is written.

9-11 Implications of the two-period binomial option pricing model

In the previous section we developed the two-period binomial option pricing model which determines the price of an option in a two-period setting with only two possible outcomes. Although this model is very simple it is nevertheless closely related to option-pricing models built on more realistic assumptions.

The first element shared by all modern option pricing models is the concept of a replicating portfolio. We saw that an option can be replicated by combining T-bills and the underlying stock in specific ways. We used the price of the replicating portfolio to determine the price of the option. This same logic carries over to richer settings in which there are more than two periods and more than two possible outcomes. The second element that the simple model shares with richer models of option pricing is the fact that the underlying determinants of option value are the same. That is: the features that make an option valuable in this simple setting will also contribute to value in more realistic environments.

Let's take another look at the binomial option pricing relationship, given below.

$$V_c = \textit{value of call option} = \left(\frac{P^u - E}{P^u - P^d}\right) \times \left(P - \frac{P^d}{1+i}\right)$$

This expression determines the value of a call option with exercise price **E**. The current price of the underlying stock is P and the stock's price may rise to P^u or it may fall to P^d. The interest rate on T-bills is denoted by i. As we discussed in section 9-7, each of these elements is important for the price of the option. The value of the call, V_c, rises with the current stock price, P; it falls with increases in the exercise price E, and increases with increases in the risk-free rate, i. The interest rate enters because it determines how much you receive today from selling T-bills with face value of E. The higher is the interest rate the lower the price you receive from selling these discount bonds, and so the higher is the cost of the portfolio that replicates the call option. These factors and their effects on V_c are summarized in Table 9-9.

Stock price volatility

In section 9-8 we discussed why increases in volatility should increase the value of options. Let's see whether this is true in the binomial option pricing model. In this simple model only two things can happen to the stock price: it can rise to P^u or it can fall to P^d. An increase in stock price volatility in this model means an increase in the range of possible outcomes, holding fixed what happens on average. Let's imagine that we increase volatility relative to our previous example by specifying that P^u = \$70.00 instead of P^u = \$60.00 and P^d = \$30.00 instead of P^d = \$40.00 as in the prior example. Let's see how this affects the value of the call option with exercise price **E** = \$50.00.

First, let's compute the hedge ratio. With the increased volatility, we have ***h*** = (\$70-\$50)/(\$70-\$30) = 0.50, which is the same as before. This is *not* a general result--typically, the hedge ratio would change with increased volatility. The increase in volatility means, however, that we need to sell fewer T-bills to construct the replicating portfolio: the face value of the necessary amount is ***h*** x P^d = 0.5 x \$30 = \$15, which is less than the earlier value of $h \times P^d = 0.5 \times \$40 = \$20$. This means that the cost of the replicating portfolio is $\boldsymbol{h} \times (P - P^d/(1+i)) = 0.5 \times (50 - 30 \times .99)) = \$10.15$. Absence of arbitrage means that the option's value is $V_c = \$10.15$, compared with only \$5.20 in our prior example. Increasing the range of possible values for the stock price--increasing its volatility--has lead to a near doubling of the value of the option. This is a general result in the binomial option pricing model: option values will always rise with increased volatility, although it is tedious to prove this mathematically. Although it's unusual for an asset's value to rise with volatility, for options volatility is a good thing. This is because higher stock price volatility is associated with the potential for larger stock price movements in a favorable direction. Since an option is a one-sided bet the option holder *likes* volatility: she wins if the stock price moves in a favorable direction, but she can tear up the option if it doesn't.

Table 9-9

Factors affecting call option value in the two-period binomial option pricing model

Factor	**Effect on Call Option Value**	**Reason**
Higher interest rate	Higher call price	NPV of bonds sold in replicating portfolio is lower: $\boldsymbol{h}$x(P^d/(1+i)) falls
Higher exercise price	Lower call price	Win by less if stock price goes up: P^u-E smaller
Higher stock price	Higher call price	Higher cost of stock in replicating portfolio: $\boldsymbol{h}$xP is larger
Higher stock price volatility	Higher call price	Higher expected return from option: get more if option is in-the-money at expiration only two periods
Time to expiration	does not appear	only two periods

What's missing?

One very surprising feature of the two-period binomial option pricing model is something that is not present: the probability that the price of the underlying stock moves in a favorable direction before the option expires. Intuitively, it seems as though the value of an option should depend on the probabilities attached to each possible outcome. The reason for the fact that probabilities don't enter is that we have created a replicating portfolio that is exact in that it matches the payouts of the option exactly no matter what happens. The probabilities of various events are already reflected in the prices of the assets comprising the replicating portfolio. This will turn out to be a very general feature of the option pricing formulas that we discuss in the rest of this chapter--the probability of changes in the price of the stock will never be important to the market valuation of the stock. It is important to stress, however, that this does not mean that one never cares about the probabilities--if you were just purchasing a call option (instead of constructing a replicating portfolio) these probabilities would be very important for what you would expect to gain from purchasing the call.

The one important determinant of option value that is not captured in the two-period model is the time value of an option. Real-world options last for periods ranging from one day through nine months--we have seen that options with more time to expiration have higher values. The next section introduces the Black-Scholes option pricing model which allows for multiple periods to an option's expiration and allows more than two possible outcomes. The Black-Scholes model is considered by many to be the crowning glory of modern finance because it is theoretically elegant, highly intuitive and--to a surprising extent--it works.

9-12 The Black-Scholes option pricing model

The binomial option pricing model assumed that there were only two periods and two possible outcomes. What happens to the option pricing formula if we relax these extreme assumptions? In 1973, Fischer Black and Myron Scholes developed an option pricing model that is much more general. Box 9-3 presents the story behind this remarkable discovery. Black and Scholes showed how to value a call option on a stock in a situation in which the length of a time period grows arbitrarily short. Think about a period first as three months, as in our earlier example, and then imagine that it shrinks to a week, then a day, then an hour, then a nanosecond. Even if there were only two possibilities for the stock price at each instant of time, say "price rises by $1" or "price falls by $1," with very short time intervals there are a wide range of potential outcomes at the end of the year. In the limit, as the time period becomes infinitesimally short, there will be an infinite number of these infinitely small periods in a year, and so there will be an infinite number of possible outcomes for the stock price at the end of the year.

Before putting the Black-Scholes option pricing model to work, we need to know more about the assumptions upon which this model builds. First, the Black-Scholes model assumes that the risk-free rate of interest is constant over the life of the option. Second, the model assumes that the returns to the underlying asset follow a particular statistical process: the returns are assumed to be normally distributed with a constant mean over time. The Black-

Scholes model also assumes that no dividends will be paid on the stock over the option's life. Finally, the Black-Scholes model gives the value for a European option, not an American option. However, since time value of an option means that it is never optimal to kill a live option on a non-dividend-paying stock this last consideration is of little practical importance.

The Black-Scholes formula for the value of a call option, V_c, is given by the following equation:

$$V_c = P \times N(D_1) - E \times N(D_2) \times e^{-RT} \tag{9-11}$$

In equation (9-11), the notation is as follows:

P: current price of the underlying asset (e.g., a share of stock)

E: the exercise price of the call option

T: the option's time to expiration, expressed as a fraction of a year

R: the continuously-compounded, annual risk-free interest rate

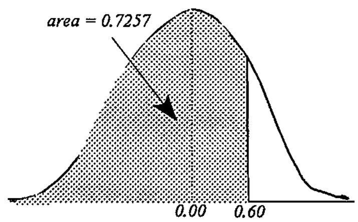
σ: the standard deviation of the continuously compounded annual rate of return on the underlying asset

$$D_1 = \frac{\ln(P/E) + (R + .5\sigma^2)T}{\sigma\sqrt{T}}$$

$$D_2 = \frac{\ln(P/E) + (R - .5\sigma^2)T}{\sigma\sqrt{T}} = D_1 - \sigma\sqrt{T}$$

The expressions $N(D_1)$ and $N(D_2)$ denote the cumulative distribution functions for a normal random variable, evaluated at D_1 and D_2, respectively. These cumulative distribution functions give the probability of an outcome less than D_1 or D_2. As you may know, the normal distribution is the familiar "bell curve" sketched in Figure 9-9. The area under a standard normal distribution (one with mean zero and standard deviation equal to 1) below a given point is the probability that a random draw from the distribution will have value less than or equal to this point. In Figure 9-10, for example, the probability that a random draw from this standard normal distribution will be less than or equal to 0.60 is the shaded area--this probability is 0.7257.

Figure 9-10: The standard normal distribution



Fortunately, it is not necessary to understand the derivation of the Black-Scholes formula in order to use it: most option traders don't. The ingredients needed to make use of the formula are given in the list above. The only component that is not readily available is σ: the standard deviation of the return on the underlying asset. One commonly-used approach to obtaining an estimate of σ is to compute the standard deviation of the returns on the underlying asset over a recent period of time, and then use this as the forecast of the underlying asset's σ over the life of the option.

Box 9-3: THE STORY BEHIND THE BLACK-SCHOLES OPTION PRICING FORMULA

Photo of
Fischer Black

Fischer Black describes how he and Myron Scholes came up with the option pricing formula:*

"Jack Treynor was at Arthur D. Little, Inc., when I started work there in 1965. He had developed, starting in 1961, a model for the pricing of securities and other assets that is now called the 'capital asset pricing model.'...

"Jack sparked my interest in finance, and I began to spend more and more time studying the capital asset pricing model and other theories of finance. The notion of equilibrium in the market for risky assets had great beauty for me. It implies that riskier securities must have higher expected returns, or investors will not hold them--except that investors do not count the part of the risk that they can diversify away.

" I started trying to apply the capital asset pricing model to assets other than common stock. One of Treynor's papers was on the valuation of cash flows within a company, and he had derived a differential equation to help in figuring this value. ...

*"With this background, I started working on a formula for valuing a warrant.*** *...The equation I wrote down said simply that the expected return on a warrant should depend on the risk of the warrant in the same way that a common stock's expected return depends on its risk. I applied the capital asset pricing model to every moment in a warrant's life, for every possible stock price and warrant value....This gave me a differential equation....*

"I spent many, many days trying to find the solution to that equation. I have a Ph.D. in applied mathematics, but had never spent much time on differential equations, so I didn't know the standard methods used to solve problems like that. I have an A.B. in physics, but I didn't recognize the equation as a version of the "heat equation." which has well-known solutions. ...

"So I put the problem aside and worked at other things. In 1969, Myron Scholes was at MIT, and I had my office near Boston, where I did both research and consulting. Myron invited me to join him in some of the research activities at MIT. We started working together on the option problem, and made rapid progress. ...The final draft of the paper (dated May 1972) as called "The Pricing of Options and Corporate Liabilities." It appeared in the May/June 1973 issue of the <u>*Journal of Political Economy*</u>*."*

* This material is excerpted from Fischer Black's article "How we came up with the option pricing formula," which appeared in *The Journal of Portfolio Management*," Winter 1989, pages 4-8.

** A warrant is very much like an option, except that it is issued by the firm that has issued the underlying stock. Further, warrants may not have expiration dates.

Let's put the Black-Scholes model to use. Suppose that we want to determine the value of a call option with six months to expiration and an exercise price of \$40.00. The current price of the underlying security is \$25.00, and the risk-free interest rate (continuously compounded) is R=0.05, or 5%. We estimate that the σ of the underlying security is $\sigma = 1.0$. Inserting these numbers into the expressions for D_1 and D_2, we have:

$$D_1 = \frac{\ln(25/40) + (0.05+0.50\times 1)\times 0.50}{1\times\sqrt{0.50}} = -0.2758$$

$$D_2 = D_1 - 1\times\sqrt{0.50} = -0.9829.$$

Next, we need to evaluate $N(D_1)$ and $N(D_2)$. This is done with a calculator that has statistical functions; or a computer program; or (as in the old days) with the help of a table like Table 9-10 which gives the cumulative distribution function for a standard normal random variable. For each value of D given in the table, the value of N(D) is given alongside. The exact values we obtained for D_1 and D_2 are not given in the table, but we can compute approximations to $N(D_1)$ and $N(D_2)$ by interpolating between values given in the table.[9] We will also need to use the fact that N(x) = 1-N(x).

For example, we have D_1 = -0.2758. The table lacks entries for this value, but it does

[9]Linear interpolation between -0.25 and -0.30 is the value given by a straight line between N(-0.25) and N(-0.30) evaluated at -0.2758. Algebraically, this is given by

$$N(-0.2758) \approx 0.3821 + \left(\frac{0.3000-0.2758}{0.3000-0.2500}\right)\times(0.4013-0.3821) = 0.3914.$$

If more precise values are necessary, these can be obtained from financial calculators or specialized computer programs.

Table 9-10

The cumulative distribution function for a normal random variable

$$\Pr(X \le x) = N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$[N(-x) = 1 - N(x)]$$

x	N(x)	x	N(x)	x	N(x)
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.960	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999

have entries for 0.025 and 0.39. The table gives us N(-0.2500) = 1-N(0.2500) = 1-0.599 = 0.401, and N(-0.300) = 1-N(0.300) = 1-0.618 = 0.382. We know that the value for N(-0.2758) must lie between these two values. Using interpolation, we obtain N(-0.2758)=0.391. By the same approach, we find that $N(D_2) = 0.1664$.

Now we are ready to compute the Black-Scholes model's prediction for the price of this call option, by inserting these numbers into equation (9-11):

$$V_c = \$25 \times 0.3914 - \$40 \times 0.1664 \times e^{-0.05 \times 0.5} = \$3.29.$$

The Black-Scholes formula give a value for this call option of $3.29. Notice that the entire amount is attributable to the time value of the option, since this call option is out-of-the-money: the exercise price is $40.00 while the current price of the underlying asset is only $25.00.

Determinants of option values in the Black-Scholes model

The Black-Scholes model highlights five factors as the determinants of option prices--these are the same factors discussed in Table 9-9 in connection with the two-period binomial option pricing model. The direction of the effect of changes in each of the first four factors is the same in the Black-Scholes model as in the binomial model. Specifically, call values are increased by (i) increases in the stock price; (ii) decreases in the exercise price; (c) higher stock price volatility; and (iv) higher interest rates. The important addition of the Black-Scholes model is time value: in this option pricing model, the value of an option increases with time to expiration.

Comparison with the binomial option pricing model

You may be wondering about the relationship between the Black-Scholes option pricing model and the two-period binomial option pricing model we developed in Section 9-9. In fact, there is a very close relationship, as we can see if we write down and compare the two option pricing formulae:

$$V_c = h \times P - h \times \frac{E}{1+i}$$ **binomial OPM**

$$V_c = P \times N(D_1) - E \times N(D_2) \times e^{-RT}$$ **Black-Scholes OPM**

Let's compare the first term on the right-hand-side of each equation. In the two-period binomial option pricing model this term is $h \times P$--the hedge ratio times the price of the underlying security. In the Black-Scholes model the first term is $P \times N(D_1)$, suggesting that $N(D_1)$ may be interpretable as a hedge ratio. In fact, it is exactly a hedge ratio--it is the number of shares of the underlying asset that an investor would purchase in constructing a portfolio to exactly replicate the multi-period call option priced by the Black-Scholes formula.

Now let's look at the second term on the right-hand-side of each equation. In the two-period binomial model this term is $-h \times E \times (1+i)^{-1}$, compared with $-E \times N(D_2) \times e^{-RT}$ in the Black-Scholes model. Remember that this term in the two-period model has the interpretation as the current value of the bonds that must be sold in constructing the replicating portfolio for a call option. The face value of the bonds is $h \times E$, and the present discounted value of these bonds is $-h \times E \times (1+i)^{-1}$. There is an analogous interpretation in the Black-Scholes model: the face value of the bonds sold as part of the replicating portfolio is $E \times N(D_2)$, and the present discounted value, or current price, of these bonds with continuous compounding is $E \times N(D_2) \times e^{-RT}$.

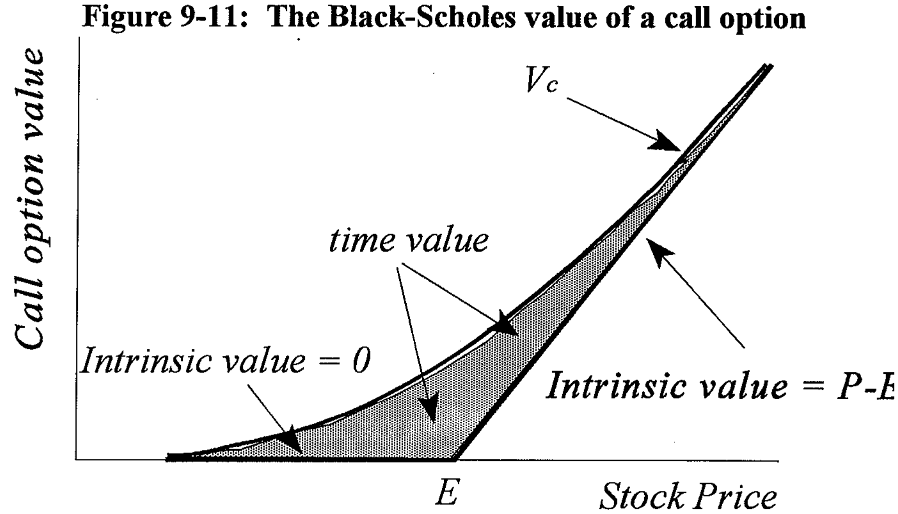
Although the Black-Scholes formula looks daunting, we see that it has the same intuitive form as the two-period binomial option pricing formula presented earlier. The reason is that the logic behind the two models is the same. In each case, the call option is priced by finding the value of a replicating portfolio for the call option: this replicating portfolio consists of a long position in the underlying security together with a short position in (the sale of) risk-free bonds.

9-13 The hedge ratio and delta hedging

The hedge ratio plays a central role in option pricing models. We learned in section 9-10 that the term "hedge ratio" refers to the fact that it gives the number of units of the underlying security that must be purchased as part of the replicating portfolio for the call option. When we studied how a call writer can hedge the risk associated with a short position in a call option, we learned that the hedge involved purchasing shares of the underlying security in an amount equal to the hedge ratio. In this section we'll take a closer look at the relationship between the hedge ratio and the values of call options and the underlying security. We'll also learn about one of the most widely used hedging strategies--delta hedging.

Figure 9-11 shows the Black-Scholes prediction for call option value as it relates to the price of the underlying security. The intrinsic value of the call option is also shown on the graph: remember that the intrinsic value of a call is zero if the price of the underlying security is below the option's exercise price, and is otherwise equal to the difference between the price of the underlying security and the exercise price. The Black-Scholes value of a call option is the line labeled V_c and the time value of the option is indicated by the shaded area.

Figure 9-11: The Black-Scholes value of a call option



The hedge ratio of the call option, $N(D_1)$, for a given stock price, is given by the slope of the Black-Scholes value line at that price. This slope--the hedge ratio--gives the change in the value of the call option as a percentage of the change in the stock's price. For very low stock prices, given the exercise price, the slope of the Black-Scholes value line is nearly zero. That is: the call option is very deep out-of-the-money at this point, and the value of the call option will change very little with small changes in the price of the stock. For very high stock prices--when the option is deep in-the-money--the value of the option is primarily intrinsic value, and intrinsic value changes one-for-one with changes in the stock price. So we see that the hedge ratio--the slope of the curve V_c-- is nearly zero for low stock prices and rises to a maximum of 1 for very high stock prices.

Delta hedging is the term given to the construction of a risk-free combination of a call option, shares of the underlying security, and risk-free bonds. The call writer who hedged his risk in section 9-9 was engaged in delta hedging. The term itself comes from the fact that the Greek letter delta (Δ) is commonly used in mathematics and the sciences to stand for the *change* in something. Here, the hedge ratio gives the change in the dollar value of an option for a \$1 change in the value of the underlying security:

$$h = \frac{\Delta V_c}{\Delta P} .$$

To perfectly insure the hedger against risk, the delta hedge must be adjusted very frequently. To see why, look at Figure 9-12. The hedge ratio is given by the slope of the V_c curve at the current stock price. Since stock prices change almost every instant that the markets are open, the hedge ratio also changes almost continuously. In practice, of course, it is not practical to adjust hedges at every instant of time--transactions costs would make this a very costly practice. But individuals and firms who use delta hedging must adjust their hedge ratios whenever movements in the value of the underlying security mean that there has been a significant change in the appropriate hedge ratio.

Summary

1. An option on a financial asset confers the right to buy or sell the underlying financial asset at the option's exercise price on or before the option's expiration date. A call option entails the right to purchase the underlying security; a put option entails the right to sell the underlying security.

2. Graphs of the option's value at expiration are called "hockey stick" diagrams because the graph has a kink which occurs at the option's exercise price. The kink arises because the option has zero value at expiration if it is out of the money. If an option expires in the money, the value of a long position is equal to the option's intrinsic value; the value of a short position is equal to the negative of the option's intrinsic value.

3. The following variables are important determinants of an option's value: exercise price; time to expiration; price of the underlying asset; volatility of the price of the underlying asset; the risk-free interest rate. For underlying assets that pay dividends, such as many common stocks, the dividend will affect the option price through the dividend's influence on the stock price.

4. Option pricing models use a replicating portfolio approach to pricing the option. The first step is to construct a portfolio of assets that duplicates the cash flows from the option position. The next step is to determine the price of this replicating portfolio. Since we assume that there are no unexploited arbitrage opportunities, the price of the option must be equal to the price of the replicating portfolio. The Black-Scholes option pricing model can be viewed as a generalization of the binomial option pricing model in which there are many time periods and the length of each time period grows arbitrarily short.

Key concepts

American option
at the money
call option
covered call
default
default option
delta hedging
European option
exercise (an option)
exercise price
expiration date
hedge ratio
in the money
index option
intrinsic value
long position
leaps
listed option
option
option holder
option premium
option writer
out of the money
over the counter market
prepayment option
put option
put-call parity
short position
strike price
stock index
time value

Questions for review

1. Think of three examples of call options and two examples of put options that occur in everyday life. Explain why each is an option, and describe the underlying security (which may be an event) in each case. Is it easier to think of examples of call options or put options? Why do you think this is the case?

2. List options that appear in bank assets and bank liabilities. To find others that we may not have covered in this chapter, you may want to refer to the discussion of the bank balance sheet in Chapter 1.

3. Describe the main differences between an index option and a stock option. What do you get if you exercise a stock option? What do you get if you exercise an index option?

4. Describe two ways that options can be used to protect the owner of a financial asset against adverse movements in the price of the asset. Describe the pros and cons of each strategy.

5. Explain the basic logic of arbitrage pricing as it applies to financial assets. Explain how arbitrage pricing was used in developing (i) the put-call parity condition; and (ii) the binomial option pricing model.

6. Explain why options are considered risky investments.

7. What are the important conceptual differences between the two-period binomial option pricing model and the Black-Scholes option pricing model? What are the important similarities?

8. What is delta hedging? What are the practical difficulties that arise in maintaining a delta hedge?

Problems and applications

1. Suppose that TJ stock is currently selling for **P**=$80.00 per share on the New York Stock Exchange, and there are listed options on TJ stock on the Chicago Board of Trade. You are considering purchasing (establishing a long position in) a call option on TJ stock with exercise price of **E**=$85.00. The current price of the option is $6.00.

   (a) Is this option in the money or out of the money?

   (b) What is the intrinsic value of this option?

   (c) What is the time value of this option?

   (d) Draw the graph at the option's expiration of this option position. Label the graph carefully.

(e) Re-draw the graph under the assumption that you have established a *short* position in the put option. What is the relationship between this graph and the one you drew in part (d)?

2. As in problem 1, TJ stock is currently selling for **P**=\$80.00. Now, you are considering purchasing (establishing a long position in) a put option on TJ stock with exercise price of **E**=\$90.00. The current price of the option is \$0.75.

(a) Is this option in the money or out of the money?

(b) What is the intrinsic value of this option?

(c) What is the time value of this option?

(d) Draw the graph at the option's expiration of this option position. Label the graph carefully.

(e) Re-draw the graph under the assumption that you have established a *short* position in the put option. What is the relationship between this graph and the one you drew in part (d)?

3. As in problem 1, TJ stock is currently selling for **P**=\$80.00 per share and a call option on TJ stock with exercise price of **E**=\$85.00 is V_c=\$6.00. The interest rate over the period ending with the option's expiration is i=0.01. Use put-call-parity to determine the value of a put with the same exercise, **E**=\$85.00, and the same expiration date.

4. The price of CavSports stock is currently **P**=\$30.00 per share. Consider a call option on CavSports stock with exercise price **E**=\$32.00 per share. The option will expire in six months, and the six-month T-bill rate is i=0.03. Over this six-month period, the stock price may rise to $\mathbf{P}^u$ = \$38.00, or it may fall to $\mathbf{P}^d$ = \$26.00. We will use the two-period binomial option pricing model to determine the value of this call option.

(a) Is this call option in the money or out of the money? What is its intrinsic value?

(b) What is the value of the hedge ratio, ***h***, relevant for this problem?

(c) Determine the replicating portfolio for this call option.

(d) Use the two-period binomial option pricing model to determine the value of this call option.

(e) Re-do the analysis assuming that the exercise price of the call option is **E**=\$28.00.

5. The price of Centel stock is currently \$110 per share. We will use the Black-Scholes option pricing model to determine the value of a call option on Centel stock that has exercise price **E**=\$115 per share and six months to expiration. The annualized interest rate for this six-month period is **R**=0.06. The volatility of the stock price is σ=0.20.

(a) Find the value of the call option using the Black-Scholes option pricing model. You will need to evaluate the CDF of the normal distribution at specific points that you will calculate. You can either use the table given in the text, or your calculator may have this function. Alternatively, many spreadsheet programs have this function available.

(b) Use put-call parity to determine the value of a put option with exercise price **E**=\$115 per share and six months to expiration. (Don't forget to adjust the interest rate from an annual rate to a six-month rate for use in the put-call-parity relationship).