BOSTON UNIVERSITY Global Development Policy Center
Economics in Context Initiative

Chapter 8: Macroeconomic Measurement

Appendix

Appendix to Chapter 8 of Essentials of Economics in Context, Second Edition

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APPENDIX: CHAINED DOLLAR REAL GDP

The key new concept in the "chained-dollar" method is an emphasis on estimating **quantity indexes** for GDP in the current year relative to the year before and relative to the reference year.

quantity index: an index measuring changes in levels of quantities produced

Chained-dollar measures of real GDP and GDP growth are based on the use of index numbers. The ratio of two values of GDP in adjacent years, measured at a common set of prices, can be used as a quantity index to measure production in one year relative to another.

The calculation of chained-dollar real GDP starts with the calculation of a **Fisher quantity index**, which measures production in one year relative to an adjacent year by using an *average* of the ratios that would be found by using first one year and then the other as the source of prices at which production is valued. The type of average used is a "geometric" average. Instead of adding two numbers and then dividing by two, as you would in calculating the most common type of average (the arithmetic mean), to get a geometric average you *multiply* the two numbers together and then take the *square root.* The formula for this Fisher quantity index is:

Fisher quantity index (*for year-to-year comparison*)

$$= \sqrt{\left(\frac{Year\ 2\ GDP\ in\ Year\ 1\ prices}{Year\ 1\ GDP\ in\ Year\ 1\ prices}\right) \times \left(\frac{Year\ 2\ GDP\ in\ Year\ 2\ prices}{Year\ 1\ GDP\ in\ Year\ 2\ prices}\right)}$$

Fisher quantity index: an index that measures production in one year relative to an adjacent year by using an average of the ratios that would be found by using first one year and then the other as the source of prices at which production is valued

This index has a value of 1 in the reference year, which we take to be Year 1.

The growth rate of real GDP between the reference year and the next year can then be calculated as:

$$growth\ rate = (Fisher\ quantity\ index - 1) \times 100$$

For example, let's take the "apples-and-oranges" economy presented in Table 8A.1 below (note that this is same as Table 8.2 in the textbook). We have the prices and quantities sold of apples and oranges in columns 2 and 3 of the table respectively. Column 4 presents the real GDP for each year in Year 1 prices, and column 5 presents the real GDP for each year in Year 2 prices.

Table 8A.1 Calculation of Constant-dollar real GDP using different base years

(1)	(2)	(3)	(4)	(5)
Description	Price per pound ($)	Quantity (pounds)	Real GDP in Year 1 prices *[prices from Year 1 × Column (3)] ($)*	*Real GDP in Year 2 prices [prices from Year 2 × Column (3)] ($)*
Year 1				
apples	$1.00	100	$1*100 = $100	$1.50*100 = $150
oranges	$2.00	50	$2 * 50 = $100	$2.00*50 = $100
			Total Real GDP = $200	Total Real GDP = $250
Year 2				
apples	$1.50	100	$1 * 100 = $100	$1.50*100 = $150
oranges	$2.00	75	$2 * 75 = $150	$2.00*75 = $150
			Total Real GDP = $250	Total Real GDP = $300

Plugging these in, we get

Fisher quantity index (for Year 2 *compared to Year* 1*)*

$$= \sqrt{\left(\frac{250}{200}\right) \times \left(\frac{300}{250}\right)} = \sqrt{1.25 \times 1.20} = \sqrt{1.5} = 1.225$$

The growth rate of real GDP for the "apples-and-oranges" economy between these two years is

$$growth\ rate = (1.225 - 1) \times 100 = 22.5\ percent$$

Note that this growth rate is *between* the two growth rates (20 percent and 25 percent) we obtained by using the constant-dollar method with various base years. The Fisher quantity index method gives us a unique *average* number for estimated growth.

A quantity index for the current year in terms of a reference year that may be several years in the past is created by "chaining together" year-to-year Fisher quantity indexes to make a **chain-type quantity index** comparing real production relative to the reference year. The chain-type quantity index has a value of 100 in the reference year. In any subsequent year, it is set equal to the chain-type quantity index from the previous year multiplied by the Fisher quantity index calculated for the current year.

chain-type quantity index: an index comparing real production in the current year to the reference year, calculated using a series of year-to-year Fisher quantity indexes

Finally, estimation of real GDP in (chained) dollar terms is made by multiplying the chain-type quantity index for a year times the level of nominal GDP in the reference year and dividing by 100.

For example, suppose that we take our "apples-and-oranges" economy, making Year 1 the reference year. Year 1's chain-type quantity index is thus set equal to 100, and its nominal and real GDP are equal. These are shown in Table 8A.2. The chain-type quantity index for Year 2 is the previous year's value (100) times the Fisher quantity index that we just calculated

(1.225). We multiply this result, the new index number 122.5, times nominal GDP in the base year ($200) and divide by 100 to get real GDP, $245. Whew!

Table 8A.2 Deriving Real GDP in Chained (Year 1) Dollars

Type of measure	Year 1	Year 2
Nominal GDP	$200	$300
Fisher quantity index (current to previous year)	---	1.225
Chain-type quantity index	100	100 × 1.225 = 122.5
Real GDP (chained Year 1 dollars)	= $200	(122.5 × $200)/100 = $245

This can be continued for many years into the future—or into the past. (For example, if the Fisher quantity index calculated for Year 3 were to come out to be 1.152, then the chain-type quantity index for Year 3 would be 122.5 × 1.152.) If you want to check to see that this method actually makes some sense, calculate the percentage change in real GDP from Year 1 to Year 2 using the values in the table above. You will find it does, in fact, equal 22.5 percent!

The new method has some other drawbacks, as well. The sum of real components of GDP in chained-dollar terms do not generally exactly add up to real GDP. Users of the data are also warned not to make comparisons of chained dollar amounts for years far away from the reference year. The BEA tries to make the data more usable by providing tables in which, for example, year to year growth rates in components of GDP are already calculated for the user.