BOSTON UNIVERSITY Global Development Policy Center
Economics in Context Initiative

Chapter 10: Fiscal Policy

Appendices

Appendices to Chapter 10 of *Essentials of Economics in Context*

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APPENDIX A: AN ALGEBRAIC APPROACH TO THE MULTIPLIER, WITH A LUMP-SUM TAX

A lump-sum tax is a tax that is simply levied on an economy as a flat amount. This amount does not change with the level of income. Suppose that a lump-sum tax is levied in an economy with a government (but no foreign sector). Consumption in this economy is:

$$C = \bar{C} + mpc\, Y_d$$

(the consumption function from Chapter 9, but using after-tax or disposable income in the formula). Since disposable income is:

$$Y_d = Y - \bar{T} + TR$$

we can write the consumption function as:

$$C = \bar{C} + mpc\,(Y - \bar{T} + TR)$$

Thus aggregate expenditure in this economy can be expressed as:

$$\begin{aligned} AE &= C + I + G \\ &= \bar{C} + mpc\,(Y - \bar{T} + TR) + I + G \\ &= (\bar{C} - mpc\,\bar{T} + mpc\,TR + I + G) + mpc\,Y \end{aligned}$$

By substituting this into the equation for the equilibrium condition, $Y = AE$, we can derive an expression for equilibrium income in terms of all the other variables in the model:

$$\begin{aligned} &\mathrm{Y} = (\bar{C} - mpc\,\bar{T} + mpc\,TR + I + G) + mpc\,Y \\ &Y - mpc\,Y = (\bar{C} - mpc\,\bar{T} + mpc\,TR + I + G) \\ &(1-mpc)\,\mathrm{Y} = (\bar{C} - mpc\,\bar{T} + mpc\,TR + I + G) \\ &Y = \frac{1}{(1 - mpc)}(\bar{C} - mpc\,\bar{T} + mpc\,TR + I + G) \end{aligned}$$

If autonomous consumption, investment, or government spending change, these each increase equilibrium income by *mult = 1/(1 – mpc)* times the amount of the original change. If the level of lump-sum taxes or transfers changes, these change Y by either negative or positive *(mult)(mpc)* times the amount of the original change.

To see this explicitly, consider the changes that would come about in Y if there were a change in the level of the lump sum tax from T_0 to a new level, T_1, if everything else stays the same. We can solve for the change in Y by subtracting the old equation from the new one:

$$Y_1 = \frac{1}{(1-mpc)}(\bar{C} + I + G - mpc\,\bar{T}_1 + mpc\,TR)$$
$$Y_0 = \frac{1}{(1-mpc)}(\bar{C} + I + G - mpc\,\bar{T}_0 + mpc\,TR)$$
$$Y_1 - Y_0 = \frac{1}{(1-mpc)}(\bar{C} - \bar{C} + I - I + G - G - mpc\,\bar{T}_1 + mpc\,\bar{T}_0 + mpc\,TR - mpc\,TR)$$

But $\bar{C}$, *I, G, TR* (and the *mpc*) are all unchanged, so most of the subtractions in parentheses come out to be 0. We are left with (taking the negative sign out in front):

$$Y_1 - Y_0 = -\frac{1}{(1-mpc)}mpc\,(\bar{T}_1 - \bar{T}_0)$$

or

$$\Delta Y = -(mult)(mpc)\Delta\bar{T}$$

As explained in the text, the multiplier for a change in taxes is smaller than the multiplier for a change in government spending, because taxation affects aggregate expenditure only to the extent that people *spend* their tax cut or pay their increased taxes by reducing *consumption*. Because people may also *save* part of their tax cut or pay part of their increased taxes out of their *savings*, not all the changes in taxes will carry over to changes in aggregate expenditure. The tax multiplier has a negative sign, since a *de*crease in taxes *in*creases consumption, aggregate expenditure, and income, while a tax increase decreases them.

APPENDIX B: AN ALGEBRAIC APPROACH TO THE MULTIPLIER, WITH A PROPORTIONAL TAX

With a proportional tax, total tax revenues are not set at a fixed level of revenues, as was the case with a lump sum tax but, rather, are a fixed *proportion* of total income. That is, $T = tY$ where t is the tax rate. The equation for *AE* becomes

$$AE = \bar{C} + mpc\,(Y - tY + TR)\,I + G$$
$$= \bar{C} + mpc\,TR + I + G) + mpc\,(Y - tY)$$
$$= (\bar{C} + mpc\,TR + I + G) + mpc\,(1 - t)\,Y$$

Substituting in the equilibrium condition, $Y = AE$, and solving yields:

$$Y = (\bar{C} + mpc\,TR + I + G) + mpc\,(1 - t)\,Y$$
$$Y - mpc\,(1 - t)\,Y = \bar{C} + mpc\,TR + I + G$$

$$(1 - mpc\,(1 - t))\,Y = \bar{C} + mpc\,TR + I + G$$

$$Y = \left[\frac{1}{1 - mpc(1 - t)}\right](\bar{C} + mpc\,TR + I + G)$$

The term in brackets is a new multiplier, for the case of a proportional tax. It is smaller than the basic (no proportional taxation) multiplier, reflecting the fact that now any change in spending has smaller feedback effects through consumption. (Some of the change in income "leaks" into taxes.) For example, if $mpc = 0.8$ and $t = 0.2$, then the new multiplier is 1/(1 – 0.64), or approximately 2.8, compared to the simple model multiplier 1/(1 – 0.8), which is 5. Changes in autonomous consumption or investment (or government spending or transfers) now have less of an effect on equilibrium income—the "automatic stabilizer" effect mentioned in the text.

Is there a multiplier for the tax rate, t? That is, could we derive from the model a formula for how much equilibrium income should change with a change in the rate (rather than level) of taxes? For example, if the tax rate were to decrease from 0.2 to 0.15, could we calculate the size of the change from Y_0 to Y_1 illustrated in Figure 10.7? Yes, but deriving a general formula for a multiplier relating the change in Y to the change in the tax rate requires the use of calculus, which we will not pursue here. (If you are familiar with calculus, you can use the last formula above to calculate the change in Y resulting from a change in t).