
# Outline

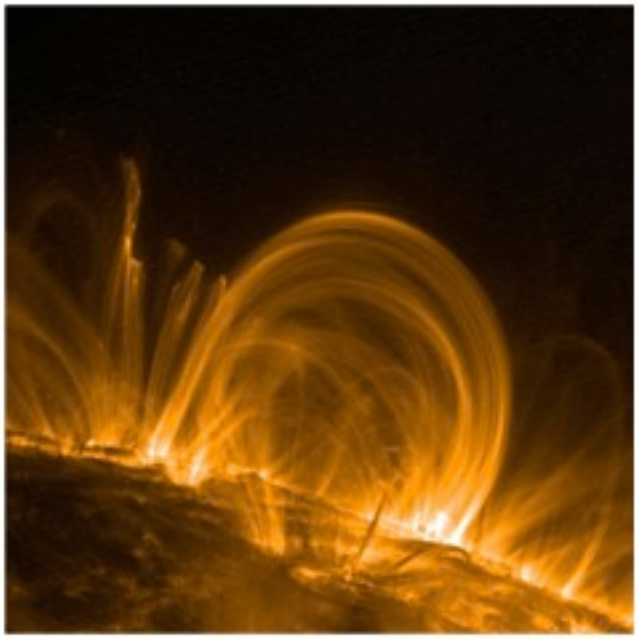
- Some motivating data
- Langmuir waves
- Ion-acoustic waves
- Alfvén waves

To fill in theoretical gaps:

*Plasma Physics* by Dwight Nicolls (1983)

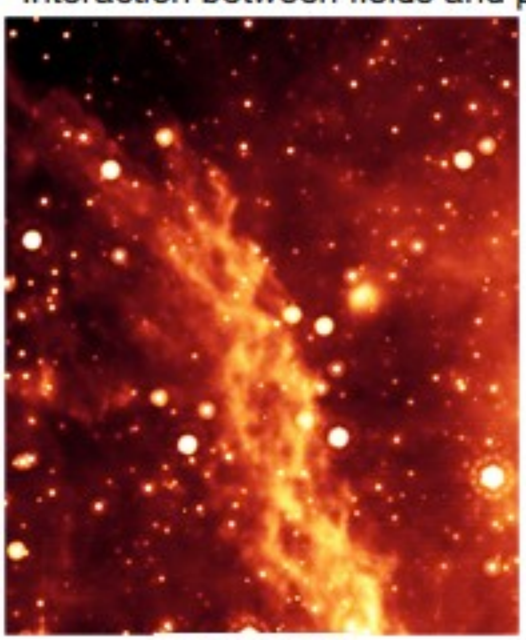
# Some universal themes

- Field structure illuminated by kinetic processes

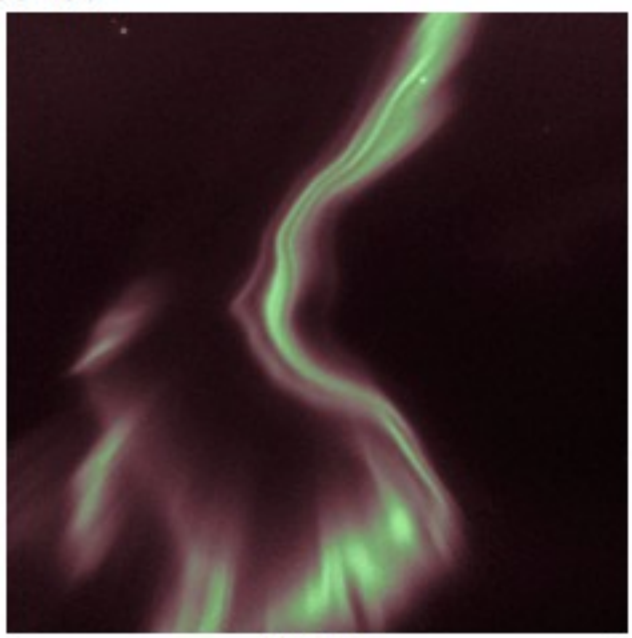


# Some universal themes

- Field structure illuminated by kinetic processes
- Interaction between fields and plasmas



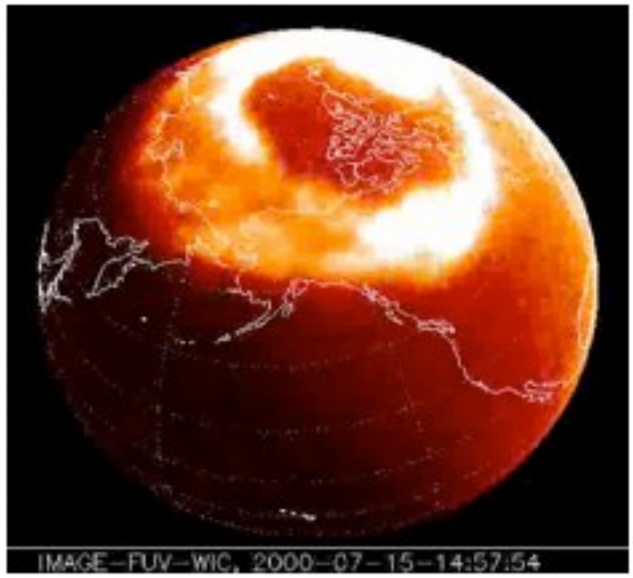
← 80 light years →



← 15 km →

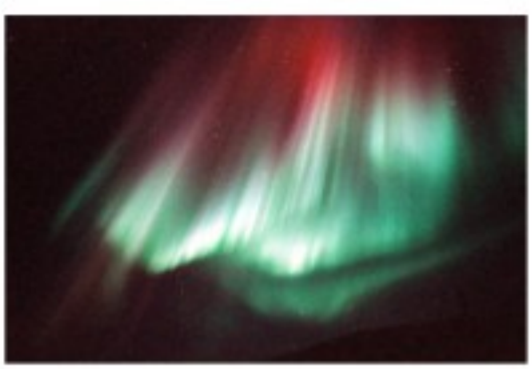
# Some universal themes

- Field structure illuminated by kinetic processes
- Interaction between fields and plasmas
- Explosive release of magnetic energy




# Some universal themes

- Field structure illuminated by kinetic processes
- Interaction between fields and plasmas
- Explosive release of magnetic energy
- Filamentation



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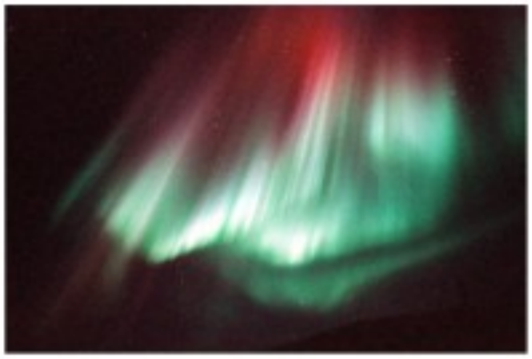
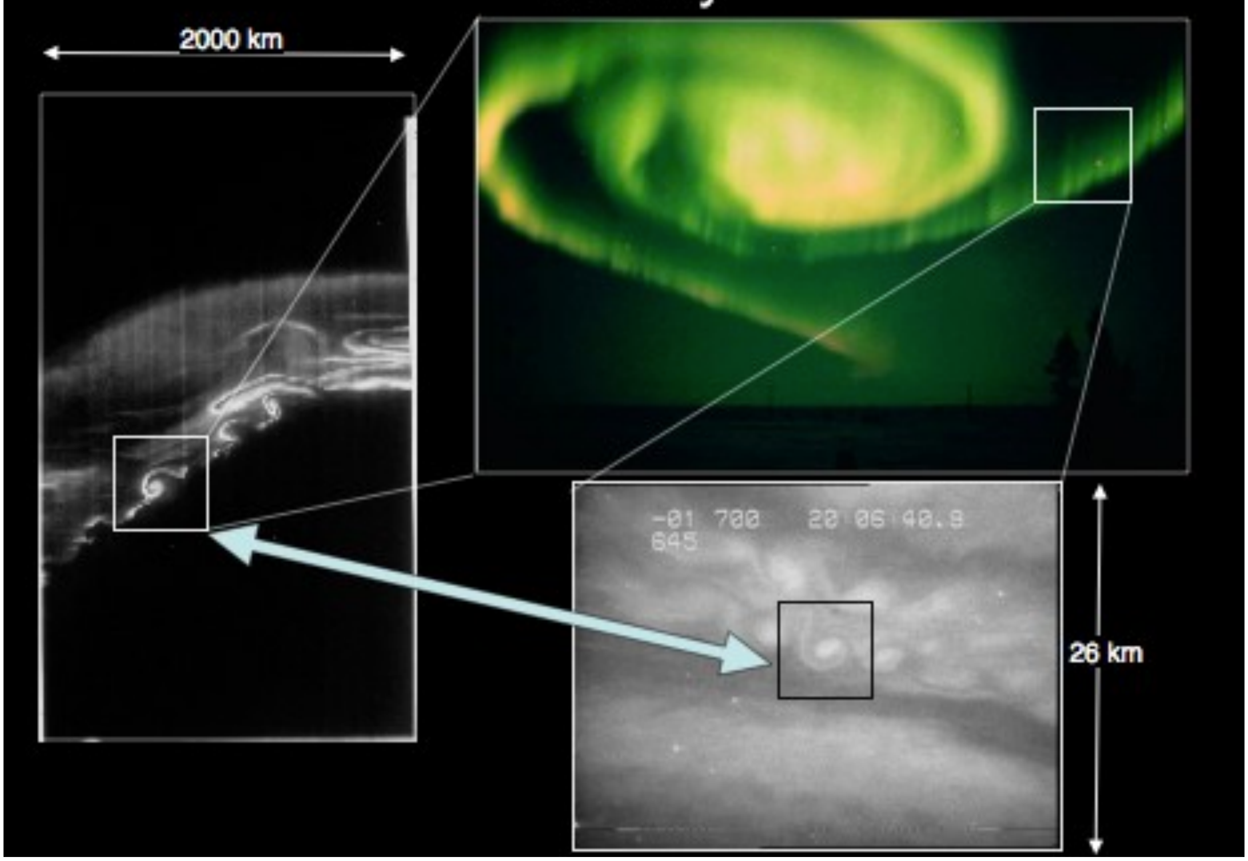
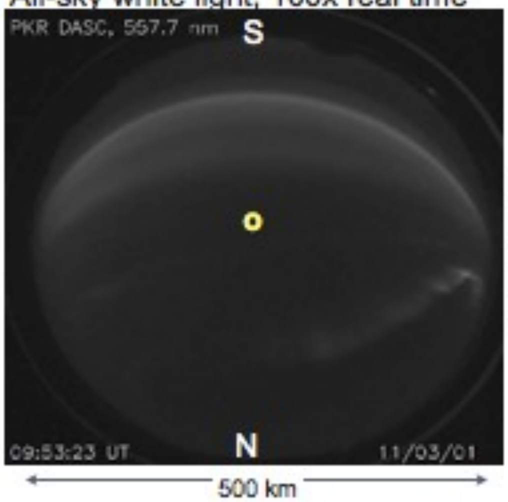


## Vorticity

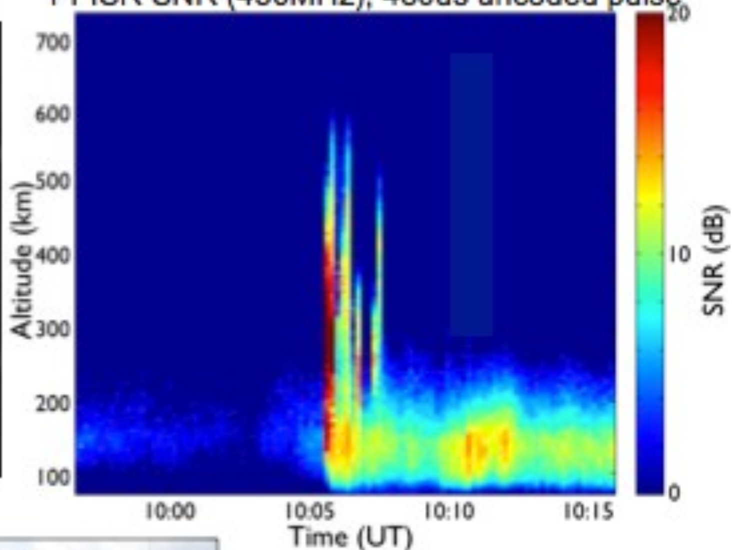


# Plasma turbulence

All-sky white light, 100x real time

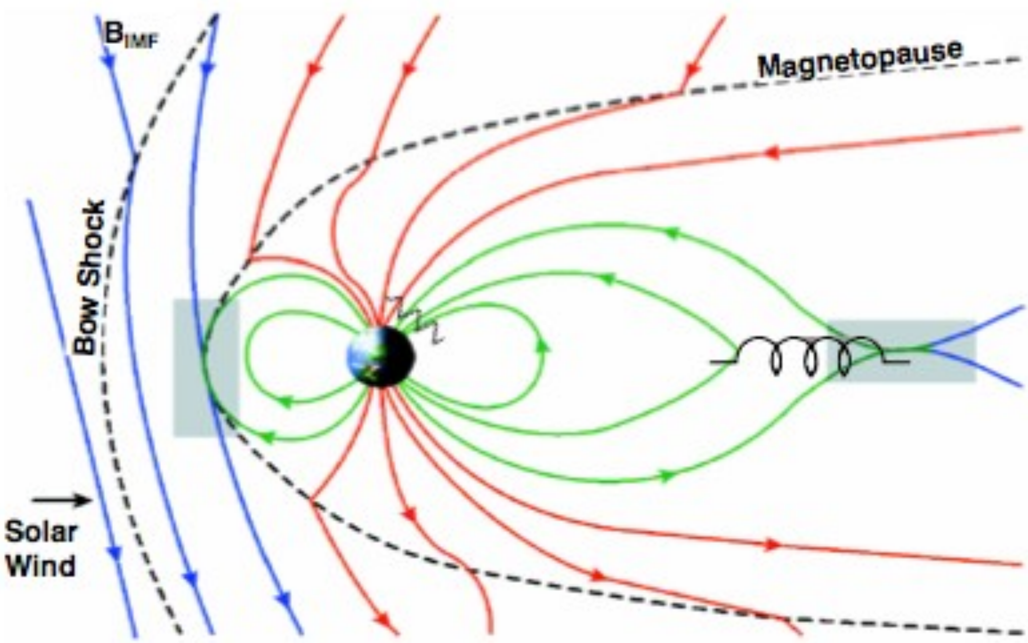



PFISR SNR (450MHz), 480us uncoded pulse




Akbari et al., 2013

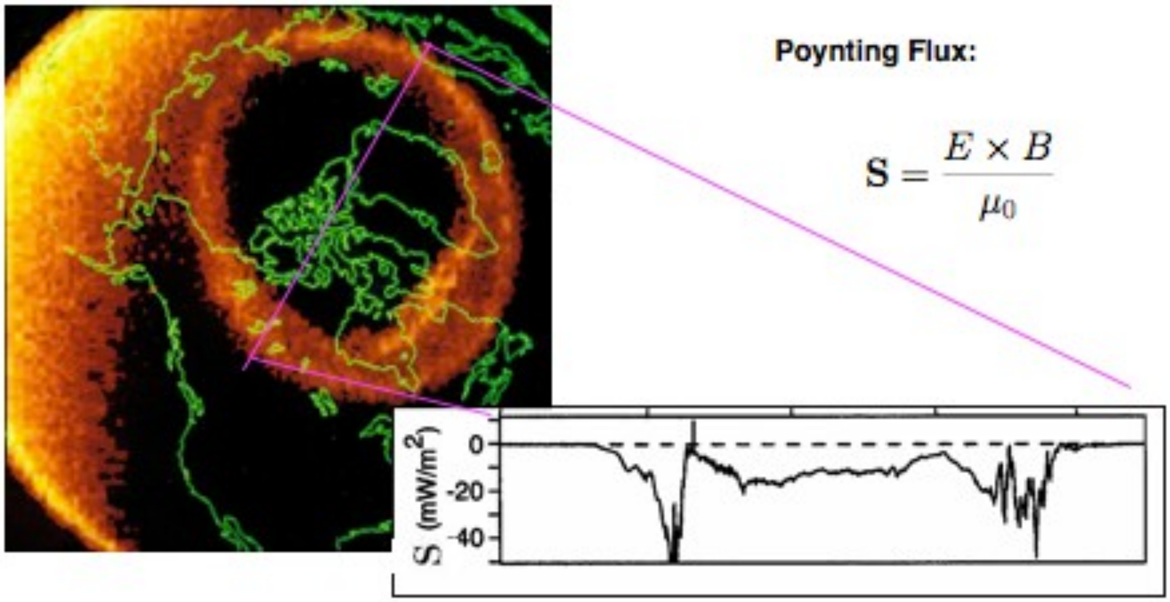
# Solar wind - Magnetosphere Coupling: Energy Storage and Release



# The EMF comes from Poynting flux delivered by the magnetosphere

**Poynting Flux:**

$$\mathbf{S} = \frac{E \times B}{\mu_0}$$

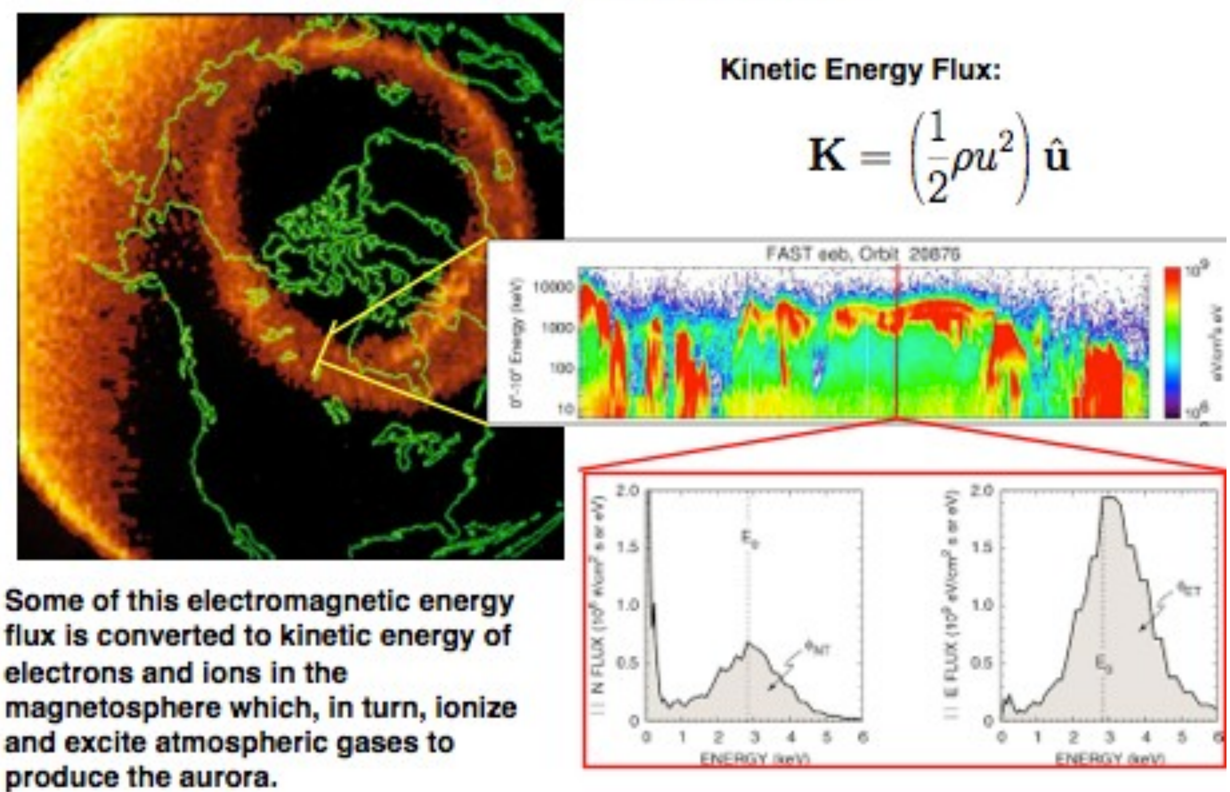



**Poynting flux is carried by magnetic field-aligned currents.**
**The currents are generated by mechanical interactions (dynamo) between the solar wind and magnetosphere.**
**The ionosphere looks like a resistive load to this current.**

# Which may be converted to kinetic energy flux of electrons and protons

**Kinetic Energy Flux:**

$$\mathbf{K} = \left(\frac{1}{2}\rho u^2\right)\hat{\mathbf{u}}$$




**Some of this electromagnetic energy flux is converted to kinetic energy of electrons and ions in the magnetosphere which, in turn, ionize and excite atmospheric gases to produce the aurora.**

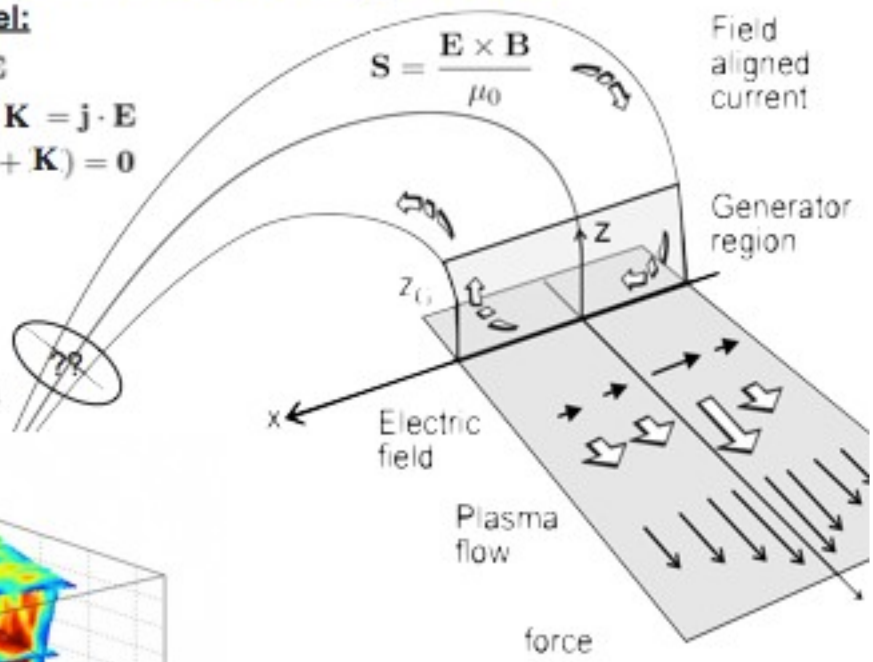
# The auroral system

**A Simple Physical model:**

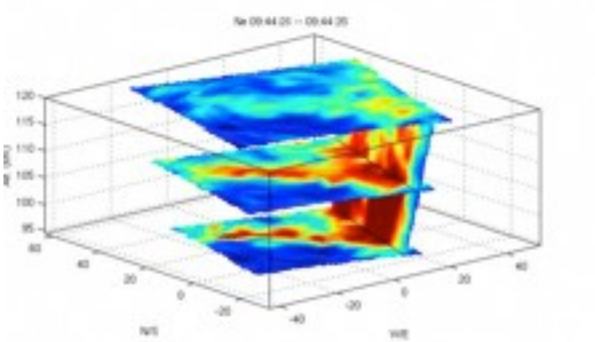
Generator: $\nabla \cdot \mathbf{S} = -\mathbf{j} \cdot \mathbf{E}$

Acceleration Region: $\nabla \cdot \mathbf{K} = \mathbf{j} \cdot \mathbf{E}$

Adiabatic System: $\nabla \cdot (\mathbf{S} + \mathbf{K}) = \mathbf{0}$




$$\mathbf{K} = \frac{1}{2}\rho u^2 \mathbf{u}$$



# Vlasov Equation

$f_s(\mathbf{x},\mathbf{v},t)$ is a probability density associated with an ensemble of systems. Particles are neither created nor destroyed, and so the density function satisfies a continuity equation in 6-dimensional phase space:

$$\frac{\partial f_s(\mathbf{x}, \mathbf{v}, t)}{\partial t} + \nabla_{\mathbf{x}} \cdot \left( \frac{d\mathbf{x}}{dt} f_s \right) + \nabla_{\mathbf{v}} \cdot \left( \frac{d\mathbf{v}}{dt} \right) = 0$$

Fluid elements subject to electromagnetic forces, so we have

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q_s}{m_s} \left[ \mathbf{E}(\mathbf{x}, t) + \frac{\mathbf{v}}{c} \times \mathbf{B}(\mathbf{x}, t) \right]$$

Substituting, and making use of the identity $\nabla \cdot (\mathbf{a}b) = b\nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla b$ we obtain the Vlasov equation

$$\boxed{\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla_x f_s + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f_s = 0}$$

# Governing equation, no applied field

Linear the equation by limiting to first-order variations. This can be done by breaking $f_s$ into two components

$$f_s = f_{s0} + f_{s1}$$

Ignore magnetic field and let $\boldsymbol{E}$ be a first-order **x**-directed perturbation. The resulting first-order terms of the Vlasov equation are

$$\frac{\partial f_{s1}}{\partial t} + v_s \frac{\partial f_{s1}}{\partial x} + \frac{q_s}{m_s} E \frac{\partial f_{s0}}{\partial v_s}$$

Let us look for solutions of the form $\exp(i\mathbf{k}\cdot\mathbf{x} - i\omega t)$

$$(-i\omega + ikv_x) f_{s1} = -\frac{q}{m_s} E \frac{\partial f_{s0}}{\partial v_s}$$

$$f_{s1} = \frac{-iq_s/m_s}{\omega - kv_s} E \frac{\partial f_{s0}}{\partial v_s}$$

# Dispersion relation

Poisson's equation for our plane wave solutions

$$ikE = 4\pi q(n_i - n_e) = 4\pi q \int d\mathbf{v}(f_{i1} - f_{e1})$$

Substitute $f_s$ and eliminate $E$ from both sides yields the dispersion relation for electrostatic waves in an unmagnetized plasma

$$\boxed{1 + \frac{\omega_{pe}^2}{k^2} \int du \frac{dg(u)/du}{\omega/k - u} = 0}$$

$$\omega_{pe}^2 = \frac{4\pi n_0 q^2}{m_e}$$

$$g(v_x) \approx \frac{1}{n_0} \int dv_y dv_z f_{e0}(\mathbf{v})$$

where for $g$ we have ignored the term associated with more massive ions. We assume ions are an immobile background.

# Langmuir waves

For now we avoid the pole by assuming $\omega/k \gg u$ for all $g(u)$ we care about. I.e., $dg(u)/du = 0$ at $u = \omega/k$. Now expand the denominator up to second order.

$$1 + \frac{\omega_{pe}^2}{k^2} \int du\, g(u) \left(1 + \frac{2uk}{\omega} + \frac{3u^2k^2}{\omega^2}\right) = 0$$

This integral can now be evaluated explicitly to obtain

$$1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{3k^2 v_e^2 \omega_{pe}^2}{\omega^4} = 0$$

which upon solving for $\omega^2$ assuming $v_e^2 \ll \omega^2/k^2$ and using $v_e = T_e/m_e$ yields the dispersion relation for Langmuir waves

$$\boxed{\omega^2 = \omega_{pe}^2 + 3k^2 v_{te}^2}$$

# Stability and Landau damping

What happens when wave velocity approaches the particle velocity? The effect of the pole now becomes important. and we need to consider the issue of system stability. This means Fourier transform in space, but Laplace transform in time. Our solution consists of normal modes, related to $\omega_r$, and transients, related to $\omega_i$

$$(-i\omega + ikv_x) f_{s1} - \frac{q}{m_s} E(\omega) \frac{\partial f_{s0}}{\partial v_s} = f_{s1}(k, \mathbf{v}, t = 0)$$

$$\omega = \omega_r + i\omega_i$$

Our solution is now of the form

$$E(k, t) \propto \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t) = \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega_r t + \omega_i t)$$

Hence, linear solution of the Vlasov-Poisson equation set is a damped oscillator. Landau damping does not come from collisions between particles but from decorrelations between particles and waves.

The imaginary part of the frequency is called the damping increment, and can be positive (wave growth, unstable system) or negative.

# Results for ions and electrons

Langmuir waves

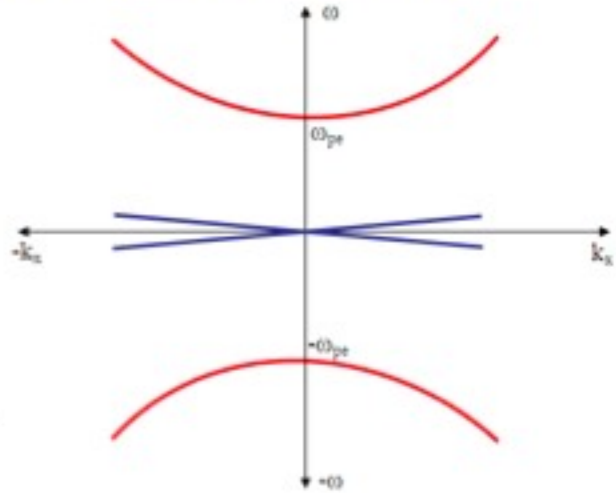
$$\omega^2 = \omega_{pe}^2 + 3k^2 v_{te}^2$$

$$\omega_i^2 = -\sqrt{\frac{pi}{8}} \frac{\omega_{pe}^3}{k^3 v_{te}^3} \exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{te}^2} - \frac{3}{2}\right) \omega$$

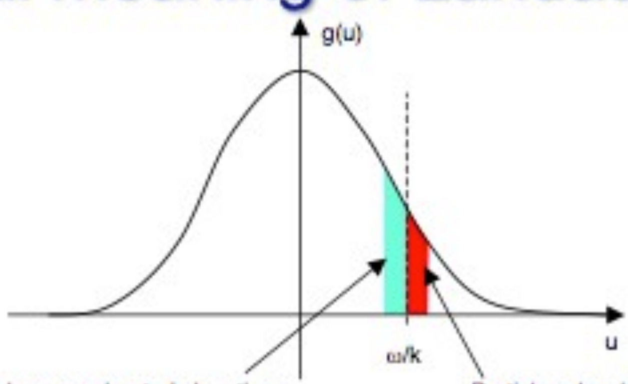
Ion-acoustic waves

$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}} \left[ \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left( -\frac{T_e}{2T_i} - \right.\right.$$




# Physical meaning of Landau damping




Change in energy that the electron distribution experiences during the interaction with a Langmuir wave.

$$\Delta W_e \propto -m_e t \frac{\omega_{pe}}{k^2} \left.\frac{\partial f}{\partial v}\right|_{v'=\omega_{pe}/k}$$

# The ISR Target: Plasma Fluctuations

Particle-in-cell (PIC):

$$\frac{d\,\mathbf{v}_i}{d\,t} = \frac{q_i}{m_i}(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i))$$

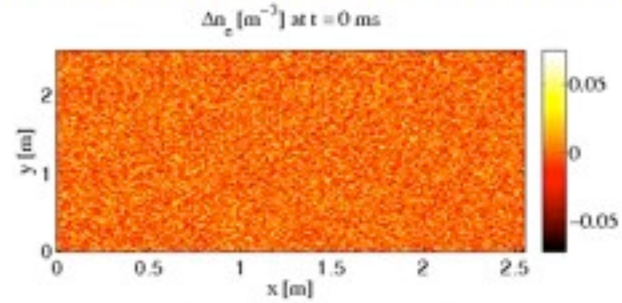
$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}$$

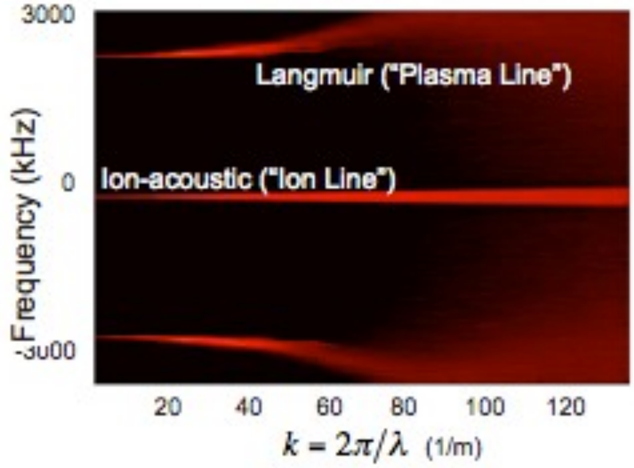
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

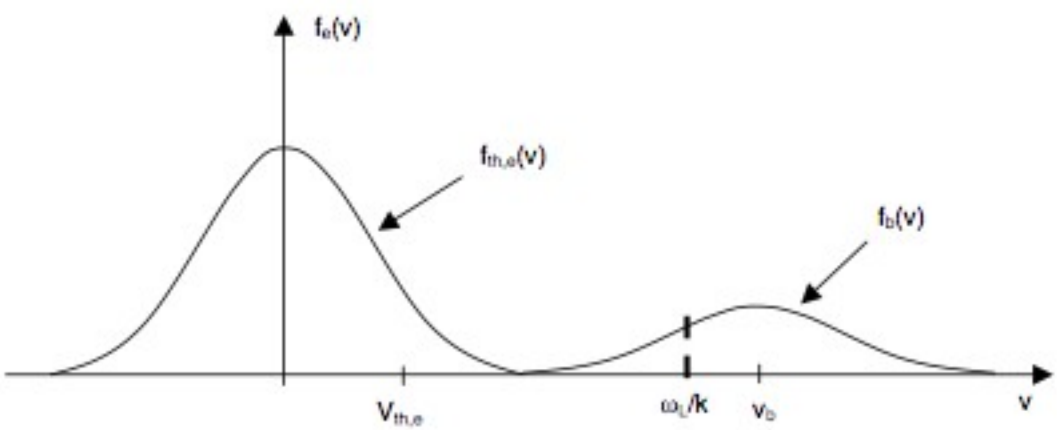
$$\nabla \cdot \mathbf{B} = 0$$

Simple rules yield complex behavior




# Beam instability



# Plasma simulation of beam driven instability

Ions

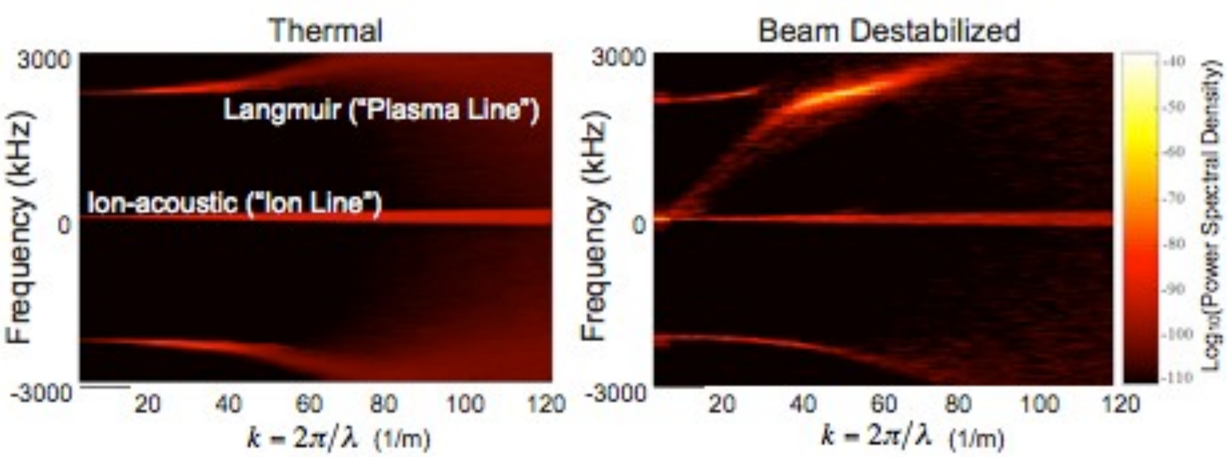
Electrons

Electric
Field

Once threshold electric field is achieved (blued line in top panel),
parametric decay to ion acoustic mode occurs

Diaz et al., Ann. Geophys. 2011

# Beam destabilized plasma




Parametric decay of Langmuir waves produces enhancement in
ion-acoustic waves

Diaz et al., JGRA 2011

# Incoherent Scatter Radar (ISR)


# How Does a Doppler Radar Work?

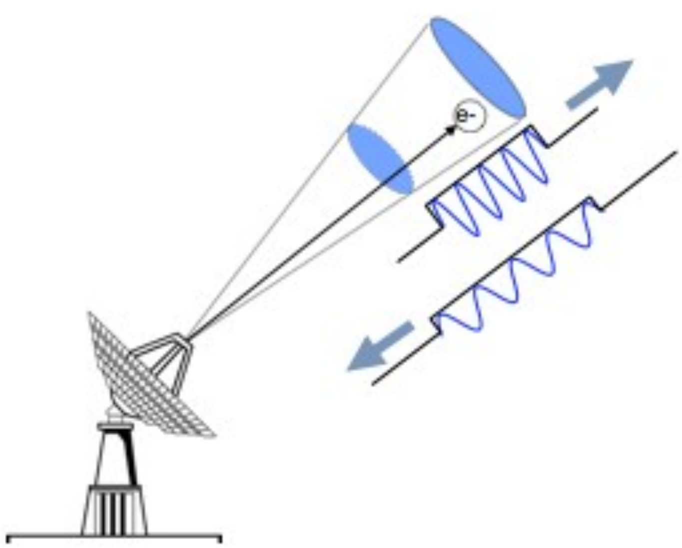
Two key concepts:

Distant ⇔ Time

$$R = c\Delta t/2$$

Velocity ⇔ Frequency

$$v = -f_D\lambda_0/2$$




A Doppler radar measures backscattered power as a function range and velocity. Velocity is manifested as a Doppler frequency shift in the received signal.

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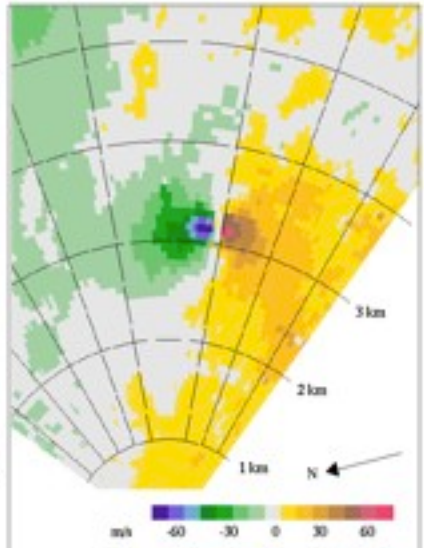


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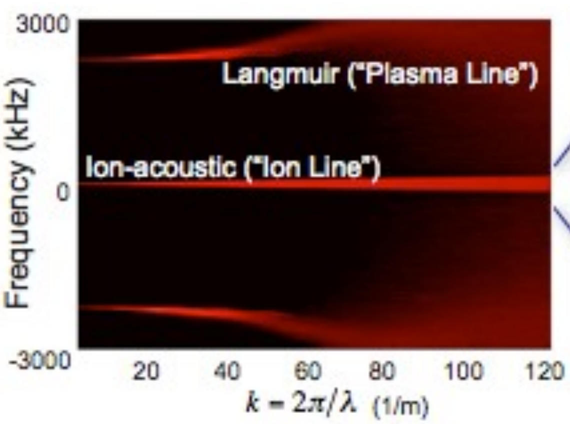
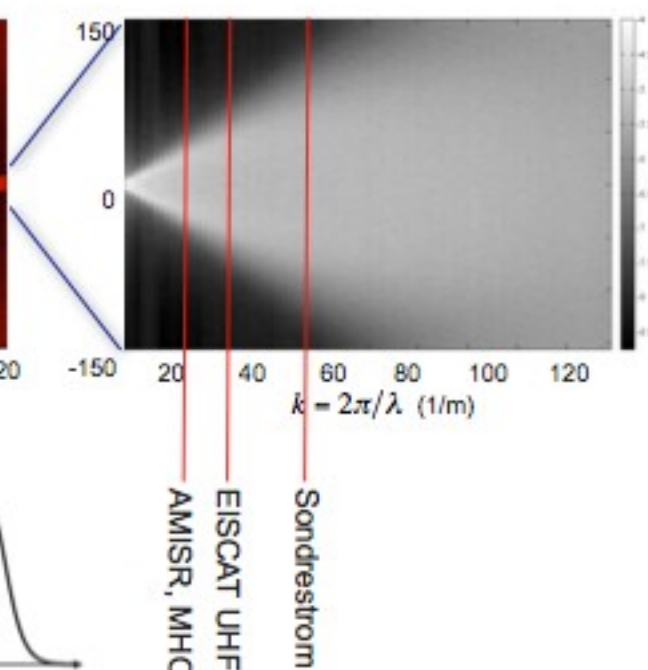
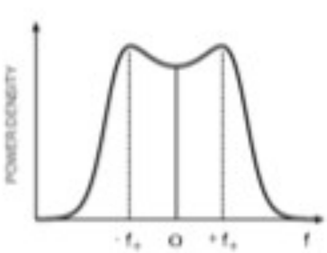
# The Ionosphere's Doppler Spectrum




Ion-acoustic "lines" are broadened by Landau damping



# Incoherent Scatter Radar (ISR)

Ion-acoustic

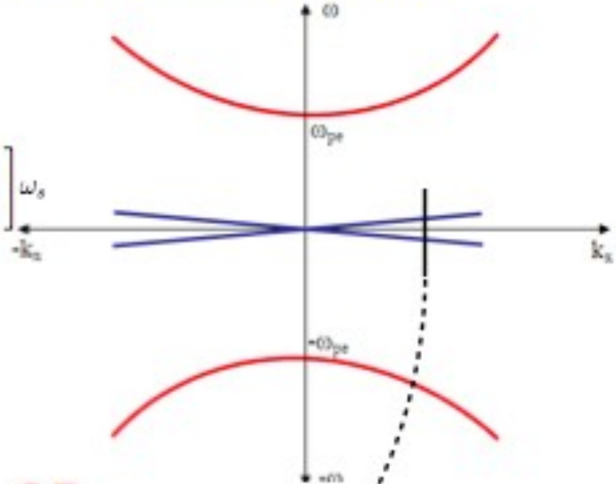
$$\omega_s = C_s k \qquad C_s = \sqrt{k_B(T_e + 3T_i)/m_i}$$

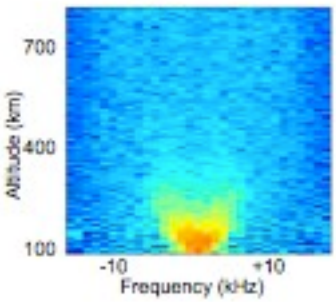
$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

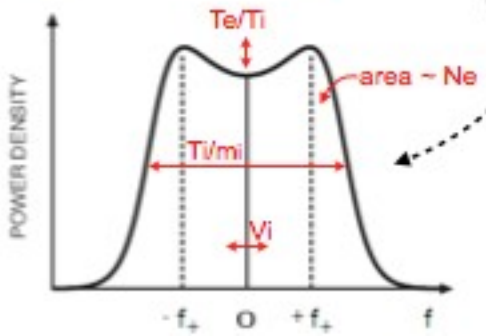
Langmuir

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the} \lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}} \frac{\omega_{pe}^3}{k^3} \frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$

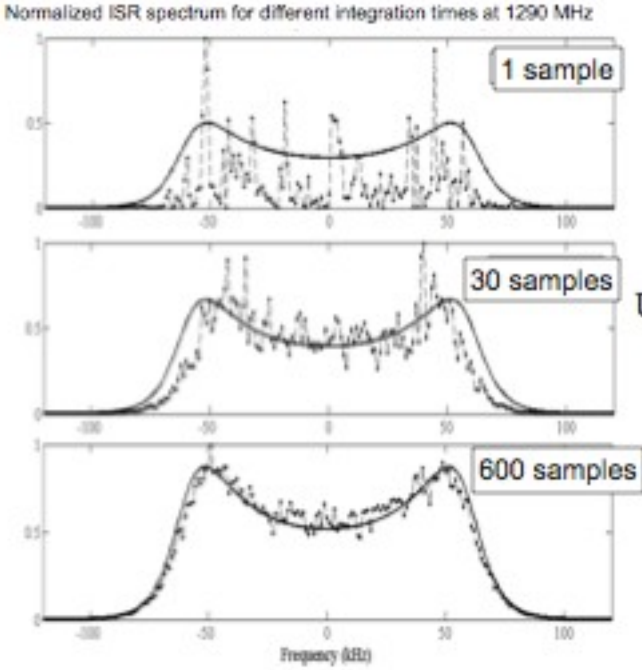



# ISR: A rich signal processing challenge

Normalized ISR spectrum for different integration times at 1290 MHz



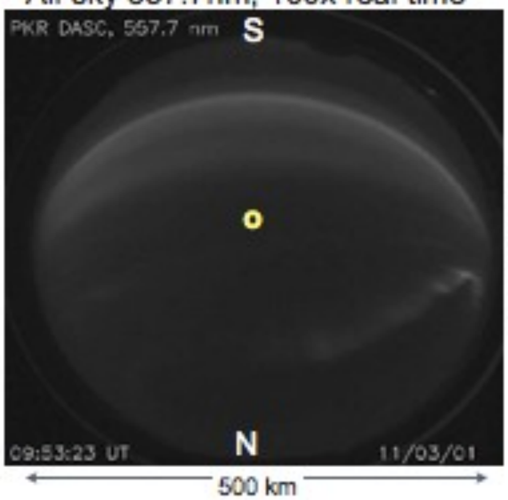

**Target is noise-like:**
We are seeking to estimate the power spectrum of a Gaussian random process. This requires that we sample and average many independent "realizations" of the process.

$$\text{Uncertainties} \propto \frac{1}{\sqrt{\text{Number of Samples}}}$$

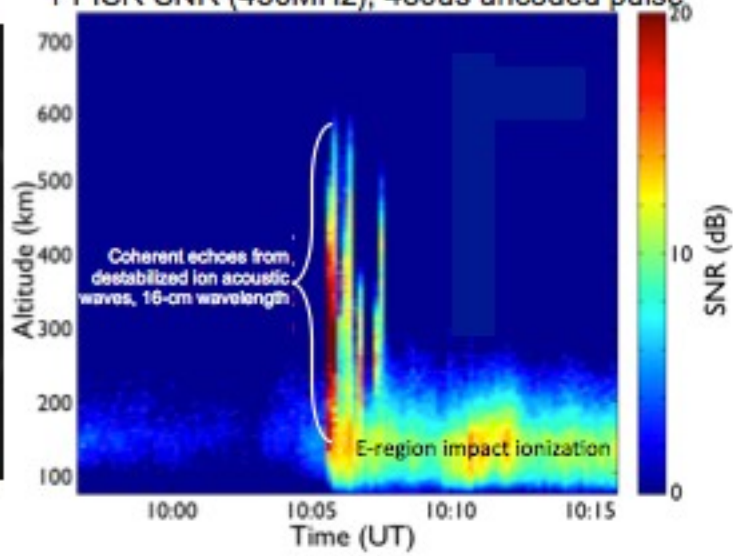
**Target is overspread:**
Plasma de-correlation time is short compared to the time between pulse transmission. Alternately, the pulse repetition frequency is much less than the width of the power spectrum. Need to come up with clever sampling strategies.

# Radar perspective

All-sky 557.7nm, 100x real time

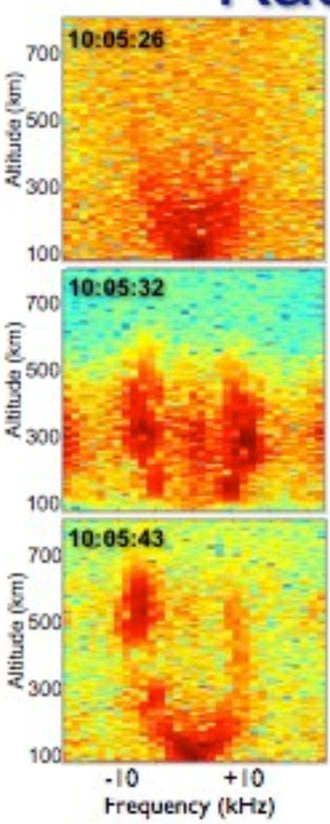



PFISR SNR (450MHz), 480us uncoded pulse

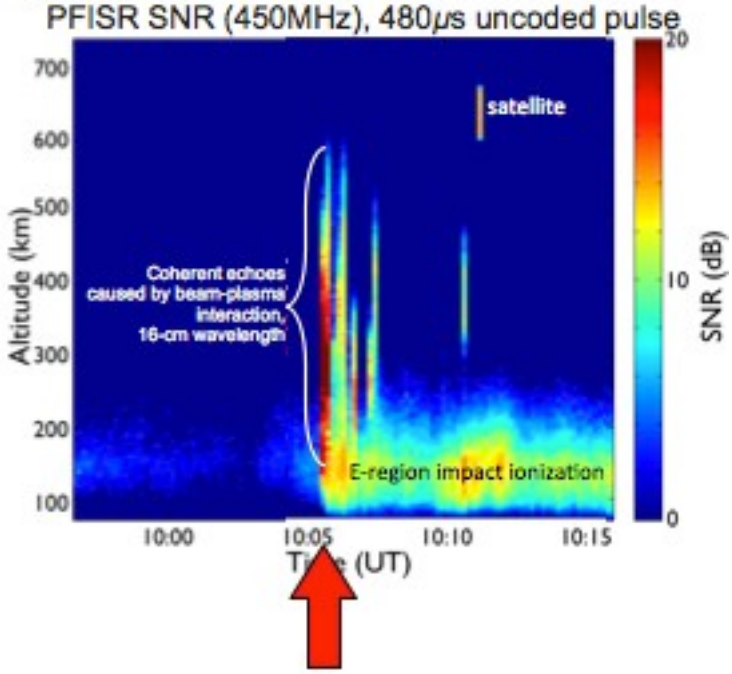



Akbari et al., 2013

# Radar perspective

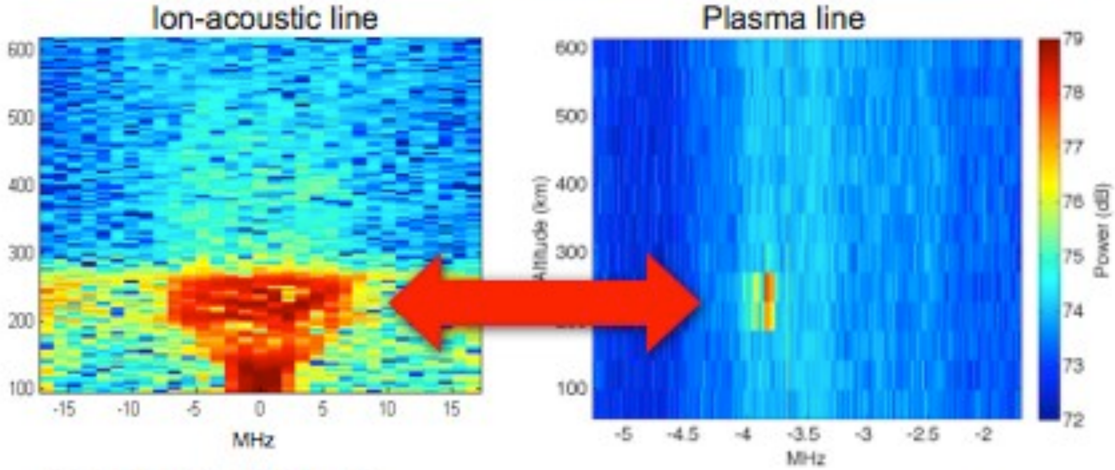



PFISR SNR (450MHz), 480$\mu$s uncoded pulse




Akbari, H., J. L. Semeter, H. Dahlgren, M. Diaz, M. Zettergren, A. Stromme, M. J. Nicolls, and C. Heinselman, Anomalous ISR echoes preceding auroral breakup: Evidence for strong Langmuir turbulence, Geophys. Res. Lett., 39, L03102, doi: 10.1029/2011GL050288, 2012.

# Evidence for Strong Langmuir Turbulence




$$\omega_s = \sqrt{k_B(T_e + 3T_i)/m_i}\, k$$

$$\omega_{si} = -\sqrt{\frac{\pi}{8}}\left[\left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} + \left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} \exp\left(-\frac{T_e}{2T_i} - \frac{3}{2}\right)\right]\omega_s$$

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the}\lambda_{De} k^2$$

$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$

Simultaneous enhancement of plasma line and ion line above thermal levels.

Akbari et al., GRL 2012

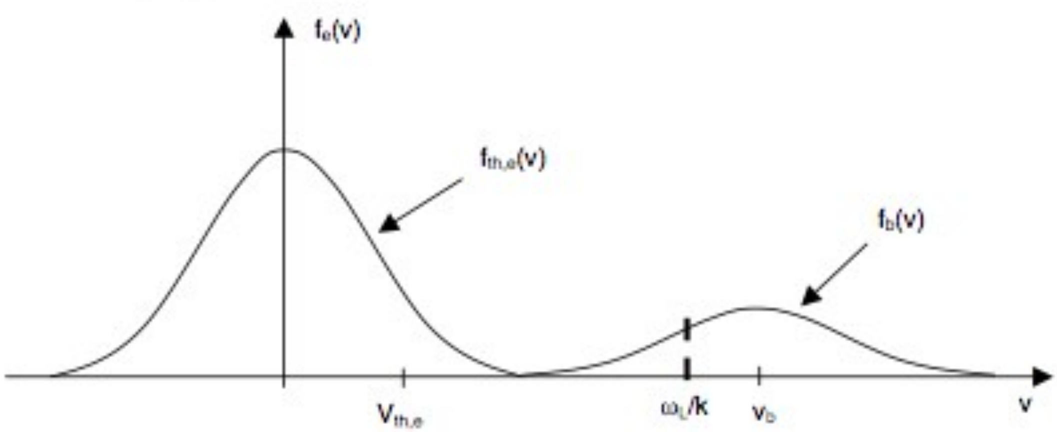
# Linear growth rate of Langmuir waves

$$\omega_L = \sqrt{\omega_{pe}^2 + 3\,k^2\,v_{the}^2} \approx \omega_{pe} + \frac{3}{2} v_{the}\lambda_{De} k^2$$

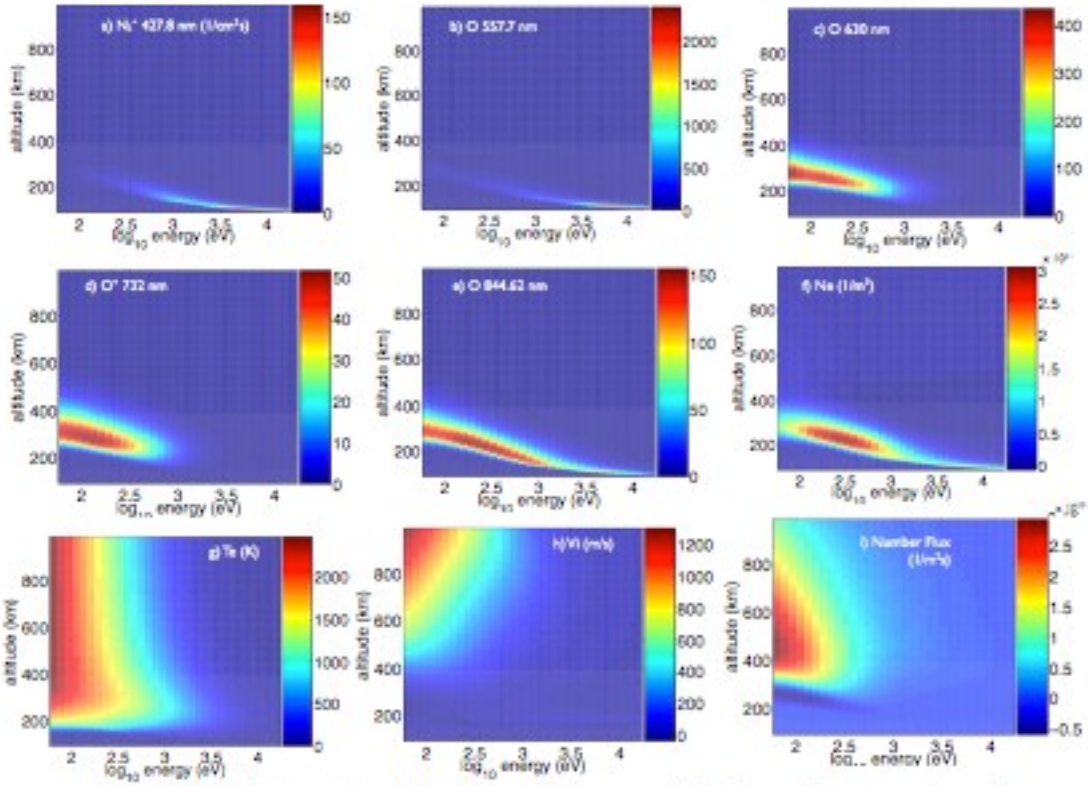
$$\omega_{Li} \approx -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}^3}{k^3}\frac{1}{v_{the}^3} exp\left(-\frac{\omega_{pe}^2}{2k^2 v_{the}^2} - \frac{3}{2}\right)\omega_L$$

$$\gamma_{\max} \propto \frac{n_b}{n_0}\left(\frac{v_b}{\Delta v_b}\right)^2 w_L$$

- $n_0$ background density
- $n_b$ beam density
- $v_b$ beam velocity
- $\Delta v_b$ beam "temperature"
- $\omega_L$ Langmuir frequency

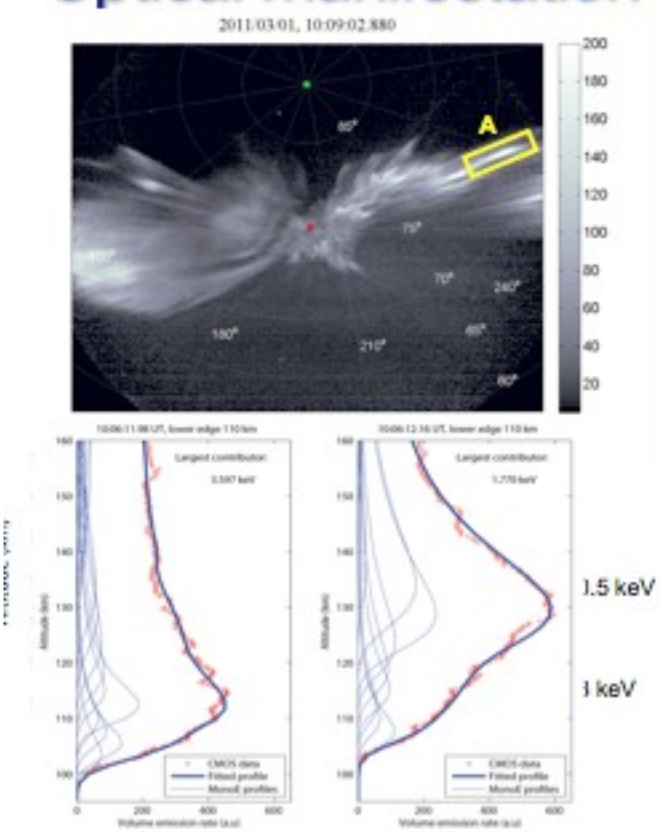


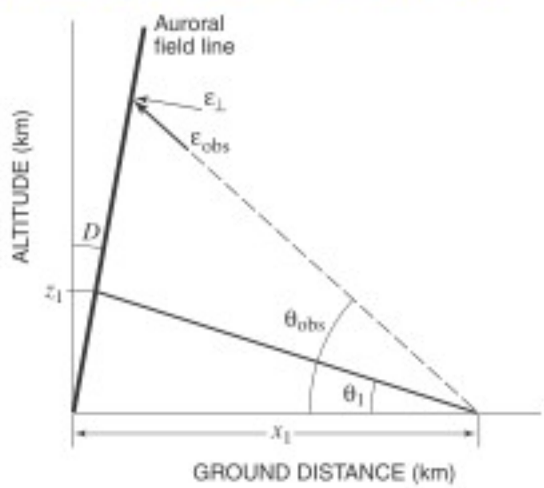
# The Green's functions of the ionospheric response



To what extent is this a linear, shift-invariant system?

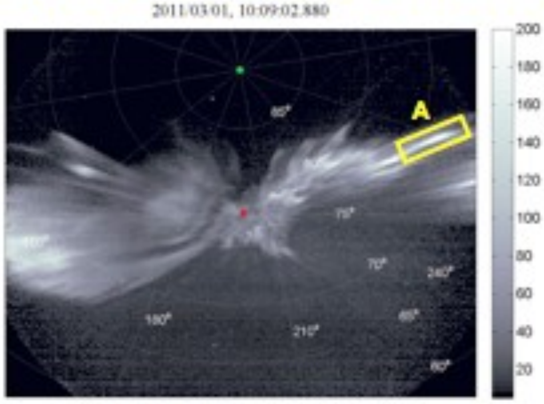
# Optical manifestation of dispersive bursts

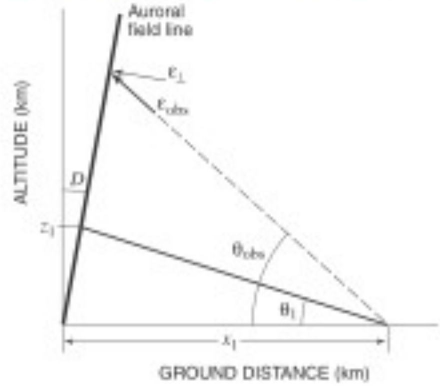


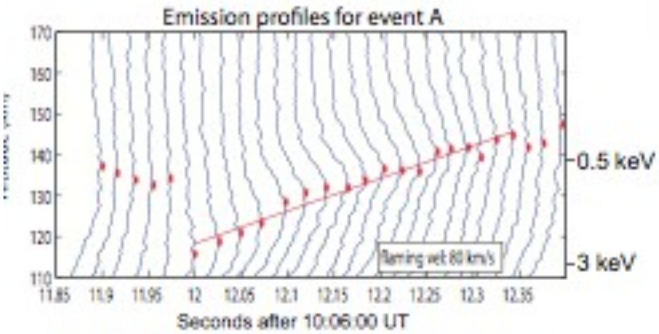


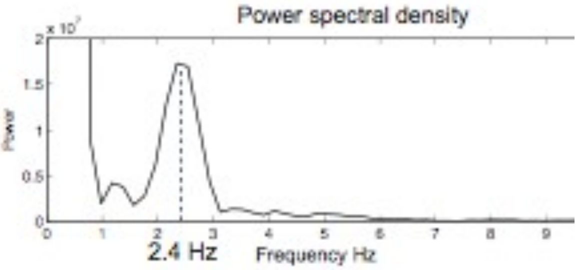

Dahlgren, H., J. Semeter, R. Marshall, and M. Zettergren, The optical manifestation of field-aligned bursts in the aurora, J. Geophys. Res. 118, doi: 10.1002/jgra.50415, 201

# Optical manifestation of dispersive bursts



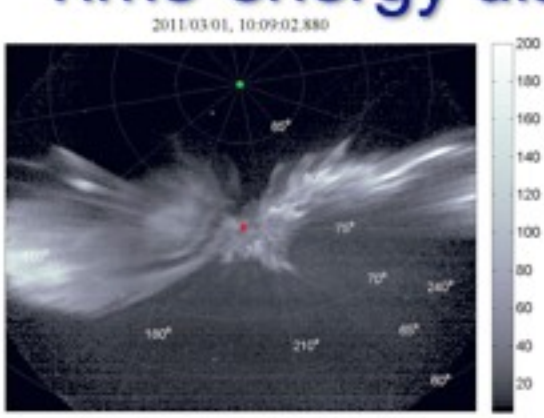



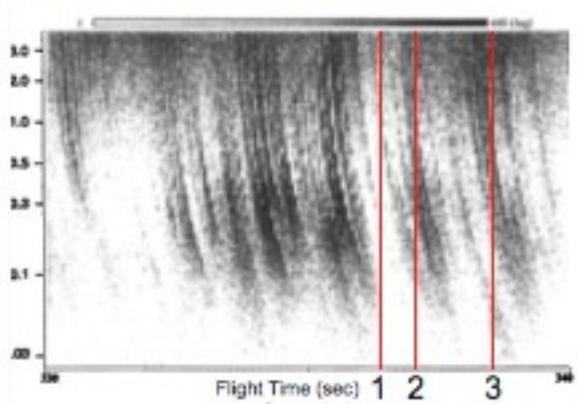

Average energy of primary electrons decreases from 3keV to <500eV in 0.2-s.

Field-aligned bursts are periodic at ~2.4Hz

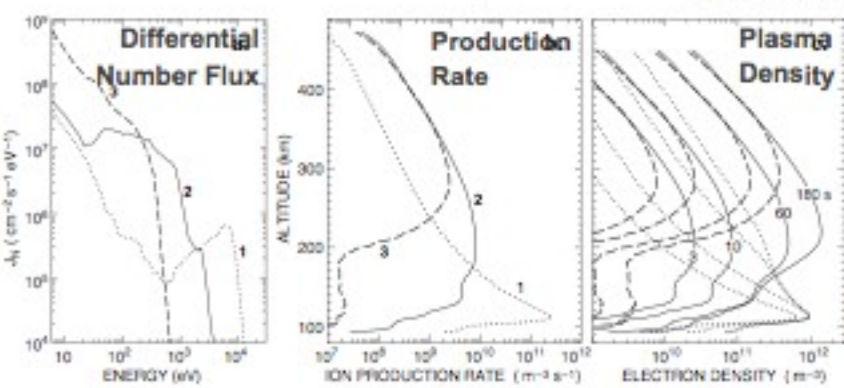
*Dahlgren et al.*, JGRA 2013.

# Time-energy dispersed electron




Arnoldy et al., 1999




Semeter et al., 2005

# Alfvén waves

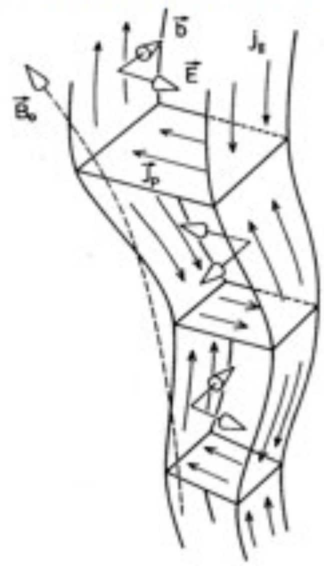
For waves with frequency less than the ion cyclotron frequency, the linear analysis of the MHD equations leads to three wave modes: the fast magnetosonic mode, the slow magnetosonic mode, and the Alfven wave.

Insofar as coupling between the magnetosphere and ionosphere is concerned, the Alfven wave is most important.

The frequency of the MHD Alfven wave is given by

$$\omega = k_z v_A$$

$$v_A = \frac{B_o}{\sqrt{\mu_o \rho}}$$



# Dispersive Alfvén wave: derivation I

Recall the basic gestalt of the Alfvén wave: ion polarization current closed by field-parallel electron current. To determine the ion drift, let's assume a time-harmonic field at a point in space, $\mathbf{E}e^{-i\omega t}$ subject to Lorentz force $m d\mathbf{v}/dt = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. We are concerned with the guiding center drift

$$\mathbf{u} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{27}$$

A time varying E field means time varying velocity (i.e., acceleration) and so in the guiding center frame of reference we feel a force

$$\mathbf{F} = -m \frac{d}{dt} \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{28}$$

This in turn gives us another drift

$$u = \frac{1}{q} \frac{\mathbf{F} \times \mathbf{B}}{B^2} = \frac{m}{qB^2} \left[ \frac{d}{dt} \frac{\mathbf{E} \times \mathbf{B}}{B^2} \right] \times \mathbf{B} \tag{29}$$

The polarization drift and polarization current are thus

$$\mathbf{u}_{\mathrm{p}} = \frac{m}{qB^2} \frac{d\mathbf{E}_\perp}{dt} \qquad \mathbf{J}_\perp = nq_i \mathbf{u}_{\mathrm{pi}} = \frac{nm_i}{B^2} \frac{d\mathbf{E}_\perp}{dt} = \frac{1}{\mu_o v_A^2} \frac{d\mathbf{E}_\perp}{dt} \tag{32}$$

# Dispersive Alfven wave: derivation II

Let us now examine the fields produced. We are dealing with electromagnetic phenomena, so we need both scalar any magnetic potential. We will simplify things by letting $\mathbf{A} = A_z\hat{\mathbf{z}}$ which presupposes that we have TEM waves guided by the Earth's field. This is justified based on the low plasma beta, strong magnetization.

$$\mathbf{E} = -\nabla\phi - \frac{\partial A_z}{\partial t}\hat{\mathbf{z}} \tag{33}$$

We can separate the parallel and perpendicular components of the wave electric field

$$E_{\parallel} = -\frac{\partial \phi}{\partial z} - \frac{\partial A_z}{\partial t} \tag{34}$$

$$\mathbf{E}_{\perp} = -\nabla_{\perp}\phi \tag{35}$$

The total wave current is given by $\mu_o\mathbf{J} = \nabla \times (A_z\hat{\mathbf{z}}) = \nabla\nabla\cdot(A_z\hat{\mathbf{z}}) - \nabla^2 A_z\hat{\mathbf{z}}$. So that the parallel and perpendicular components of the current densities are

$$\mu_o\mathbf{J}_{\parallel} = -\nabla_{\perp}^2 A_z \tag{36}$$

$$\mu_o\mathbf{J}_{\perp} = \nabla_{\perp}\frac{\partial A_z}{\partial z} \tag{37}$$

# Dispersive Alfven wave: derivation III

To see what is happening to the particles, we need an equation of motion. The motion is inversely proportional to mass, and so we only consider the electrons. Also because we are assuming that the wave phase speed is much larger than electron thermal speed, we can ignore pressure gradients. We thus have

$$m_e\frac{\partial u_{e\parallel}}{\partial t} = q_e E_{\parallel} \tag{38}$$

Noting that $J_{\parallel} = nq_e u_{e\parallel}$, we combine the above equations involving field-parallel parameters to arrive at

$$(1 - \lambda_e^2)\frac{\partial A_z}{\partial t} = \frac{\partial \phi}{\partial z}$$

We eliminate the scalar potential by combining equations involving field-perpendicular parameters, which gives

$$\frac{\partial A_z}{\partial z} = -\frac{1}{v_A^2}\frac{\partial \phi}{\partial t}$$

We combine the above to obtain an equation in *A*, and take its Fourier Transform to obtain the Alfven wave dispersion relation with electron inertia correction:

$$(1 - \lambda_e^2\nabla_{\perp}^2)\frac{\partial^2 A_z}{\partial t^2} = v_A^2\frac{\partial^2 A_z}{\partial z^2} \quad \overset{\text{F.T.}}{\Longleftrightarrow} \quad \boxed{\omega^2 = \frac{k_{\parallel}^2 v_A^2}{1 + k_{\perp}^2\lambda_e^2}}$$

# Energy flow and particle acceleration

Insofar as energy propagation is concerns it is the group velocity that matters.

$$\frac{\partial \omega}{\partial \mathbf{k}} = \hat{\mathbf{z}}\frac{v_A}{(1+k_\perp^2\lambda_e^2)^{1/2}} - \hat{\mathbf{x}}\omega\lambda_e\frac{k_\perp\lambda_e}{1+k_\perp^2\lambda_e^2}$$

From the dispersion relation, we can see that $\omega^2 \leq k_\parallel^2 v_A^2$ so the dispersive wave propagates inside a conical region ("Alfven cone") with apex angle given by

$$\tan\theta_r = \frac{\omega}{\omega_{ci}}\left(\frac{m_e}{m_i}\right)^{1/2}$$

Spreading of the wave energy means there must be a field-aligned component of the electric field. We derive that by combining these equations.

$$m_e\frac{\partial u_{e\parallel}}{\partial t} = q_e E_\parallel$$

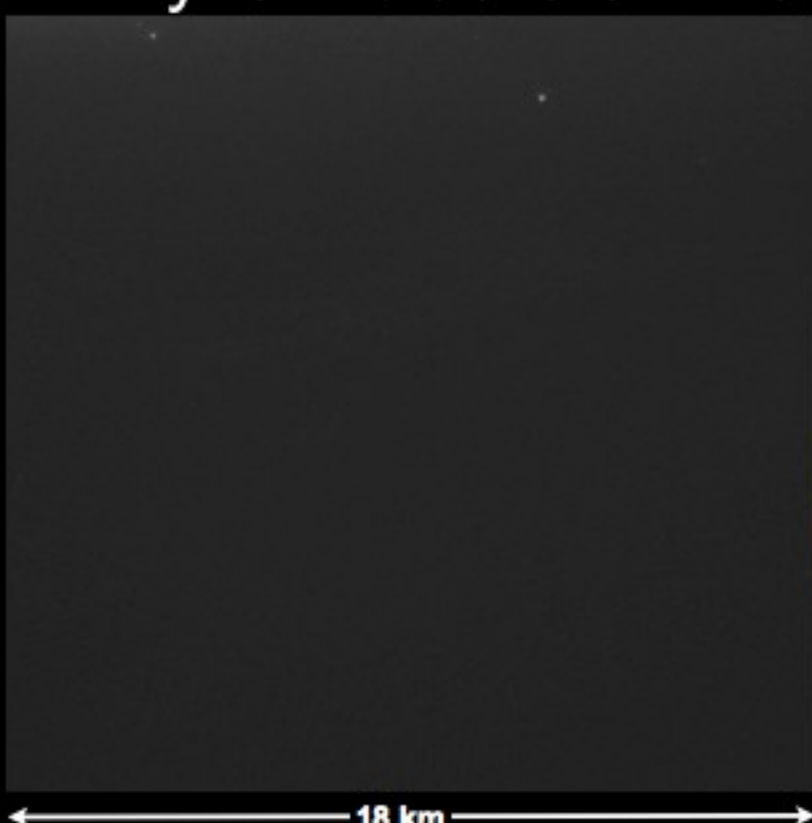
$$J_\parallel = n q_e u_{e\parallel}$$

$$\lambda_e = \frac{c}{\omega_{pe}}$$

$$\omega_{pe}^2 = \frac{n_0 q^2}{m_e \epsilon_o}$$

Combine, take F.T. → $\tilde{E}_\parallel = i\omega\lambda_e^2\mu_o\tilde{J}_\parallel$ → aurora

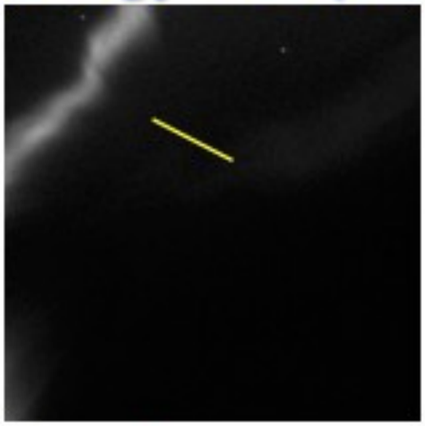
# Dynamic aurora in real time

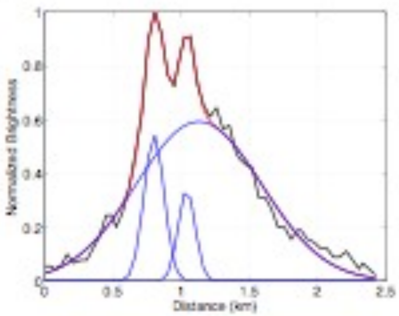




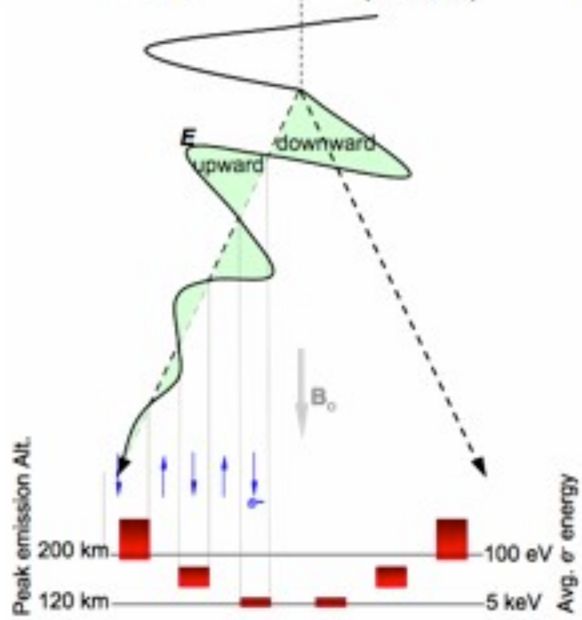
- Real Time
- 9º X 9º FOV
- 19 X 19 km
- Prompt emissions
- 30 Frames/s

# Energy dissipation via wave dispersion






$$\omega^2 = \frac{k_\parallel^2 v_A^2}{1+k_\perp^2\delta_e^2} \qquad v_g = \hat{z}\frac{v_A}{\left(1+k_\perp^2\delta_e^2\right)^{1/2}} - \hat{x}\frac{k_\perp \omega \delta_e^2}{1+k_\perp^2\delta_e^2}$$




Semeter, J., S. Mende, M. Zettergren, and M. Diaz, Wave dispersion and the discrete aurora: New constraints derived from high resolution imagery, J. Geophys. Res., 113, A12208, doi:10.1029/2008JA013122, 2008.