Shocks in Heliophysics

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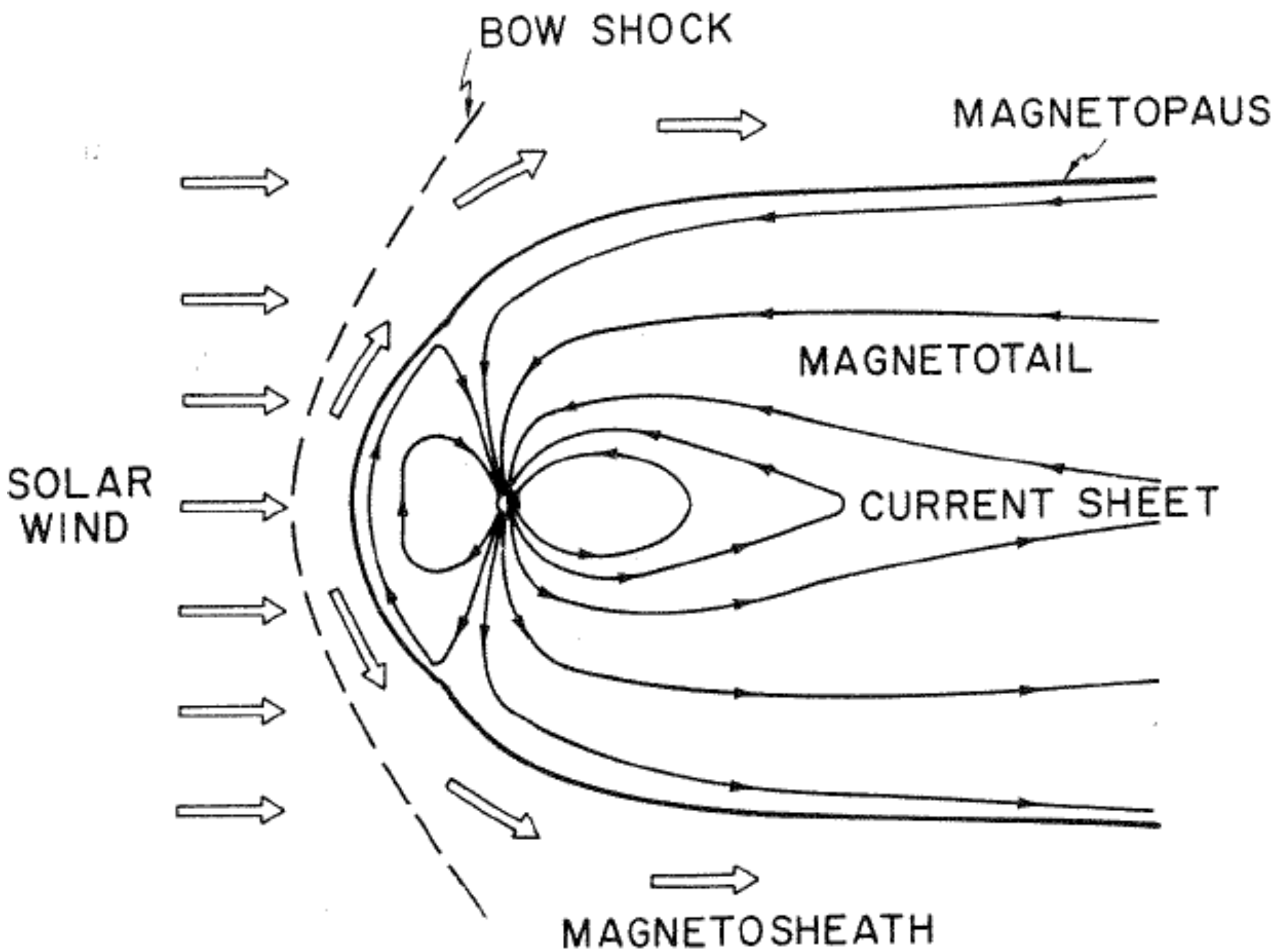
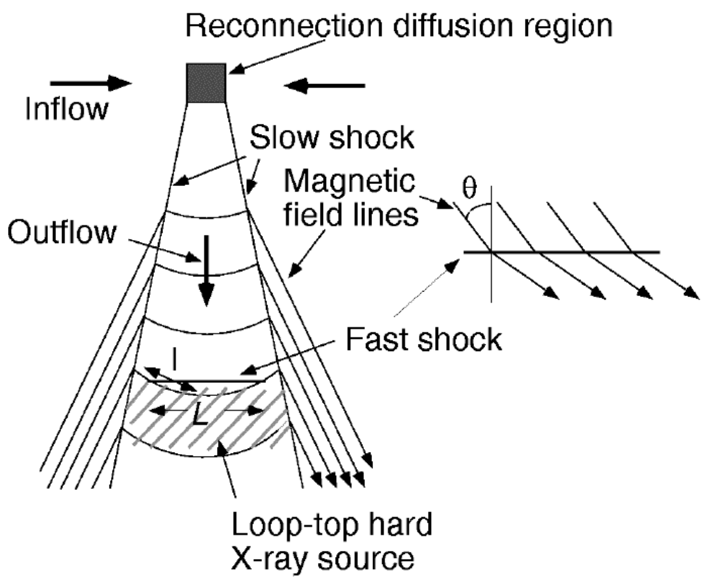
Shocks in Heliophysics

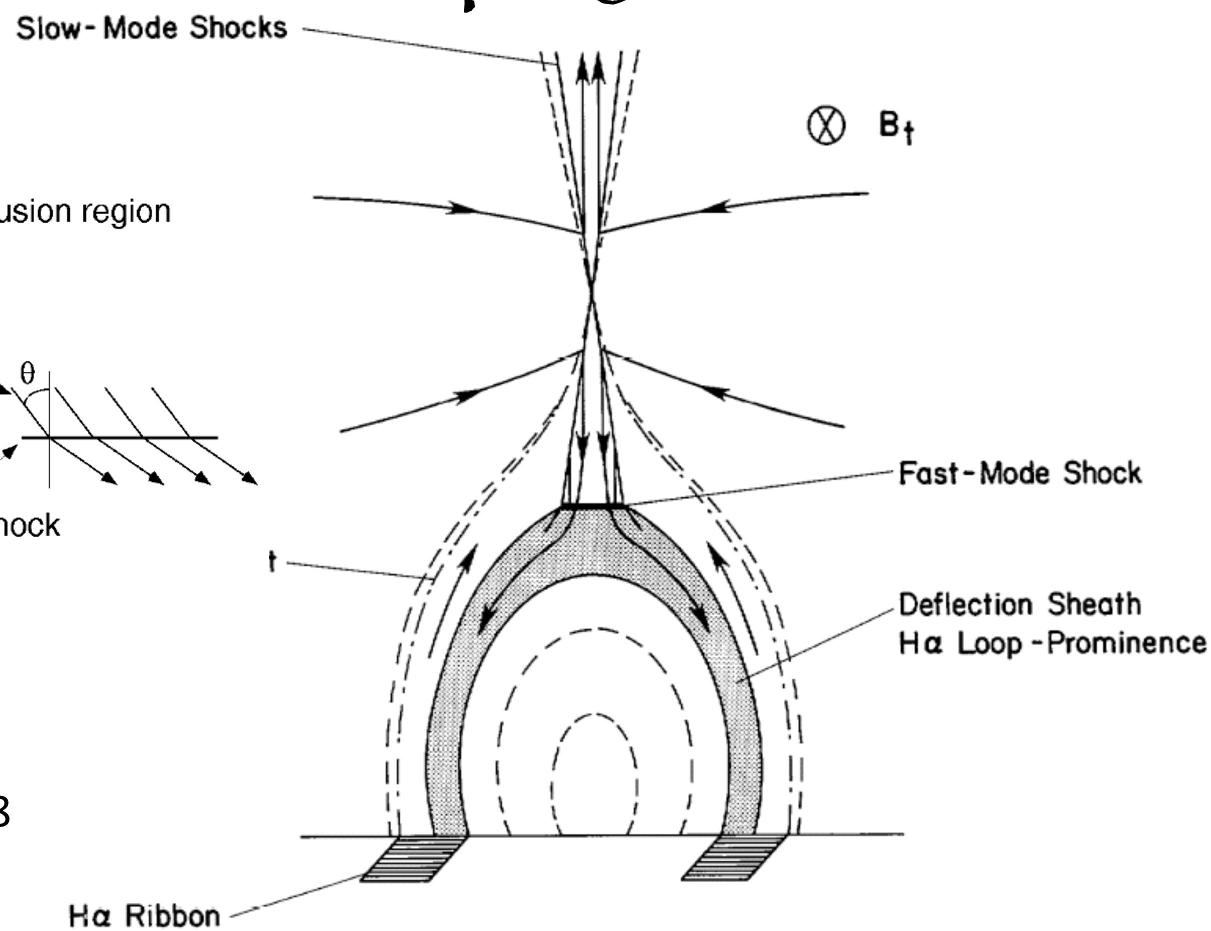


Vasyliunas, v.1 ch.10

Shocks in Heliophysics



Tsuneta and Naito 1998



Forbes, T.G., and Malherbe 1986 (v.2 ch. 6)

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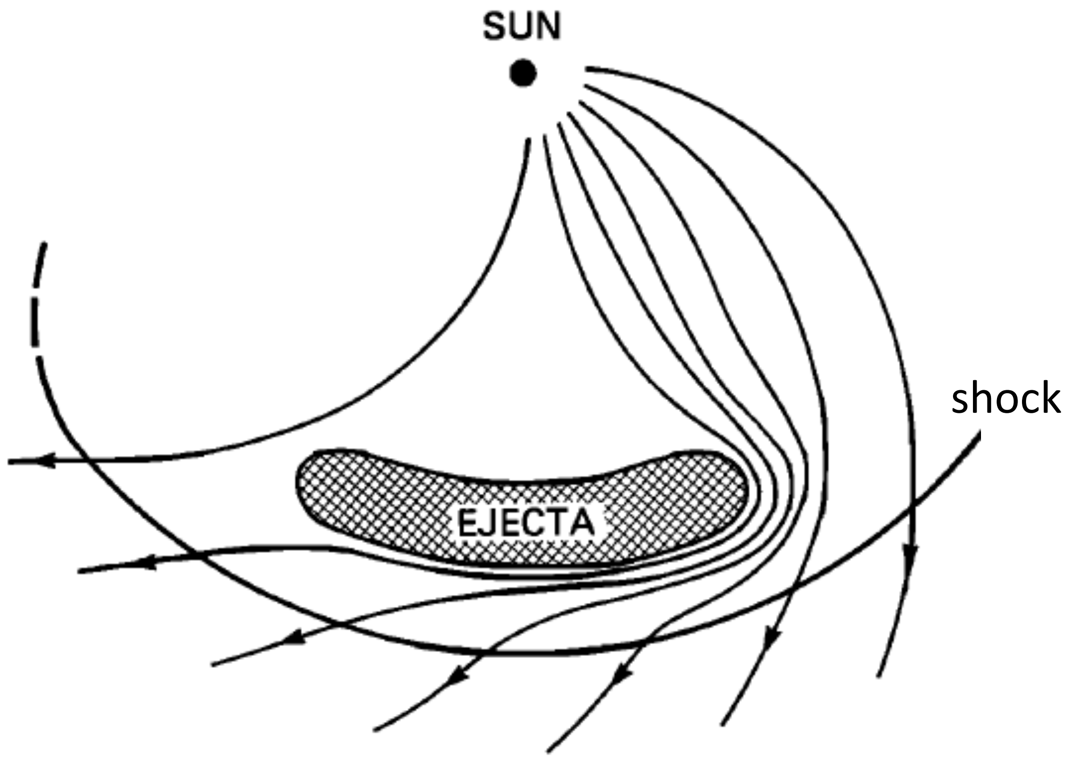
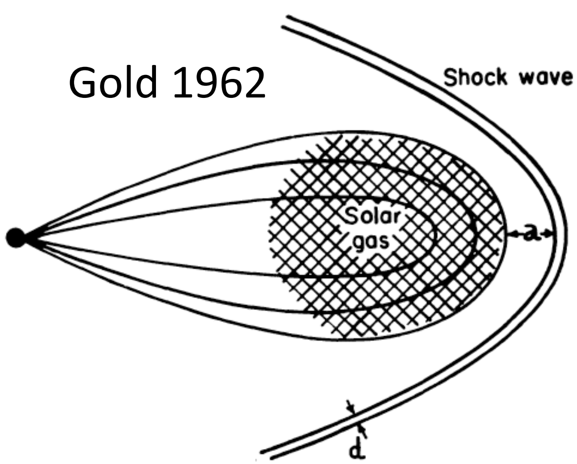
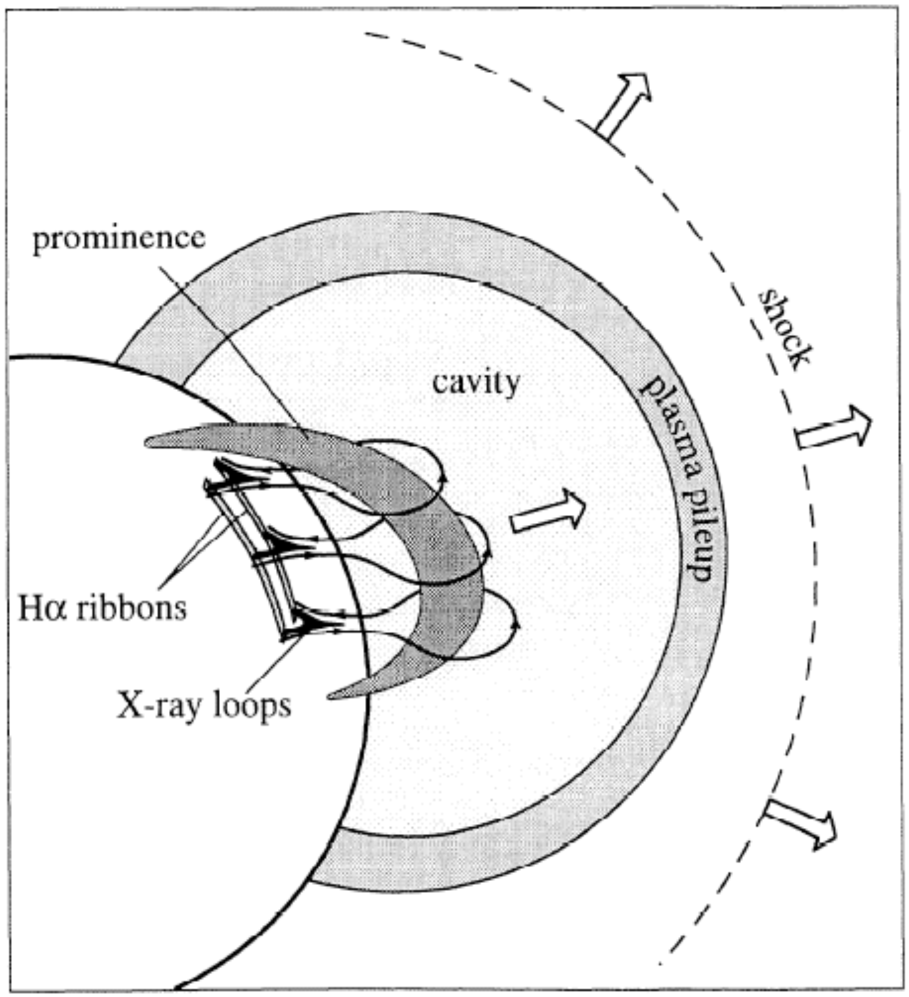


Gold 1962



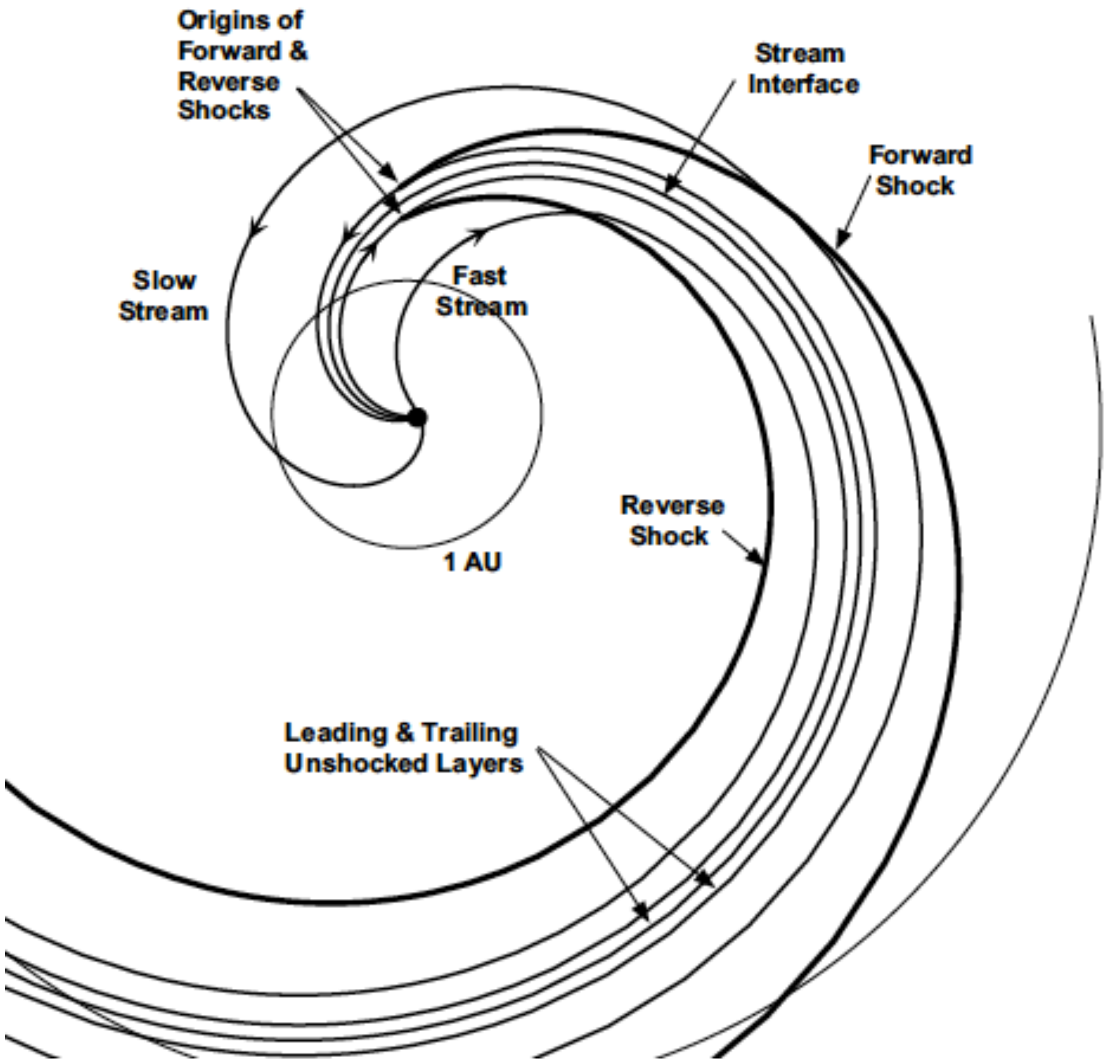
Cane, Reames, and von Rosenvinge 1988 (v. 2 ch. 7)

Shocks in Heliophysics



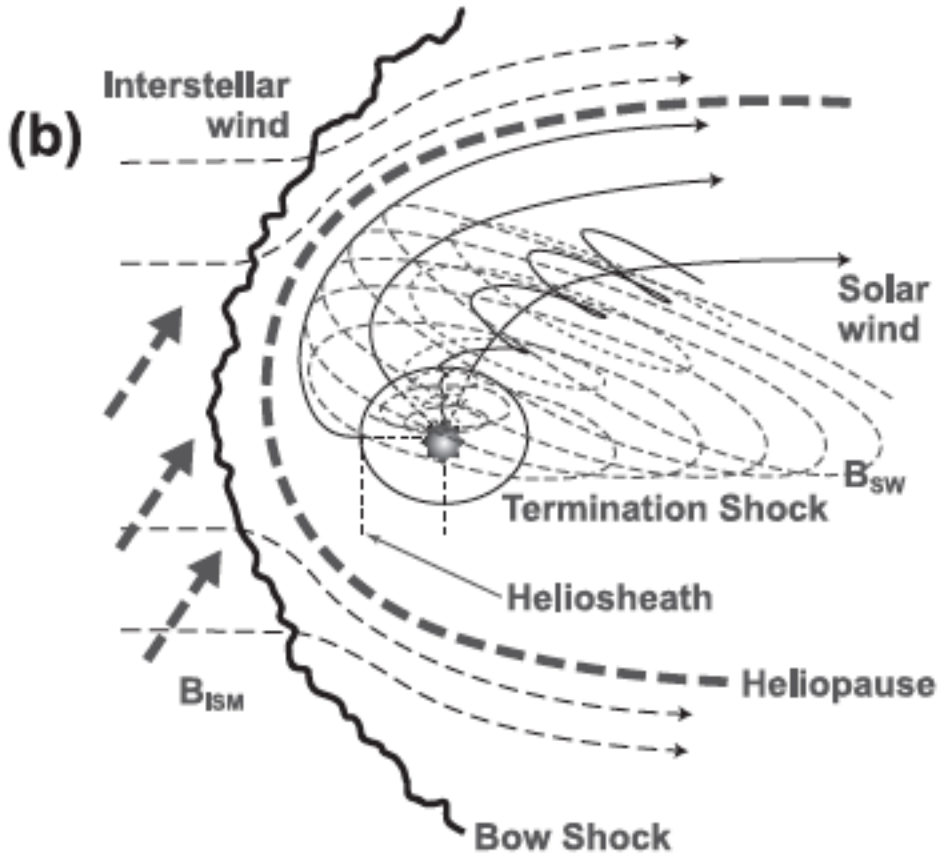
Forbes 2000 (v. 2, ch. 6)

Shocks in Heliophysics

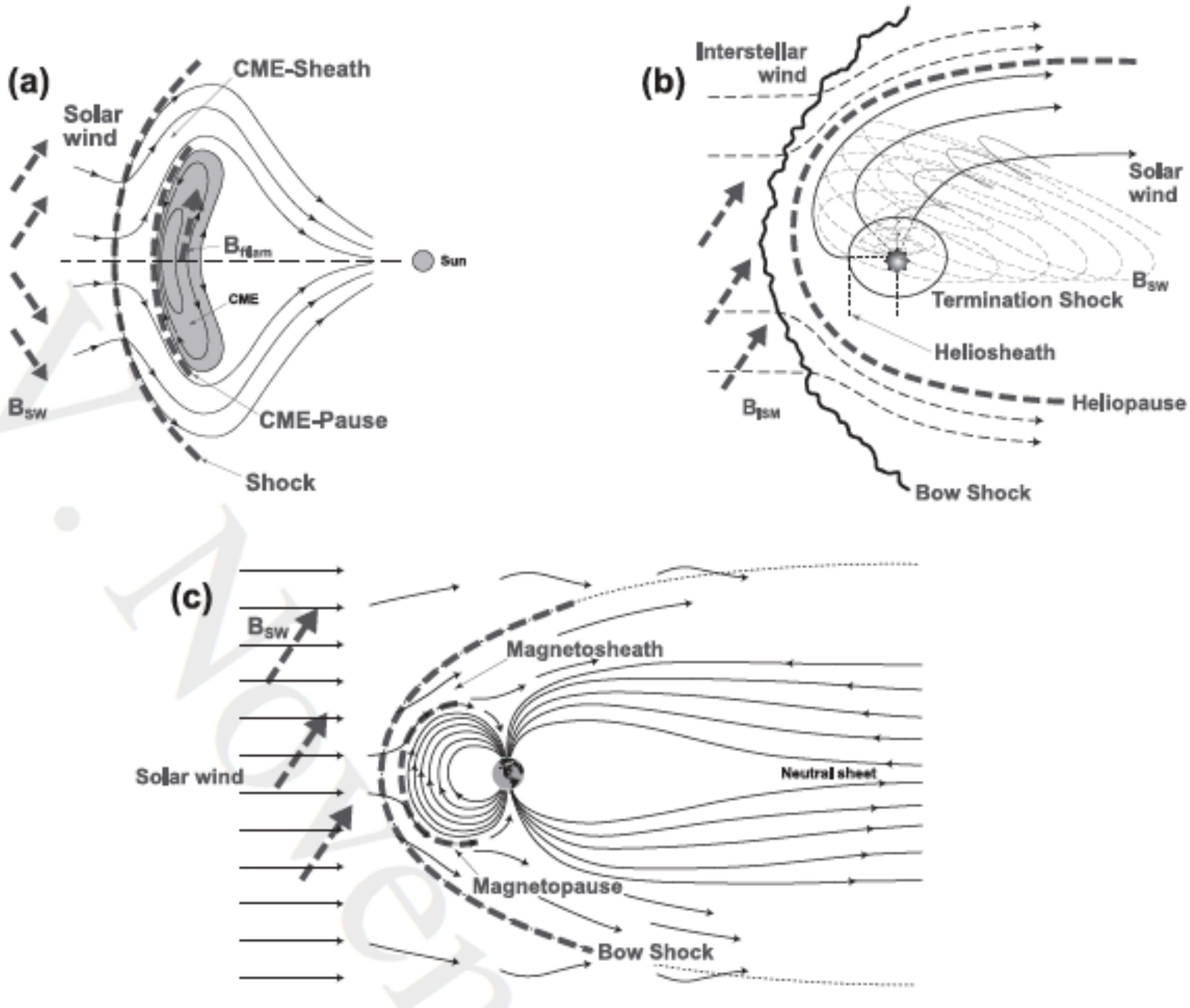


Crooker, Gosling, et al. 1999 (v3, ch. 8)

Shocks in Heliophysics



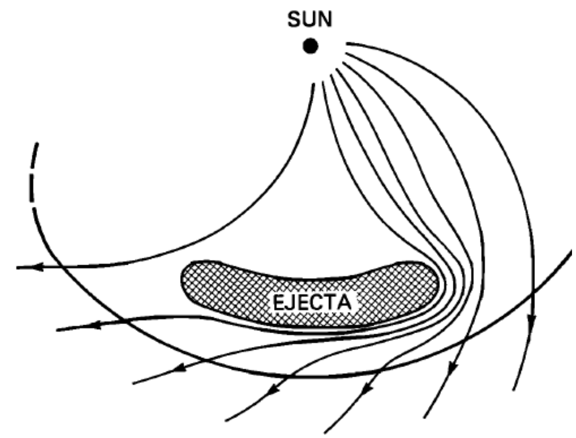
Opher v2, ch. 7


(a)
CME-Sheath
Solar
wind
B_{ISM}
Sun
CME
B_{SW}
CME-Pause
Shock
(b)
Interstellar
wind
Solar
wind
B_{SW}
Termination Shock
Heliosheath
Heliopause
B_{ISM}
Bow Shock
(c)
B_{SW}
Magnetosheath
Solar wind
Neutral sheet
Magnetopause
Bow Shock

Shocks in the air

Shocks are…

- Collision between 2 different plasmas



- A hydrodynamic surprise

Shocks in Hydrodynamics

Viscosity, exchanges thermal & kinetic energy

mass $$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v} \right]$$

momentum $$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + p\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma} \right]$$

energy $$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} \right) = -\nabla \cdot \left[\tfrac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \overleftrightarrow{\sigma} \cdot \mathbf{v} \right]$$

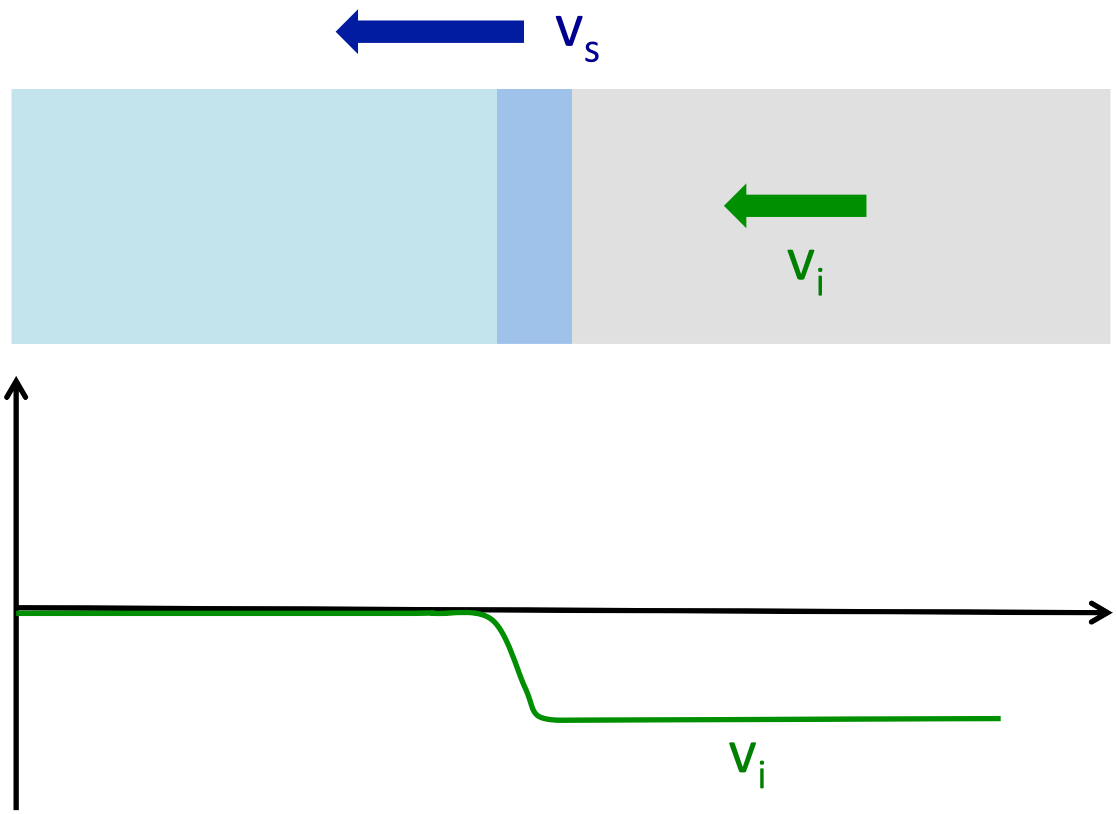
kinetic thermal

viscous stress tensor $$\sigma_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij} \nabla \cdot \mathbf{v} \right)$$

mass $$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v} \right]$$

momentum $$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + p\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma} \right]$$ $$\sigma_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij} \nabla \cdot \mathbf{v} \right)$$

energy $$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} \right) = -\nabla \cdot \left[\tfrac{1}{2}\rho v^2 \mathbf{v} + \frac{\gamma}{\gamma - 1} p\mathbf{v} + \overleftrightarrow{\sigma} \cdot \mathbf{v} \right]$$



collision

mass

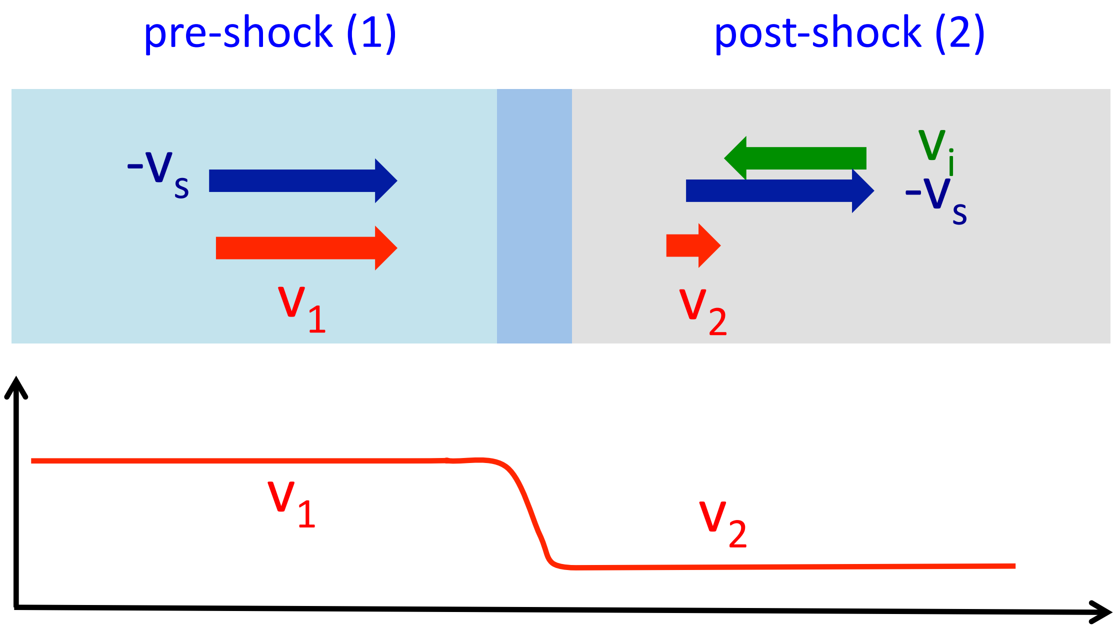
$$\cancel{\frac{\partial \rho}{\partial t}} = -\nabla\cdot\left[\rho\mathbf{v}\right]$$

momentum

$$\cancel{\frac{\partial (\rho\mathbf{v})}{\partial t}} = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} + p\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right] \qquad \sigma_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right)$$

energy

$$\cancel{\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma-1}\right)} = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \frac{\gamma}{\gamma-1}p\mathbf{v} + \overleftrightarrow{\sigma}\cdot\mathbf{v}\right]$$



transform to ref. frame co-moving w/ shock **- shock frame**

Assume

1. Solution is stationary in shock frame

2. Regions 1 & 2 are uniform:

ρ_1 v_1 p_1 ρ_2 v_2 p_2

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right]$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} + p\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \frac{\gamma}{\gamma-1}p\mathbf{v} + \overleftrightarrow{\sigma}\cdot\mathbf{v}\right]$$

jump

$$[[f]] = f_2 - f_1$$

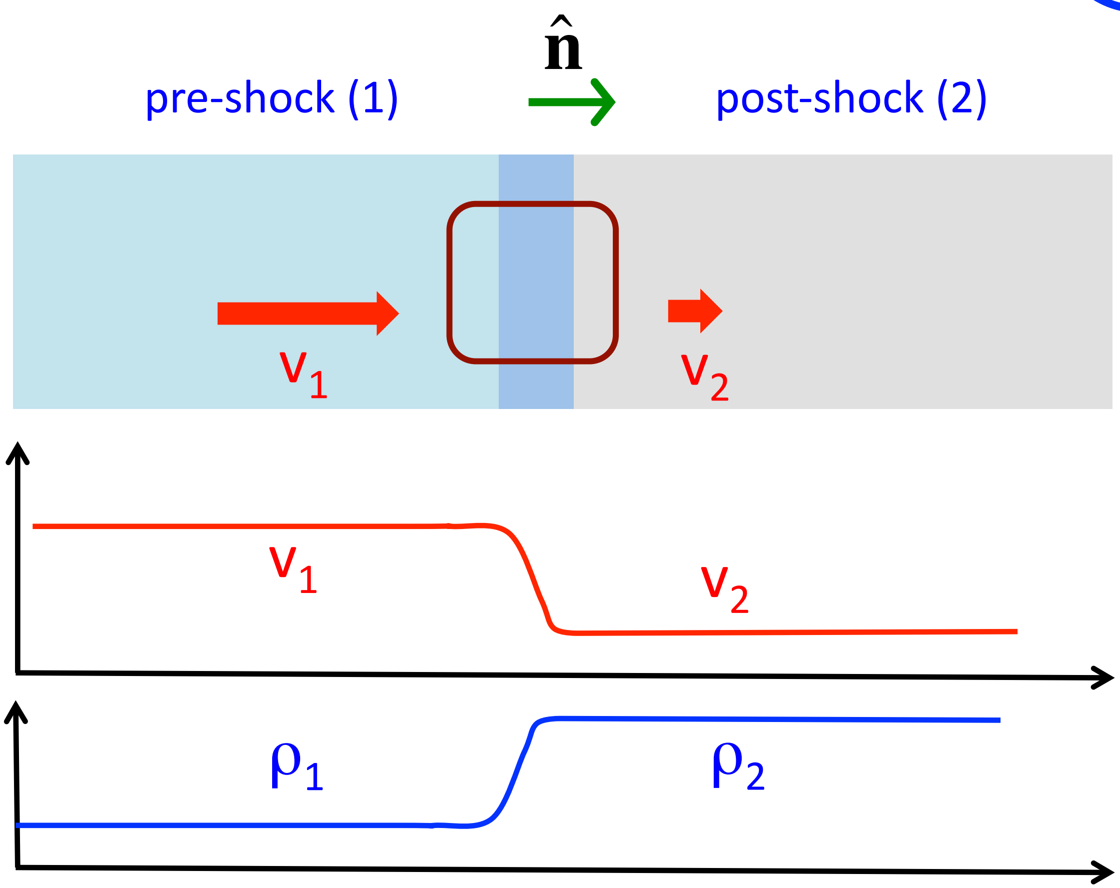
$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n\mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1}pv_n]]$$

Jump conditions

(Rankine-Hugoniot)



2. Regions 1 & 2 are uniform:

$$\sigma_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \tfrac{2}{3}\delta_{ij}\nabla\cdot\mathbf{v}\right) = 0$$

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

$$\rho v_n = \text{const.}$$

$$\rho v_n [[\mathbf{v}]] + [[p]]\hat{\mathbf{n}} = 0$$

$$\rho v_n [[\tfrac{1}{2} v^2 + \tfrac{\gamma}{\gamma-1}(p/\rho)]] = 0$$

Case 1:

$\rho v_n = 0$ - no flow across interface

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

$$\rho v_n = \text{const.}$$

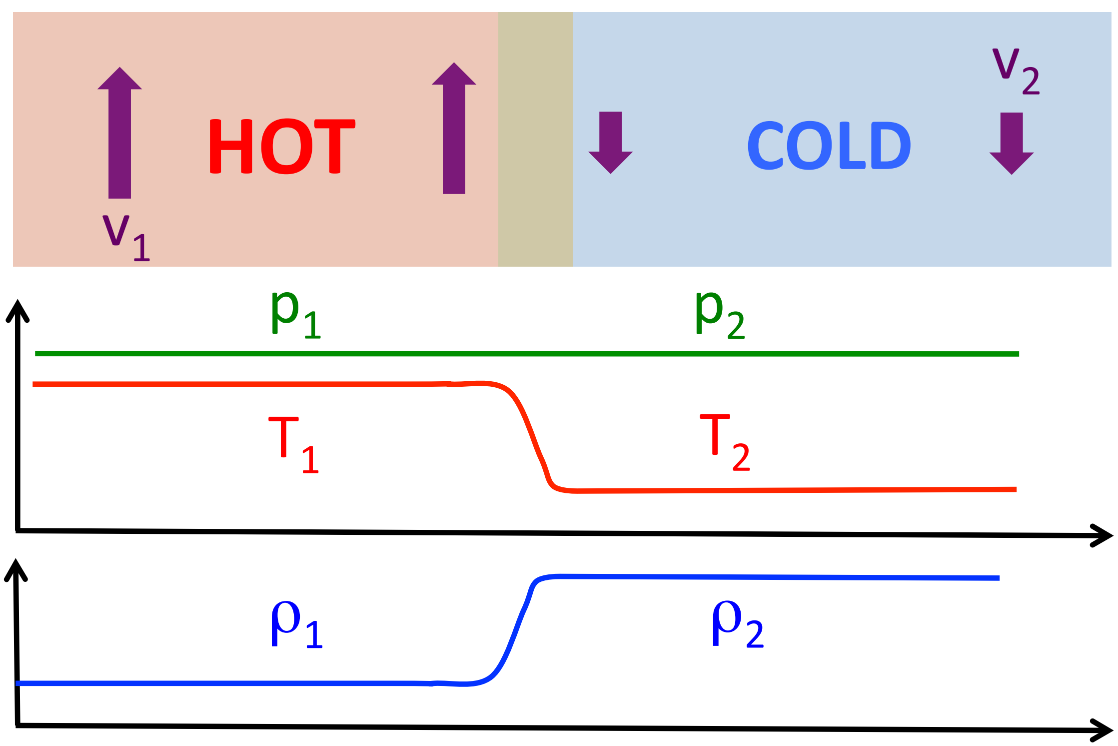
$$\rho v_n [[\mathbf{v}]] + [[p]]\hat{\mathbf{n}} = 0$$

$$\rho v_n [[\tfrac{1}{2} v^2 + \tfrac{\gamma}{\gamma-1}(p/\rho)]] = 0$$

Case 1:

$\rho v_n = 0$ - no flow across interface

➔ $p_2=p_1$ pressure balance



Contact discontinuity

related to linear waves w/ $\omega=0$

entropy mode: $[[\rho]]\neq 0$

shear mode: $[[v_t]] \neq 0$

$$0 = [[\rho v_n]]$$

$$0 = [[\rho v_n \mathbf{v} + p\hat{\mathbf{n}}]]$$

$$0 = [[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]]$$

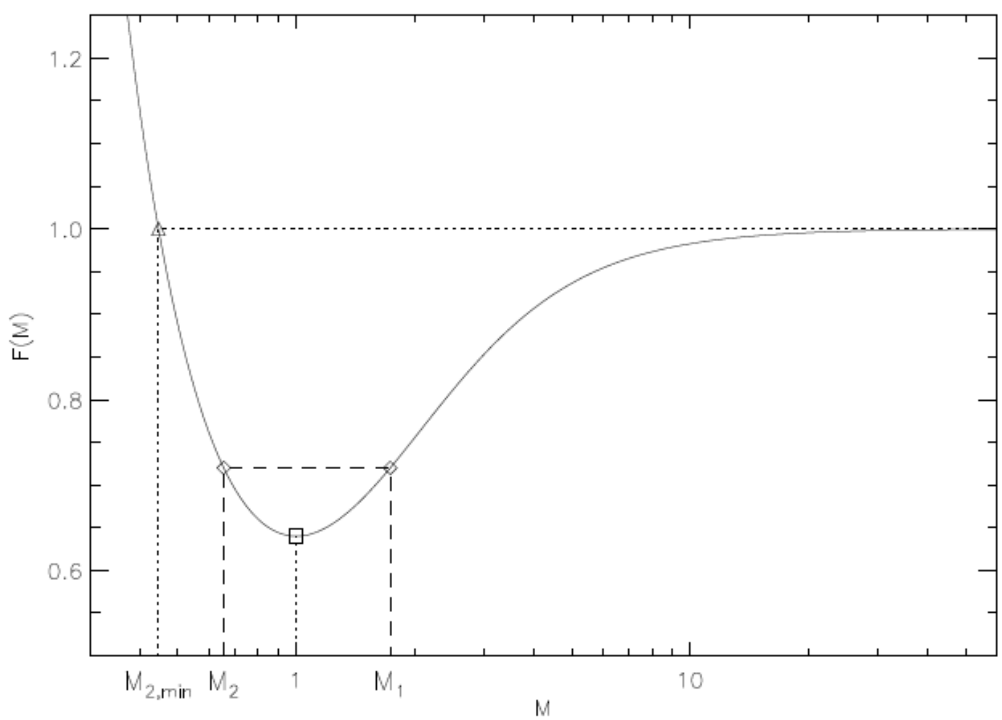
Case 2: $\rho v_n = \text{const.} \neq 0$

$$\rho v_n [[\mathbf{v}_\perp]] = 0$$

$$\Rightarrow \mathbf{v}_{\perp,1} = \mathbf{v}_{\perp,2} = \mathbf{v}_\perp$$

transform away

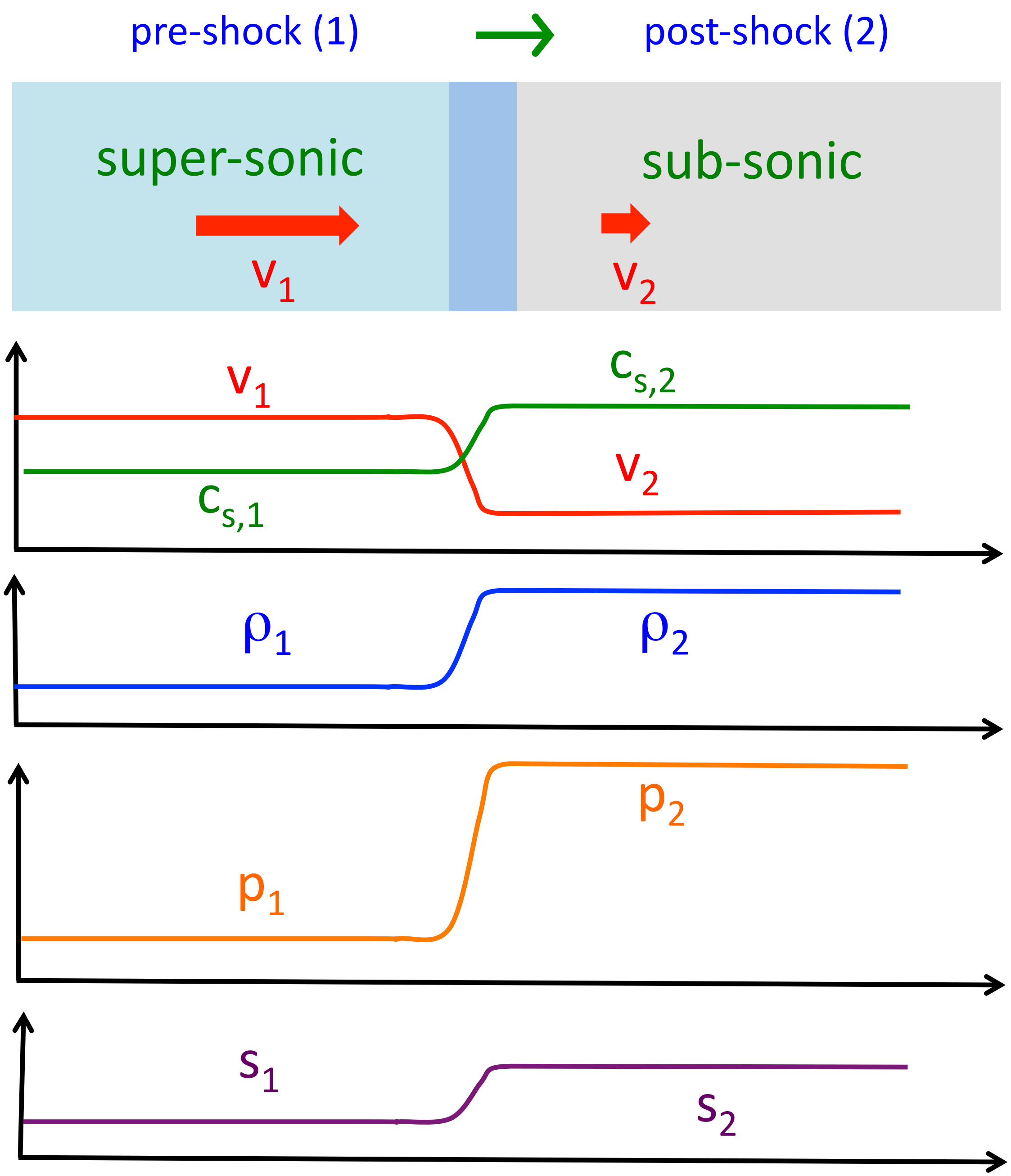
$$\frac{(\rho v^2 + p)^2}{2(\rho v)\left(\frac{1}{2}\rho v^3 + \frac{\gamma}{\gamma-1} p v\right)} = \frac{\gamma - 1}{\gamma^2} \frac{(\gamma M^2 + 1)^2}{M^2\left[(\gamma - 1)M^2 + 2\right]} = F(M)$$



$$M = \frac{v}{\sqrt{\gamma p / \rho}}$$ Mach number

$$F(M_1) = F(M_2)$$

$$M_2 = F^{-1}\left(F(M_1)\right)$$



$$M_2 = F^{-1}\left(F(M_1)\right)$$

$$M_2 = \sqrt{\frac{(\gamma - 1)M_1^2 + 2}{2\gamma M_1^2 - \gamma + 1}}$$

$$\frac{v_2}{v_1} = \frac{(\gamma - 1)M_1^2 + 2}{(\gamma + 1)M_1^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma + 1}$$

$$s_2 - s_1 = \ln\left(\frac{p_2}{p_1}\right) - \gamma \ln\left(\frac{\rho_2}{\rho_1}\right)$$

Hypersonic limit

$M_1 >> 1$ $\gamma = 5/3$

$$M_2 = \sqrt{\frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-\gamma+1}} \qquad M_2 \to \sqrt{\frac{(\gamma-1)}{2\gamma}} \qquad = \frac{1}{\sqrt{5}}$$

$$\frac{v_2}{v_1} = \frac{(\gamma-1)M_1^2+2}{(\gamma+1)M_1^2} \qquad \frac{v_2}{v_1} \to \frac{(\gamma-1)}{(\gamma+1)} \qquad = \frac{1}{4}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \qquad \frac{\rho_2}{\rho_1} \to \frac{(\gamma+1)}{(\gamma-1)} \qquad = 4$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2-\gamma+1}{\gamma+1} \qquad \frac{p_2}{p_1} \to \frac{2\gamma}{\gamma+1}M_1^2 \qquad p_2 \to \tfrac{3}{2} \times \tfrac{1}{2}\rho_1 v_1^2$$

Entropy's tale

$s_2 \geq s_1$ ➔ pre-shock must be supersonic
➔ fluid goes from super- to sub-sonic
➔ fluid slows down crossing shock
➔ fluid **compresses** crossing shock

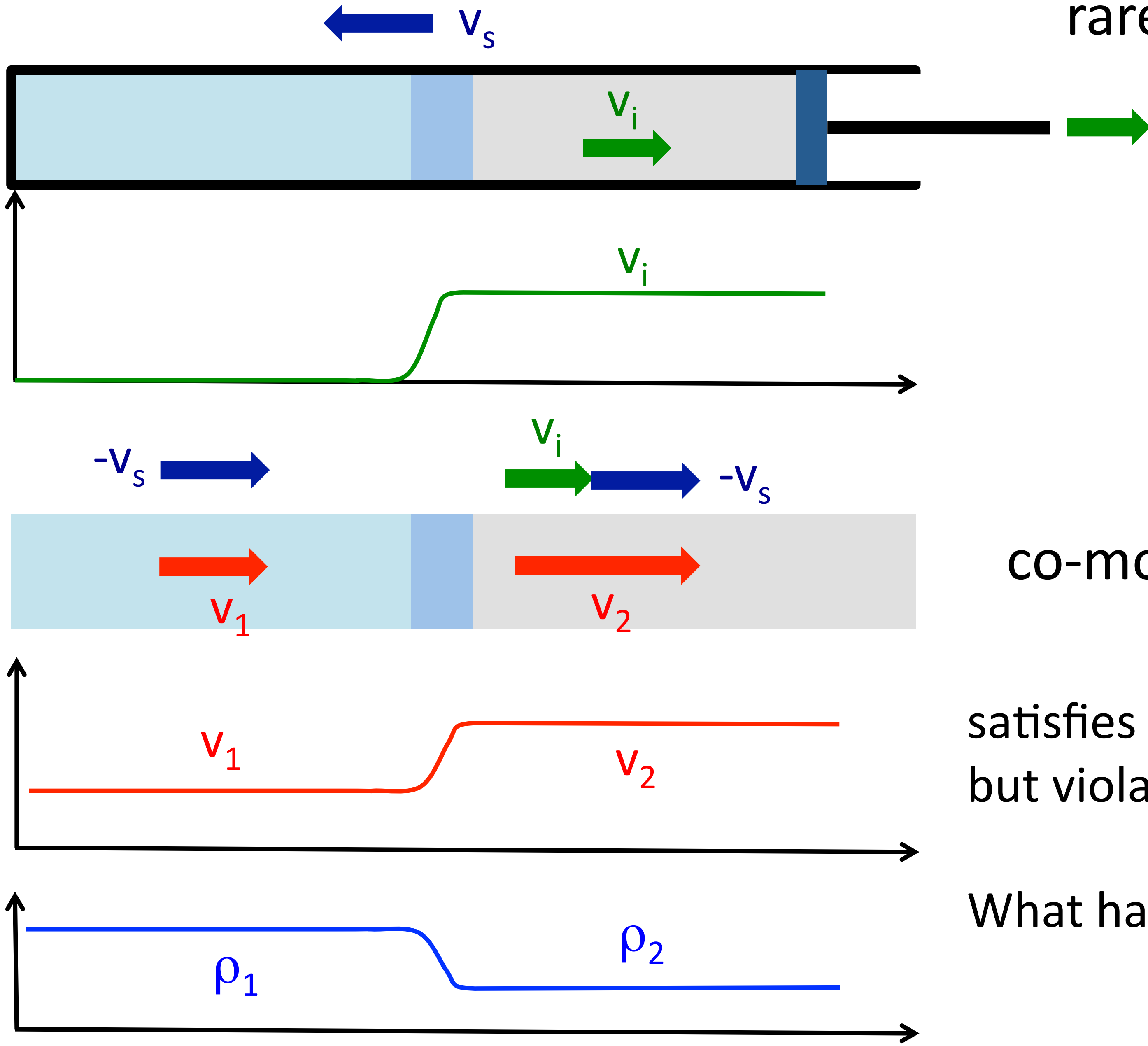
viscosity = 0 ➔ $s_2 = s_1$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \frac{p}{\gamma - 1}\right) = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \frac{\gamma}{\gamma - 1}p\mathbf{v} + \boxed{\overleftrightarrow{\sigma}\cdot\mathbf{v}}\right]$$

viscosity does not appear in jump conditions (conservation laws)
but it is necessary for transition from 1 to 2

Q: what goes wrong if we set $\mu = 0$?
Q: is that diff. from taking $\mu \rightarrow 0$?

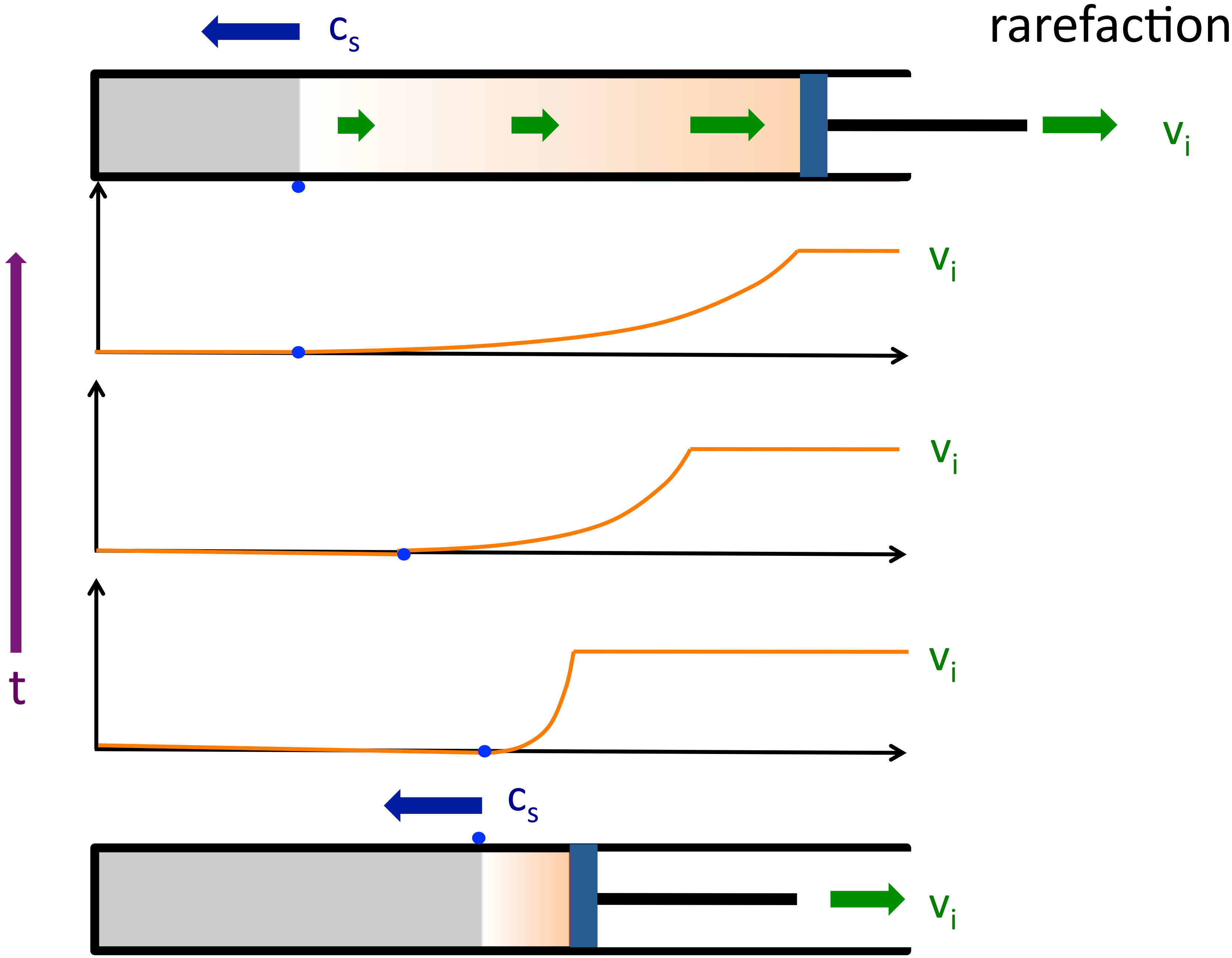
rarefaction



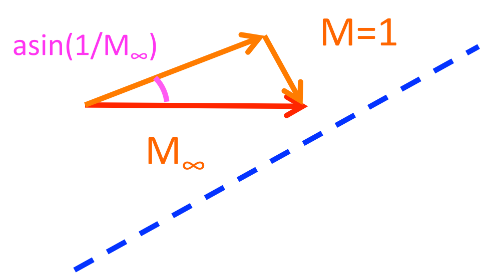
co-moving frame

satisfies conservation
but violates 2nd law TD

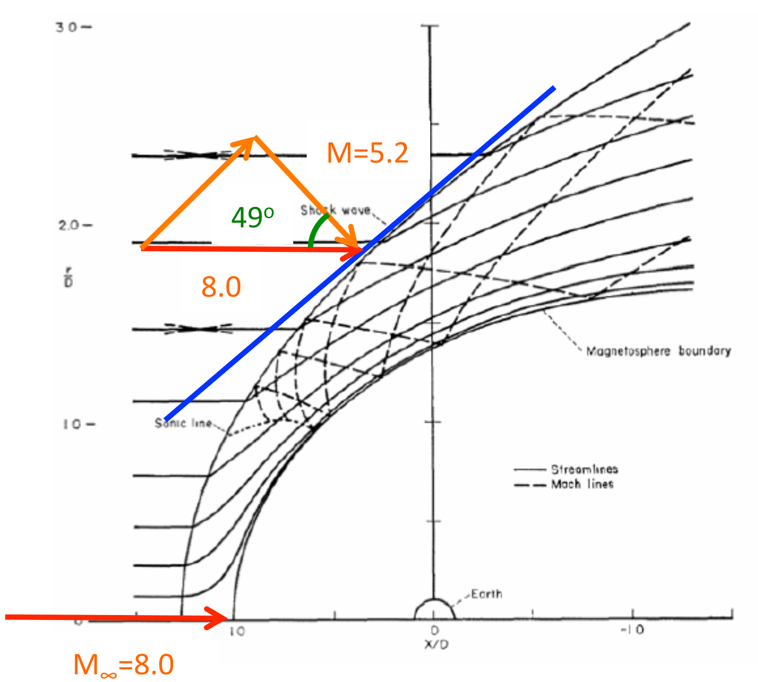
What happens instead?

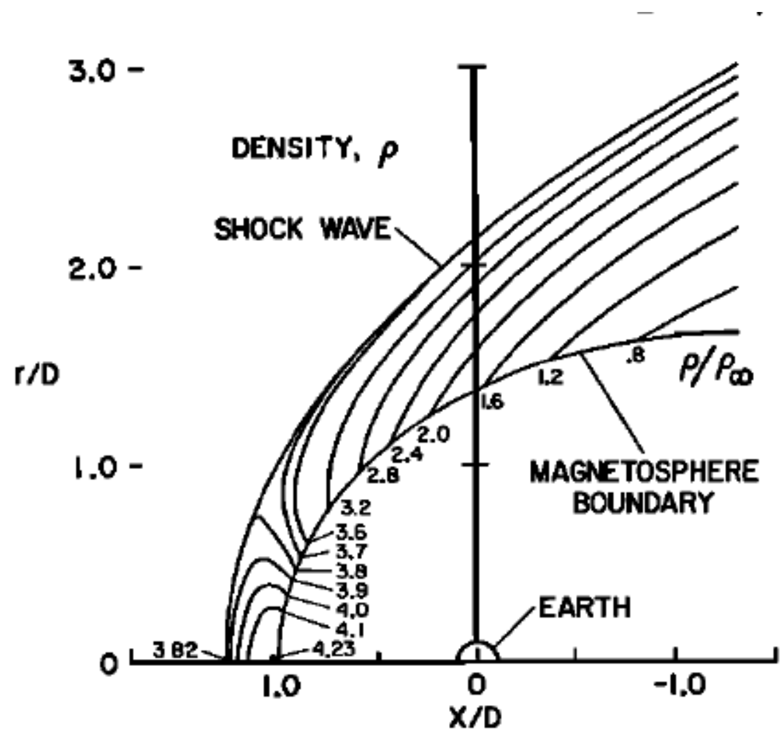

rarefaction
c
s
v
i
v
i
v
i
v
i
t
c
s
v
i

A nearby shock



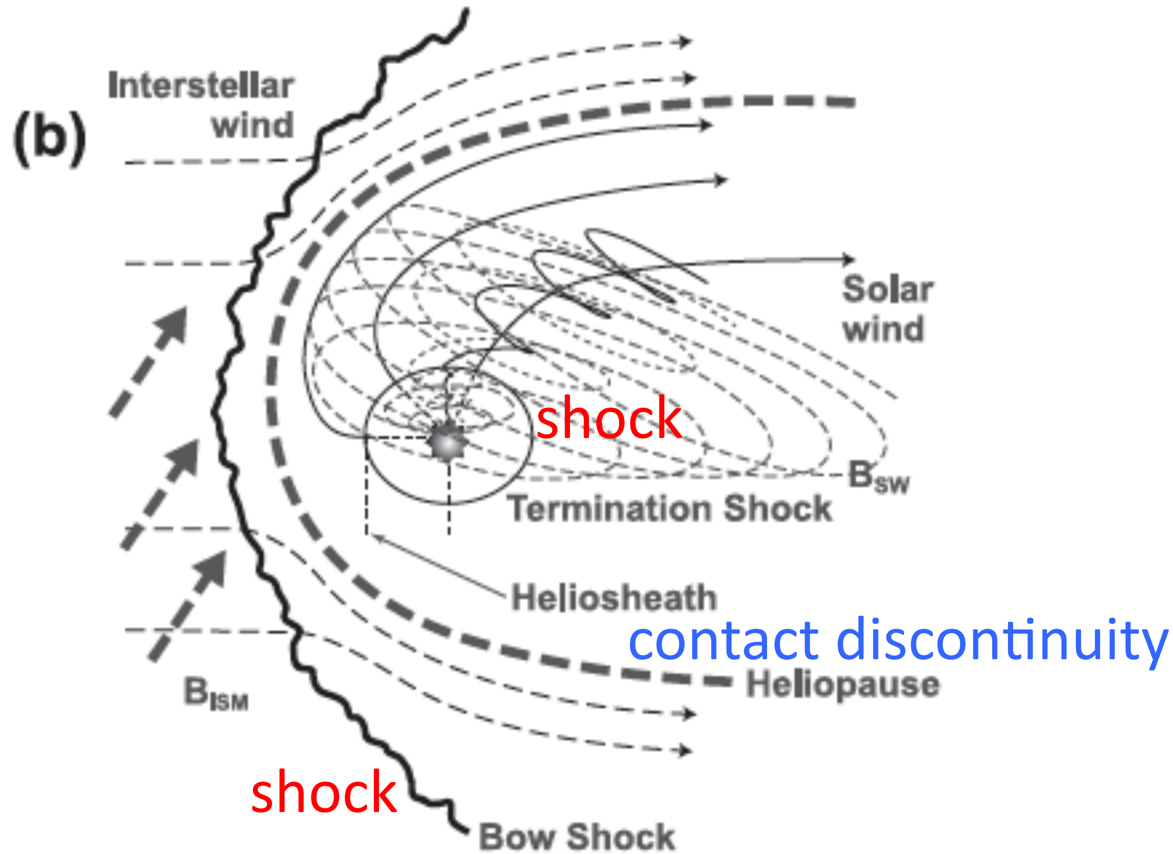
M_∞=8 γ = 5/3





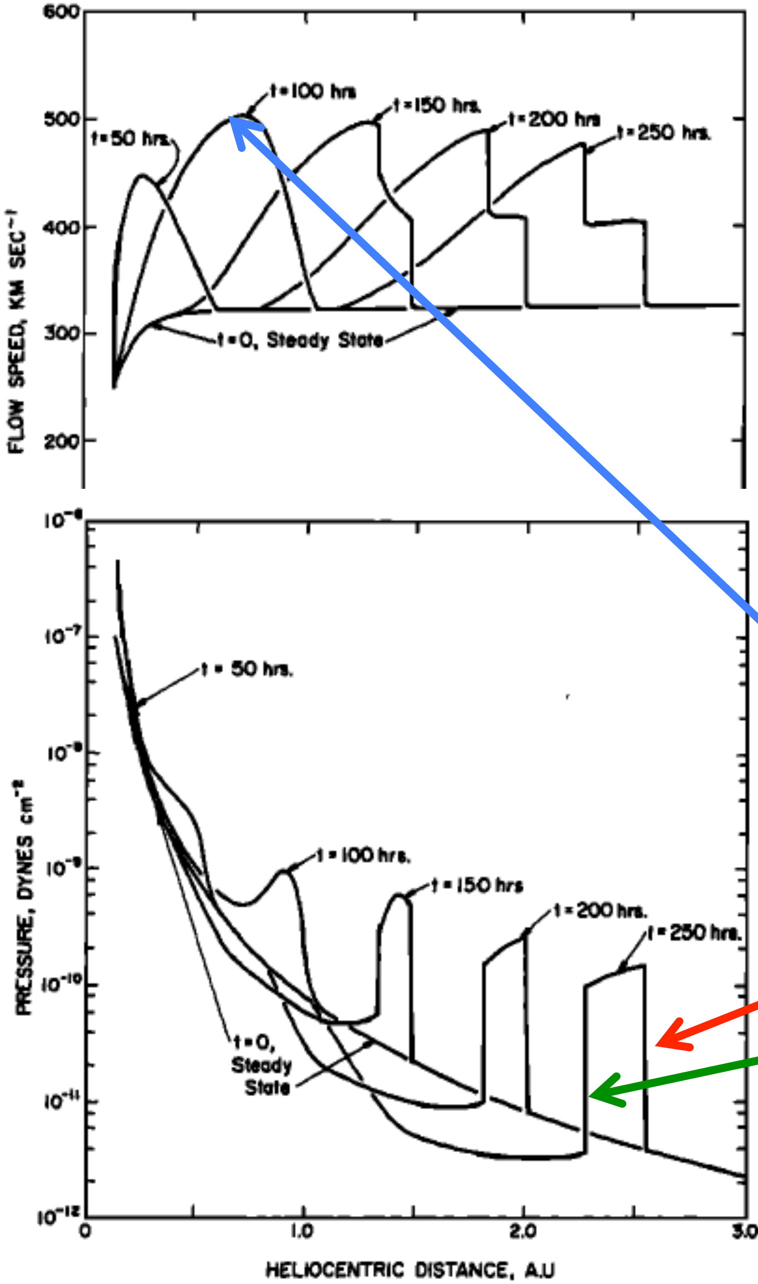
Spreiter, Summers & Alksne 1966

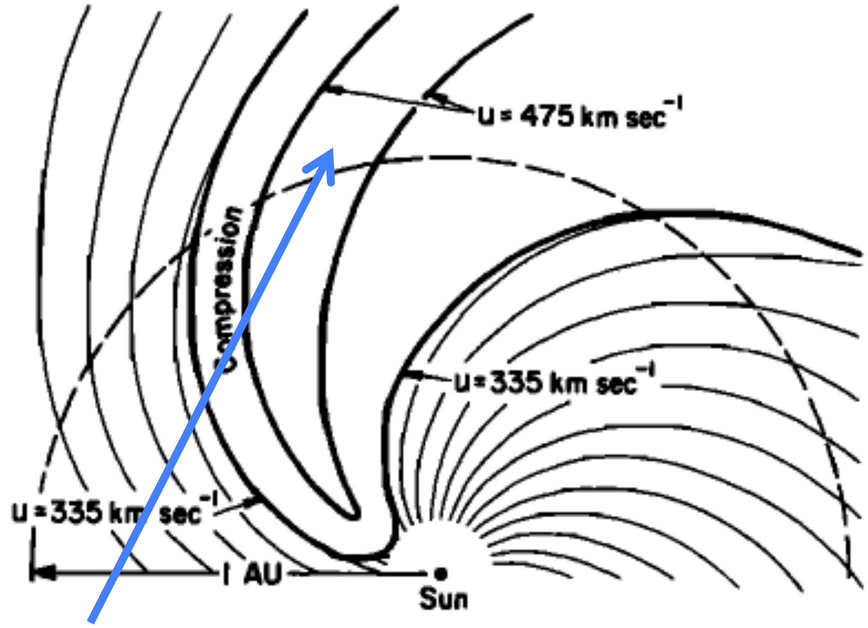
Farther afield…



Opher v2, ch. 7

Hundhausen 1973 (v.3, ch.8)





High speed stream colliding with slower wind creates 2 shocks:

Forward shock

Reverse shock

Both move outward ($v_s > 0$)

Q: which way is $\hat{\mathbf{n}}$ for each?

Shocks in MHD

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v} \right]$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi} B^2 \right) \overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma} \right]$$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\rho v^2 + \tfrac{1}{\gamma - 1} p + \tfrac{1}{8\pi} B^2 \right) = -\nabla \cdot \left[\tfrac{1}{2}\rho v^2 \mathbf{v} + \tfrac{\gamma}{\gamma - 1} p \mathbf{v} - \tfrac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} + \overleftrightarrow{\sigma} \cdot \mathbf{v} \right]$$

kinetic thermal magnetic

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\mathbf{v} \times \mathbf{B} \right]$$

$$0 = \nabla \cdot \mathbf{B}$$

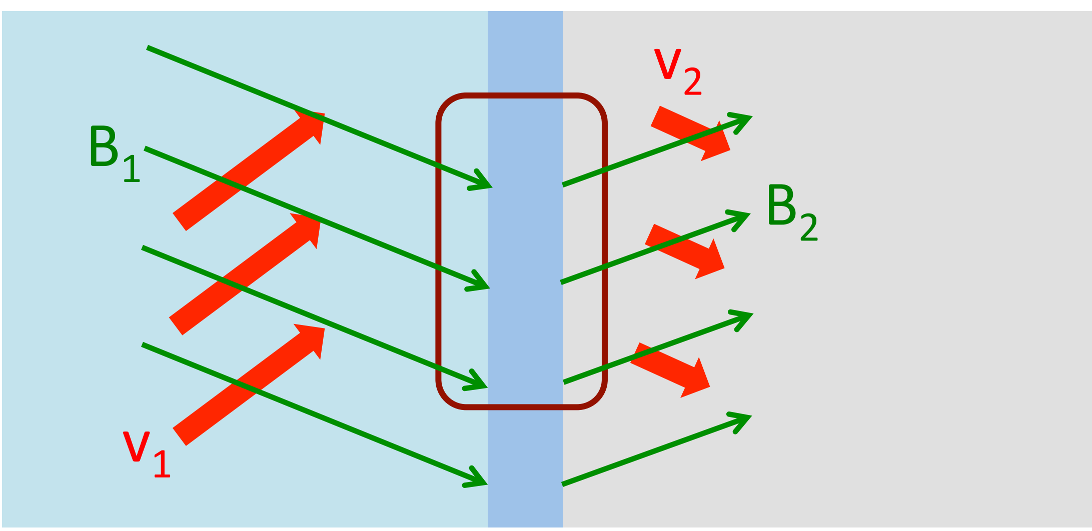
$$0 = -\nabla \cdot \left[\rho \mathbf{v} \right]$$

$$0 = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} - \tfrac{1}{4\pi} \mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi} B^2 \right) \overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma} \right]$$

$$0 = -\nabla \cdot \left[\tfrac{1}{2} \rho v^2 \mathbf{v} + \tfrac{\gamma}{\gamma - 1} p \mathbf{v} - \tfrac{1}{4\pi} (\mathbf{v} \times \mathbf{B}) \times \mathbf{B} + \overleftrightarrow{\sigma} \cdot \mathbf{v} \right]$$

$$0 = -\nabla \times \left[\mathbf{v} \times \mathbf{B} \right]$$

$$0 = \nabla \cdot \mathbf{B}$$



$$0 = -\nabla \cdot \left[\rho \mathbf{v} \right]$$

$$\rho v_n = \text{const.}$$

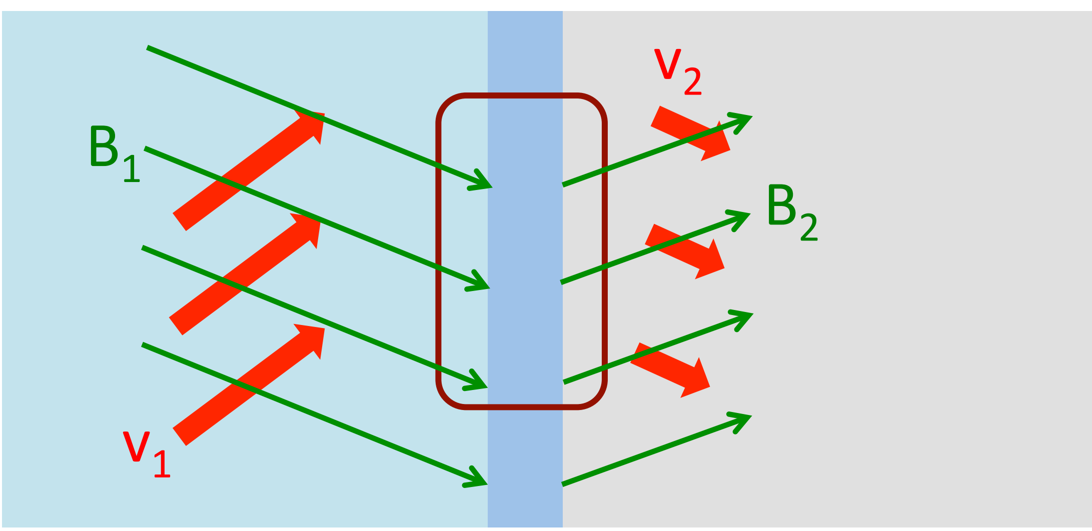
$$0 = \nabla \cdot \mathbf{B}$$

$$B_n = \text{const.}$$

$$0 = -\nabla \cdot \left[\rho \mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi} B^2 \right) \overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma} \right]$$

$$\rho v_n [[\mathbf{v}]] - B_n [[\mathbf{B}]]/4\pi + [[p + B^2/8\pi]]\hat{\mathbf{n}} = 0$$

$$\rho v_n [[\mathbf{v}_\perp]] = B_n [[\mathbf{B}_\perp]]/4\pi$$



$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

$$\rho v_n[[\mathbf{v}_\perp]] = B_n[[\mathbf{B}_\perp]]/4\pi \qquad 0 = B_n[[\mathbf{B}]]$$

Case 1: $\rho v_n = 0$

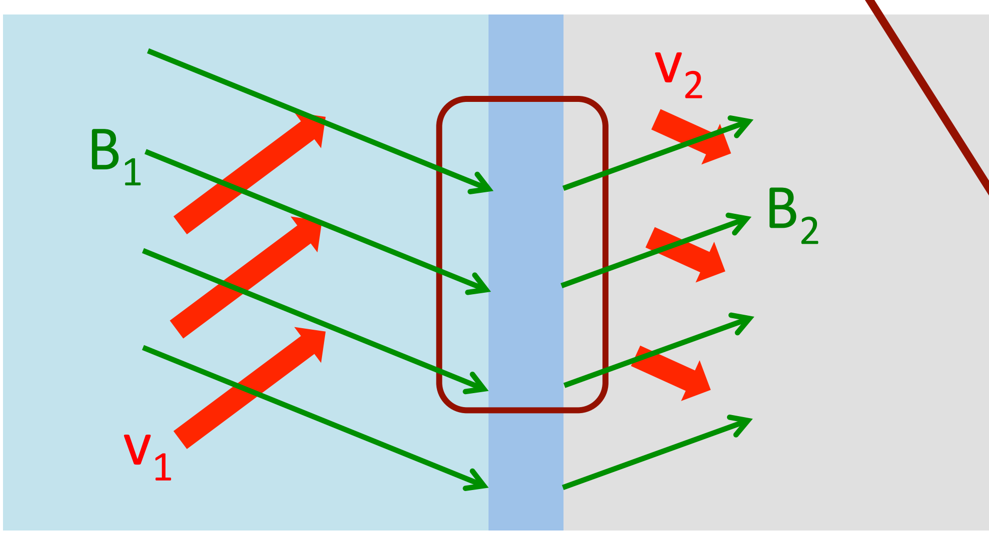
Case 2: $B_n = 0$

Case 1a: $B_n = 0$

Tangential Discontinuity

Case 1b: $[[\ \mathbf{B}\ ]] = 0$

Contact Discontinuity



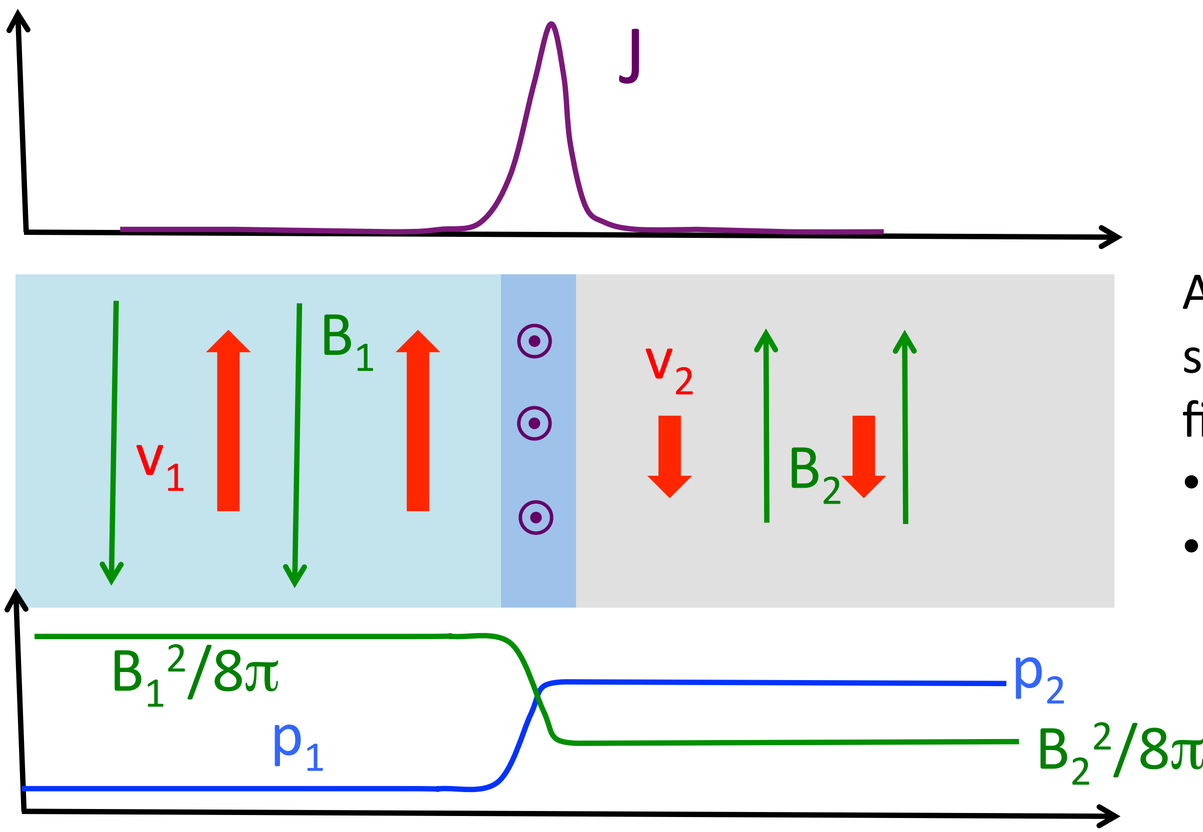
Case 1a: Tangential Disc.

$v_n = 0, \quad B_n = 0$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathbf{I}} + \overleftrightarrow{\sigma}\right] \qquad [[p + B^2/8\pi]]\hat{\mathbf{n}} = 0$$



An equilibrium current sheet separating uniform **B** fields w/ different angles.

- total pressure balance
- possible shear

Case 1b: Contact Disc.

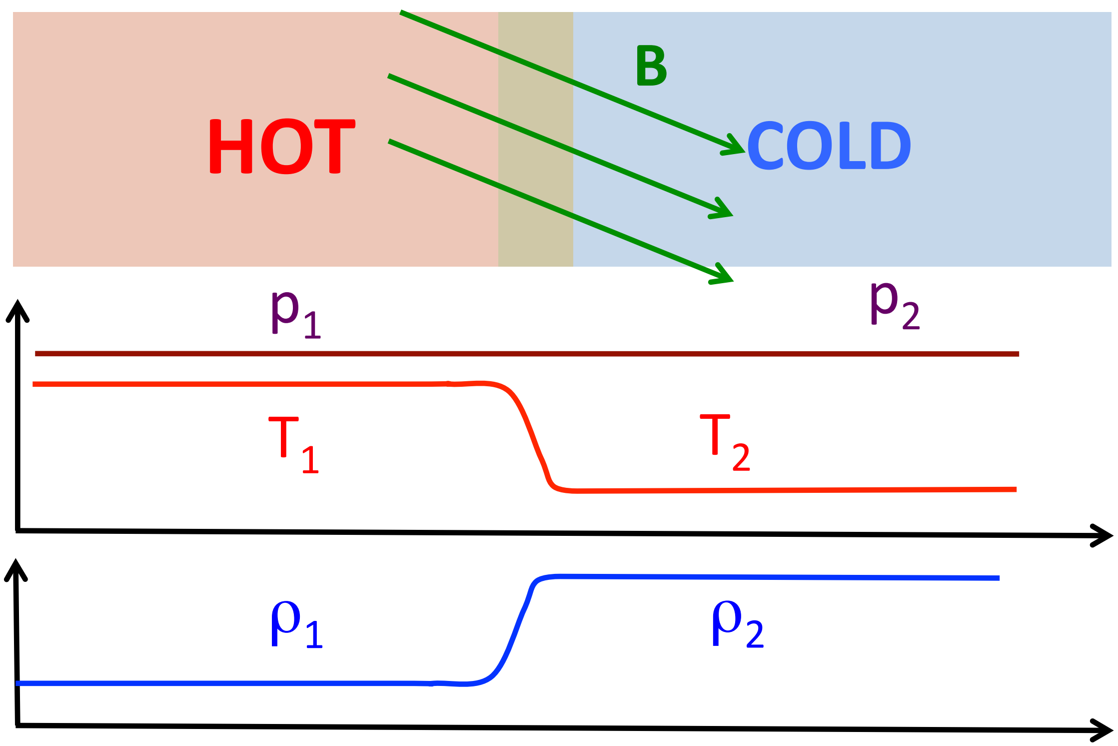
$v_n = 0$, [[**B**]] = 0

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.}$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.}$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right] \qquad [[p]]\hat{\mathbf{n}} = 0$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right] \qquad [[\mathbf{v}_\perp]] = 0$$



related to linear waves w/ $\omega=0$

entropy mode: $[[\rho]]\neq 0$

~~shear mode: $[[v_t]]\neq 0$~~

Case 2: perpendicular shock

$B_n = 0, \quad v_n \neq 0$

$$\rho v_n [[\mathbf{v}_\perp]] = B_n [[\mathbf{B}_\perp]]/4\pi$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} = \mathbf{v}_{\perp,2} = \mathbf{v}_\perp$$

transform away

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

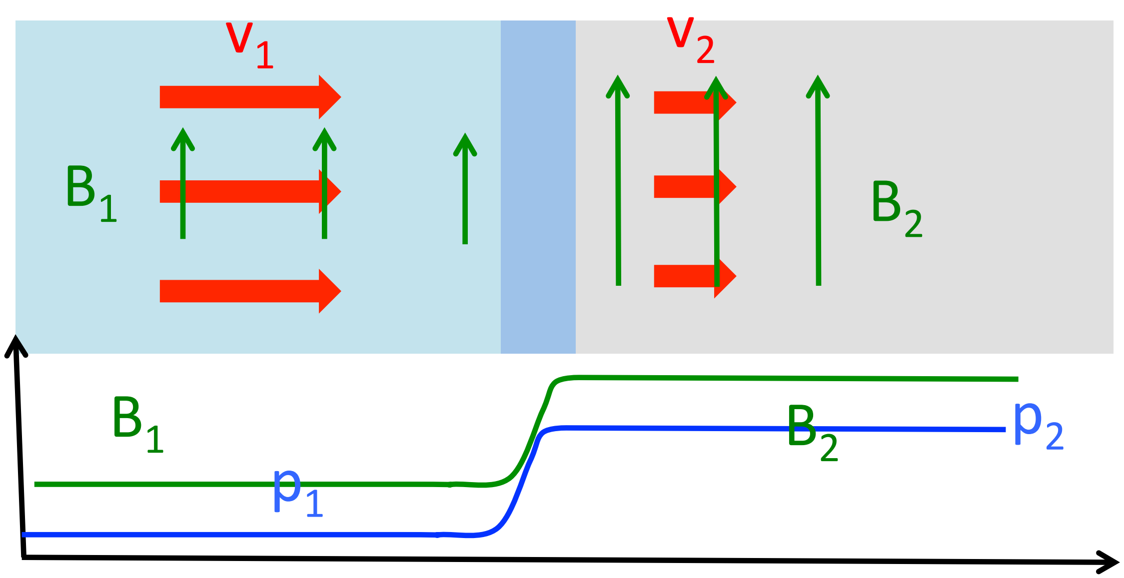
$$[[\rho v^2 + p + B^2/8\pi]] = 0$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \overleftrightarrow{\sigma}\cdot\mathbf{v}\right]$$

$$[[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1}p v_n + \tfrac{1}{4\pi}B^2 v_n]] = 0$$

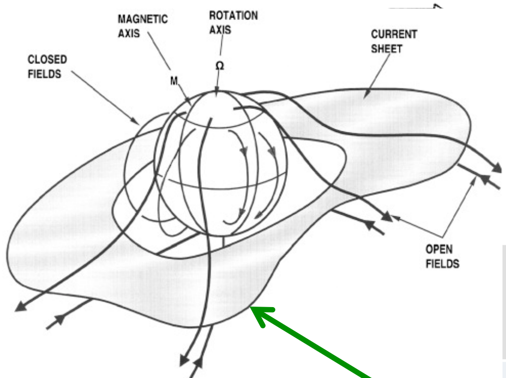
$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]$$

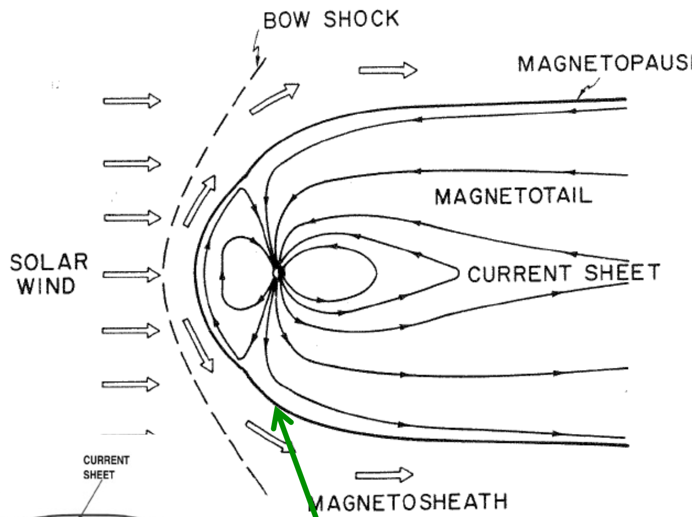
$$[[B v_n]] = 0$$

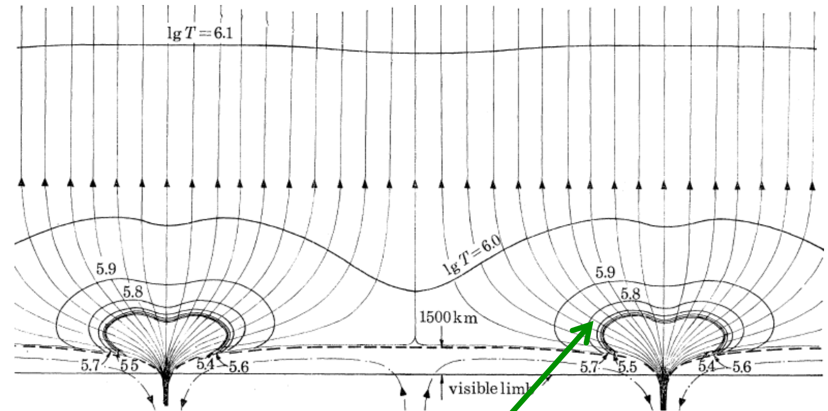


HD shock w/ 2 ``fluids''

- plasma: $\gamma = 5/3$
- mag. field: $\gamma = 2$

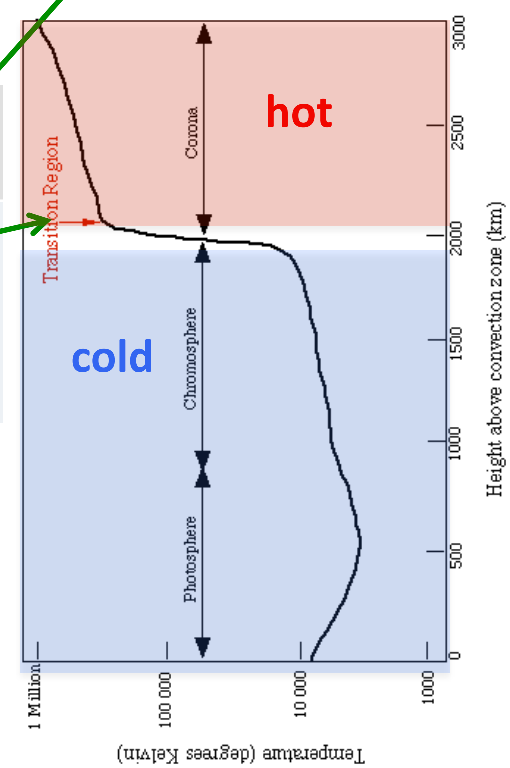






Gabriel 1976

	$B_n = 0$	$B_n \neq 0$
$v_n = 0$	Tangential discontinuity	Contact discontinuity
$v_n \neq 0$	Perpendicular shock	



Case 3: General MHD shock

$B_n \neq 0, \quad v_n \neq 0$

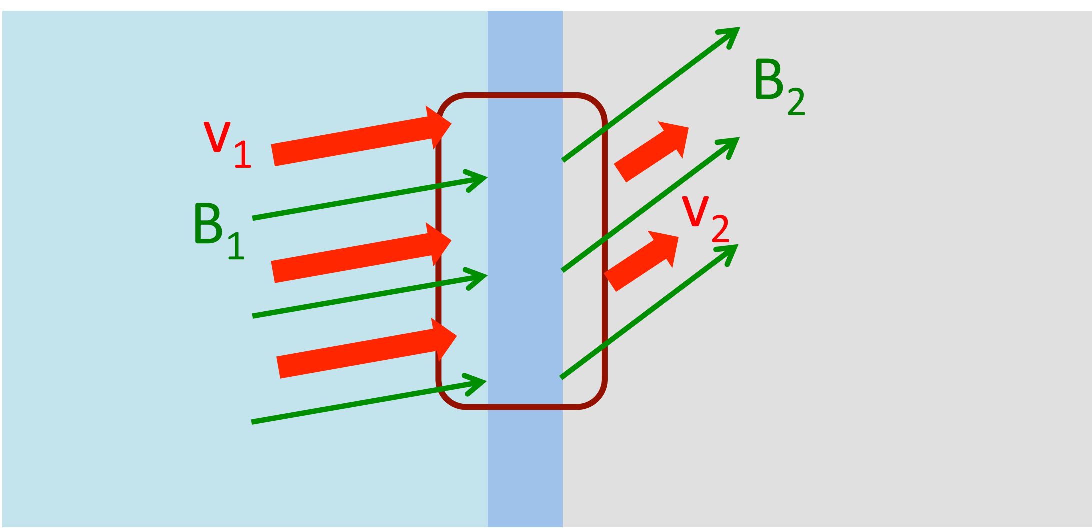
$$0 = -\nabla\cdot\left[\rho\mathbf{v}\right] \qquad \rho v_n = \text{const.} \neq 0$$

$$0 = \nabla\cdot\mathbf{B} \qquad B_n = \text{const.} \neq 0$$

$$0 = -\nabla\cdot\left[\rho\mathbf{v}\mathbf{v} - \tfrac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(p + \tfrac{1}{8\pi}B^2\right)\overleftrightarrow{\mathrm{I}} + \overleftrightarrow{\sigma}\right]$$

$$0 = -\nabla\cdot\left[\tfrac{1}{2}\rho v^2\mathbf{v} + \tfrac{\gamma}{\gamma-1}p\mathbf{v} - \tfrac{1}{4\pi}(\mathbf{v}\times\mathbf{B})\times\mathbf{B} + \overleftrightarrow{\sigma}\cdot\mathbf{v}\right]$$

$$0 = -\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]$$



$$\rho v_n\,[[\mathbf{v}_\perp]] = B_n[[\mathbf{B}_\perp]]/4\pi$$

$$\Rightarrow \quad \mathbf{v}_{\perp,1} \neq \mathbf{v}_{\perp,2}$$

cannot transform away $\mathbf{v}_\perp$

can transform to deHoffman-Teller frame

$$\mathbf{v}_{\mathrm{dHT}} = \mathbf{v}_{\perp 1} - \frac{v_{n,1}}{B_{n,1}}\mathbf{B}_{\perp,1}$$

In dH-T frame:

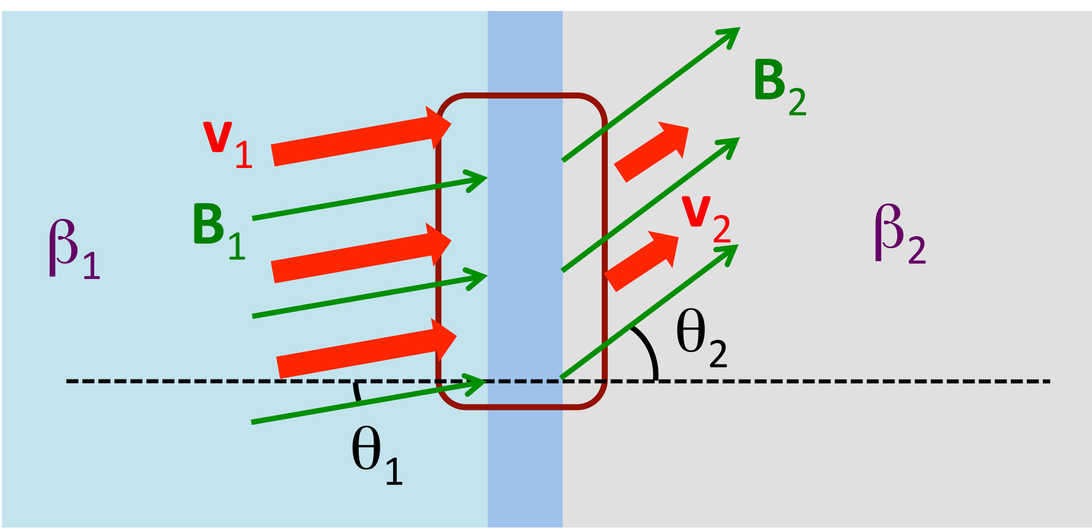
- $\mathbf{v}_1 \;||\; \mathbf{B}_1$
- $\mathbf{E}_1 = \mathbf{v}_1 \times \mathbf{B}_1 = 0$
- $\mathbf{E}_2 = \mathbf{v}_2 \times \mathbf{B}_2 = 0$
- $\mathbf{v}_2 \;||\; \mathbf{B}_2$
- flow is parallel to field lines on both sides

Jump conditions in de Hoffman-Teller frame

$$\rho v_n\,[[\mathbf{v}_\perp]] - B_n[[\mathbf{B}_\perp]]/4\pi = 0$$

$$[[\rho v_n^2 + p + B_\perp^2/8\pi]] = 0$$

$$[[\tfrac{1}{2}\rho v_n v^2 + \tfrac{\gamma}{\gamma-1} p v_n]] = 0$$



Shock characterized by 3 dimensionless pre-shock parameters:

$$M_{1A} = \frac{|\mathbf{v}_1|}{|\mathbf{B}_1|/\sqrt{4\pi\rho_1}} = \frac{v_{1n}}{B_{1n}/\sqrt{4\pi\rho_1}}$$

$$\theta_1 = \tan^{-1}\left(\frac{B_{\perp 1}}{B_{n1}}\right)$$

$$\beta_1 = \frac{p_1}{B_1^2/8\pi}$$

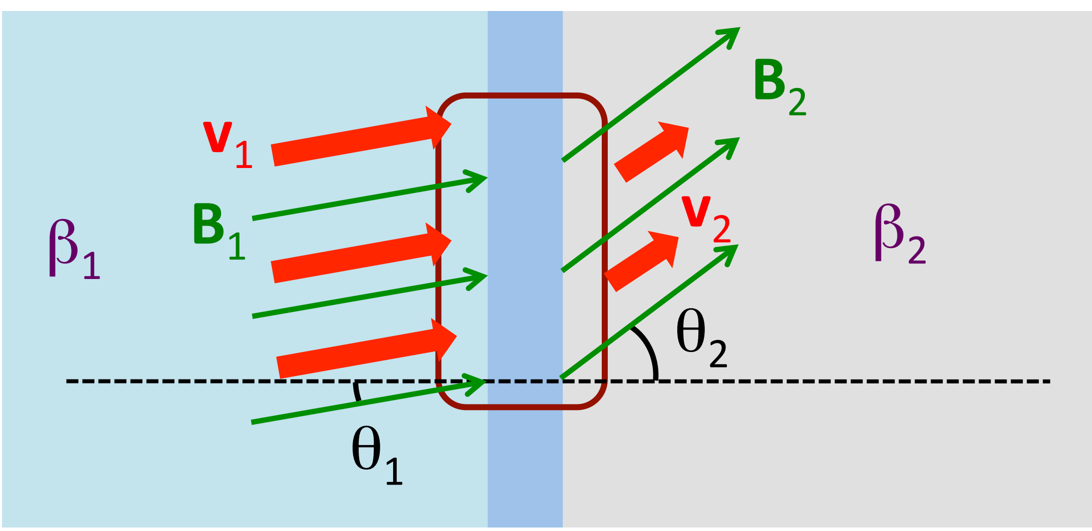
Use 3 jump. conds to solve for downstream parameters

M_{A2} θ_2 β_2

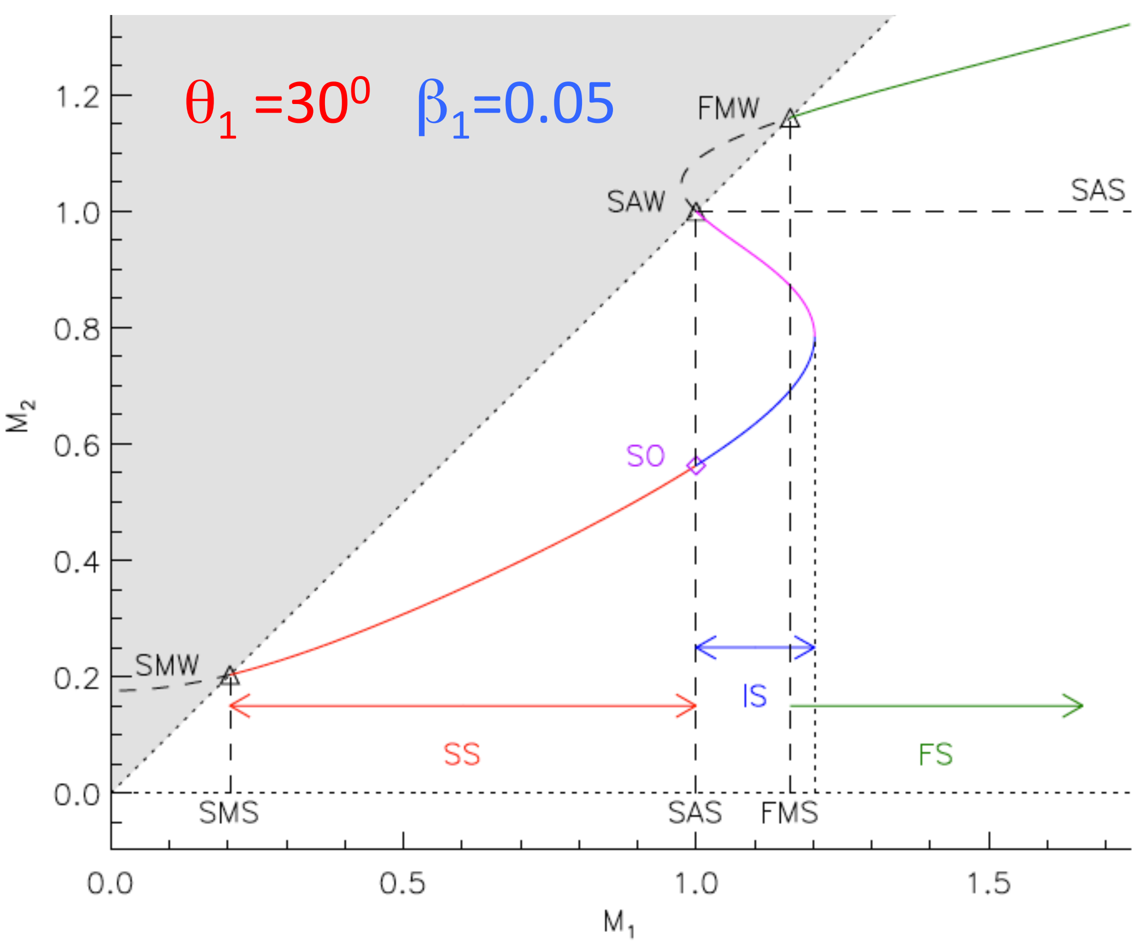
$$\rho v_n\,[[\mathbf{v}_\perp]] - B_n[[\mathbf{B}_\perp]]/4\pi = 0 \qquad\qquad [[(M_A^2 - 1)\tan^2\theta]] = 0$$

eliminate θ_2 in terms of θ_1, ${M_{A1}}^2$ and ${M_{A2}}^2$
etc. until...

$$\begin{aligned} F(M_1, M_2) \;=\; & -(\gamma+1)M_2^6 + \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4 \\ & - \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2 \\ & + \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} \;=\; 0 \end{aligned}$$

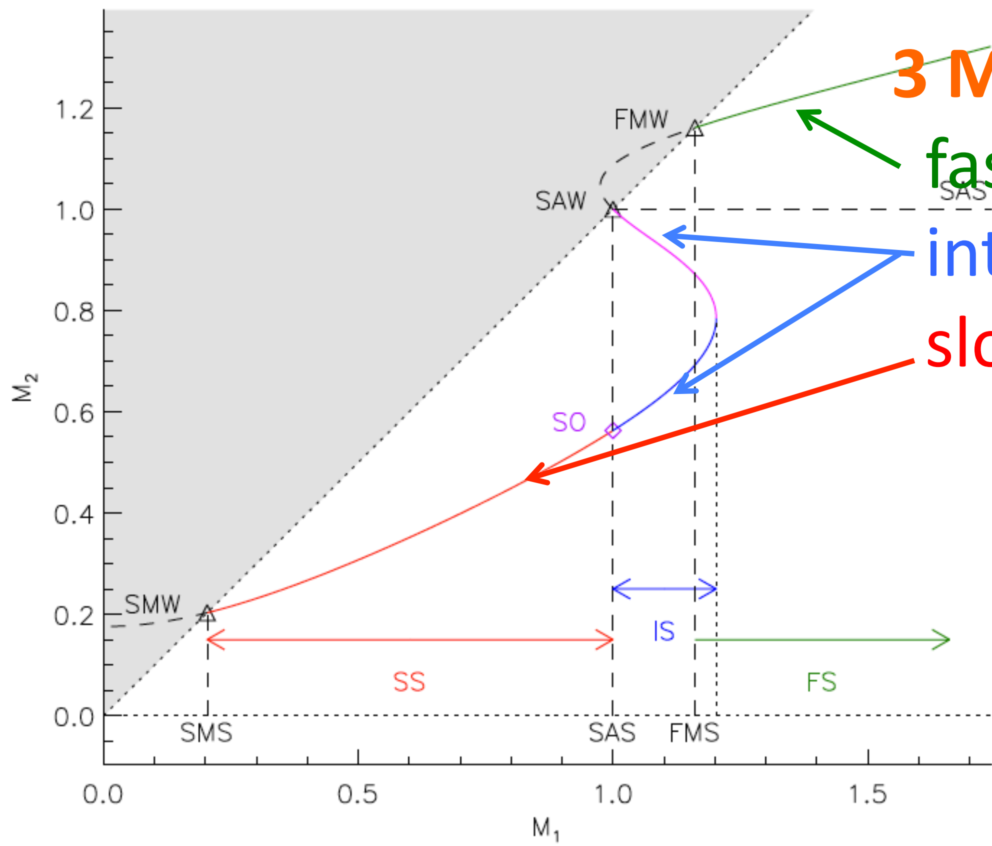


$$
\begin{aligned}
F(M_1, M_2) \;=\; & -(\gamma+1)M_2^6 + \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4 \\
& - \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2 \\
& + \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} \;=\; 0
\end{aligned}
$$



NB:

- $\rho_2/\rho_1 = M_{A1}^2 / M_{A2}^2$
- region $M_{A2} > M_{A1}$ ($\rho_2 < \rho_1$) disallowed by 2nd law thermo.
- $M_{A2} = M_{A1}$ reps. 3 linear waves of MHD
- each shock: flow **faster (slower)** than lin. wave in region **1 (2)**

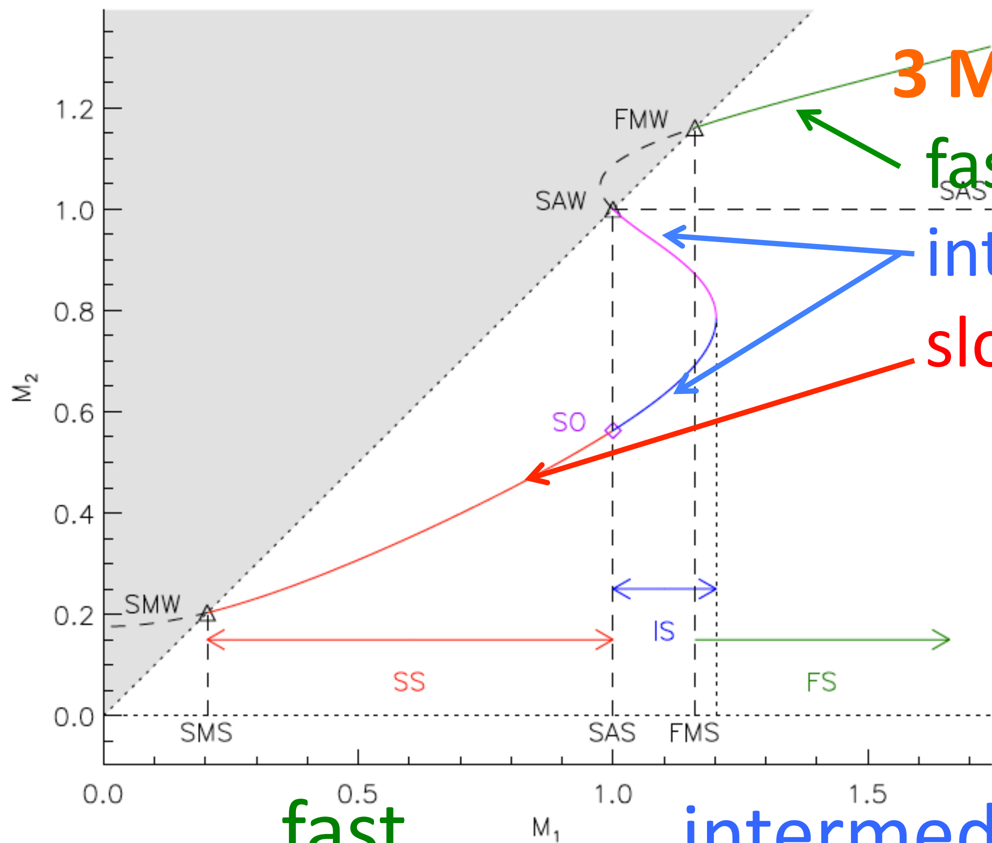


3 MHD shocks

fast: $M_{A1} > M_{A2} > 1$

intermediate: $M_{A1} > 1 > M_{A2}$

slow: $1 > M_{A1} > M_{A2}$



3 MHD shocks

fast: $M_{A1} > M_{A2} > 1$

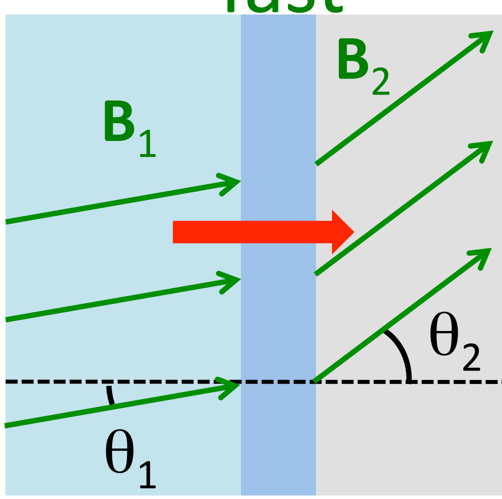
intermediate: $M_{A1} > 1 > M_{A2}$

slow: $1 > M_{A1} > M_{A2}$

$$[[(M_A^2 - 1)\tan^2\theta]] = 0$$

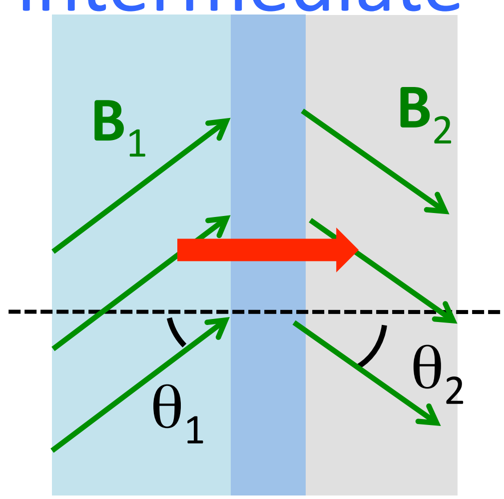
$$\frac{B_{\perp 2}}{B_{\perp 1}} = \frac{\tan\theta_2}{\tan\theta_1} = \frac{M_{A1}^2 - 1}{M_{A2}^2 - 1}$$

fast



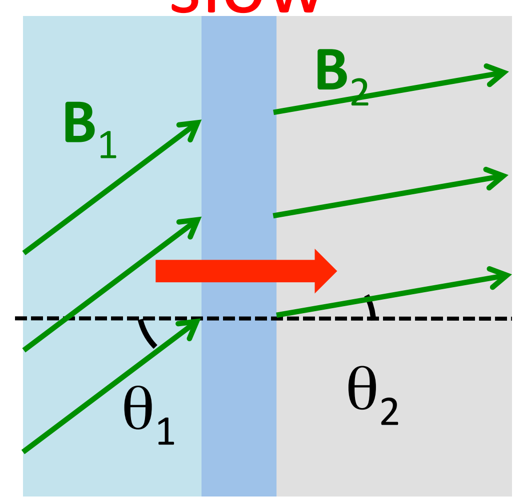
$\theta_2 > \theta_1 > 0$ $B_1 < B_2$

intermediate

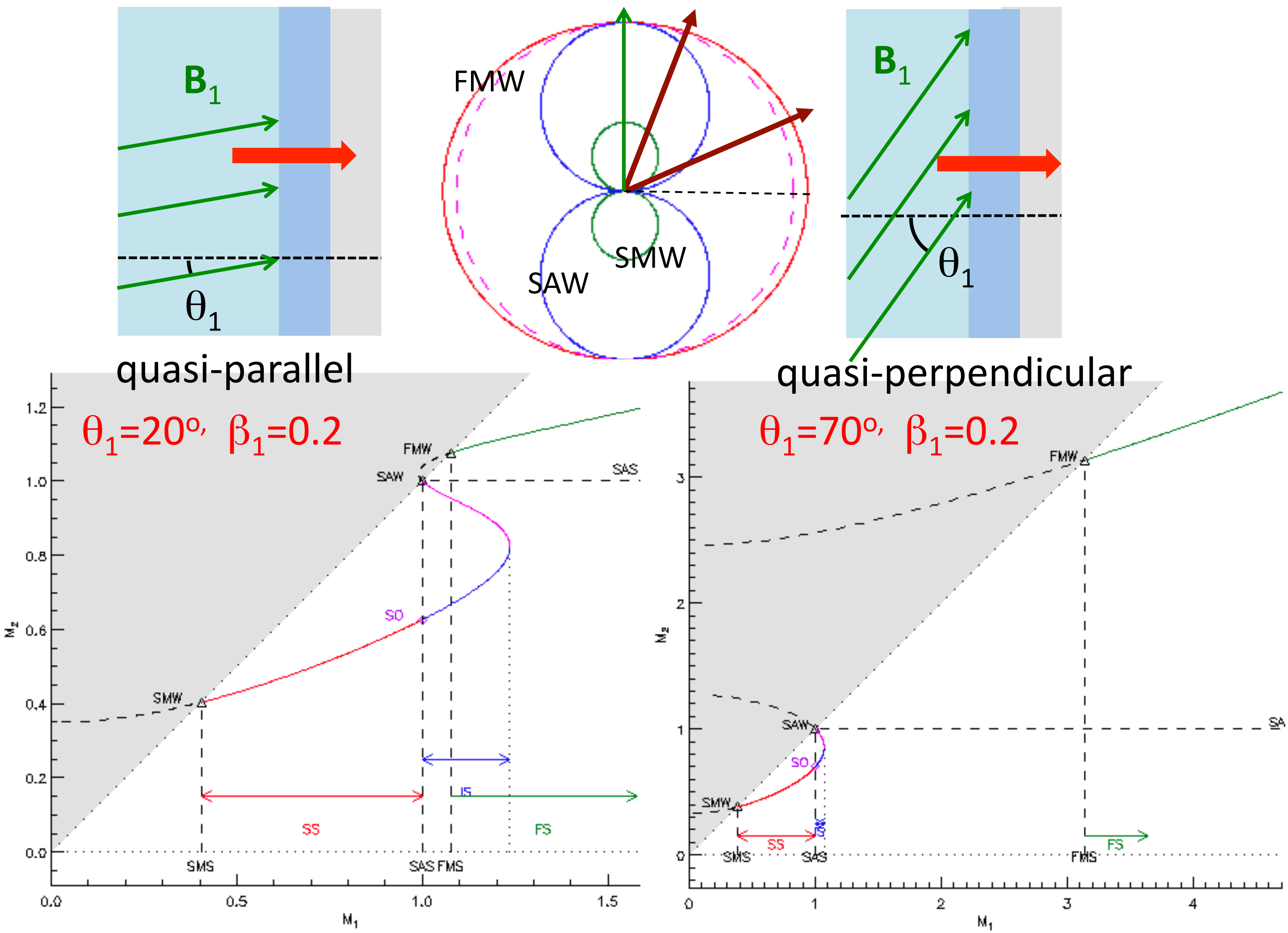


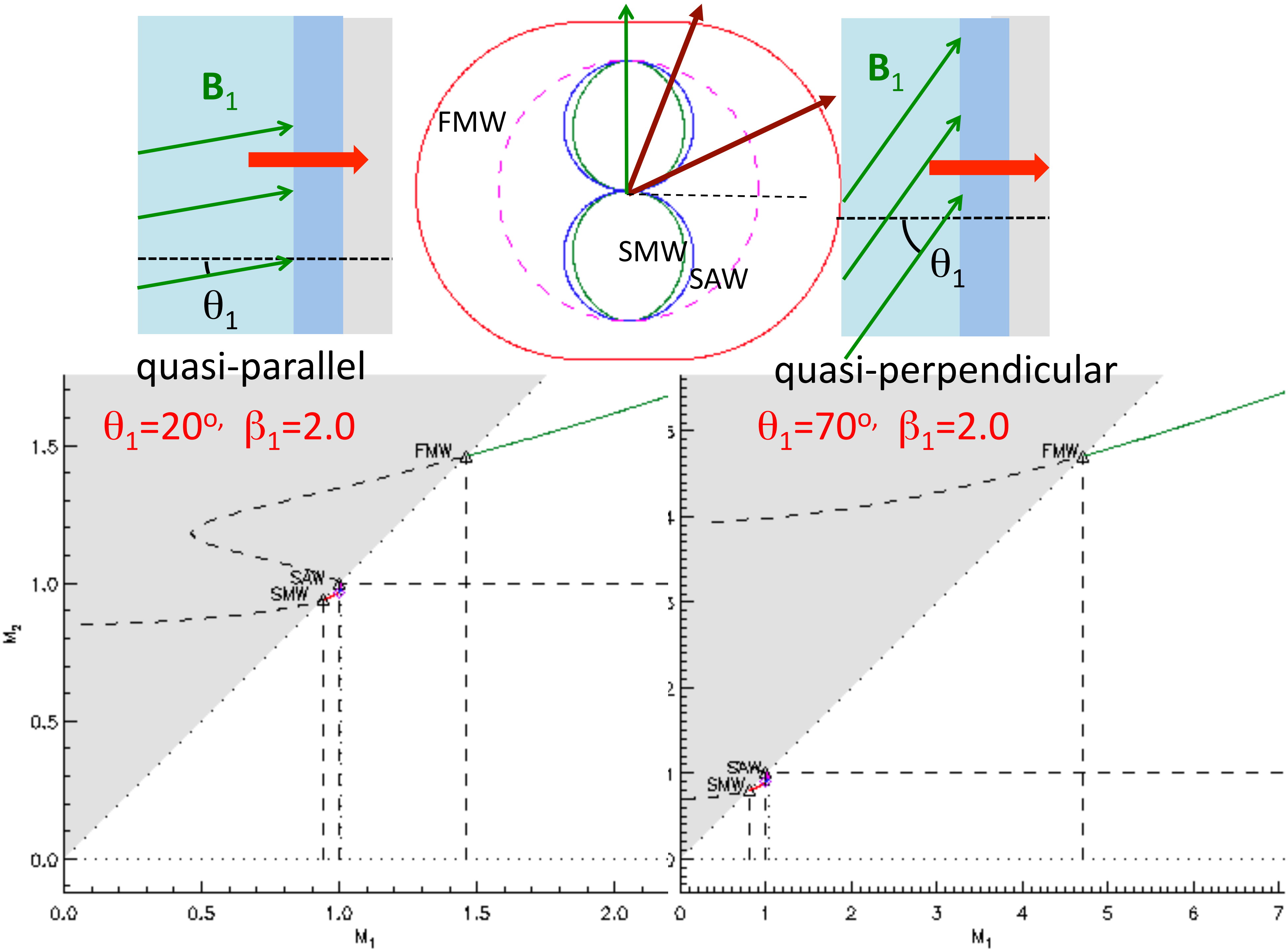
$\theta_1 > 0 > \theta_2$

slow



$\theta_1 > \theta_2 > 0$ $B_1 > B_2$


B1
θ1
quasi-parallel
FMW
SAW
SMW
B1
θ1
quasi-perpendicular
θ1=20°, β1=0.2
FMW
SAW
SAS
SO
SMW
IS
SS
FS
SMS
SAS FMS
M2
M1
θ1=70°, β1=0.2
FMW
SAW
SA
SO
SMW
SS
IS
FS
SMS
SAS
FMS
M2
M1


B1
θ1
FMW
SMW
SAW
quasi-parallel
quasi-perpendicular
θ1=20°, β1=2.0
θ1=70°, β1=2.0
M1
M2

Hypersonic limit

$$F(M_1, M_2) = -(\gamma+1)M_2^6 + \left[(\gamma-1)M_1^2 + 2(\gamma+1) + \frac{\gamma\beta_1}{\cos^2\theta_1} + \gamma\tan^2\theta_1\right] M_2^4$$
$$- \left\{2(\gamma-1)M_1^2 + \frac{(\gamma+1)+2\gamma\beta_1}{\cos^2\theta_1} + (\gamma-2)M_1^2\tan^2\theta_1\right\} M_2^2$$
$$+ \frac{(\gamma-1)M_1^2 + \gamma\beta_1}{\cos^2\theta_1} = 0$$

$$F(M_1, M_2) \simeq -(\gamma+1)M_2^6 + (\gamma-1)M_1^2 M_2^4 + \cdots = 0 \ .$$

$$\frac{M_1^2}{M_2^2} \simeq \frac{\gamma+1}{\gamma-1} \simeq \frac{\rho_2}{\rho_1} \simeq \frac{B_{t,2}}{B_{t,1}} \ , \quad \rightarrow 4$$

Magnetic pressure: only x16

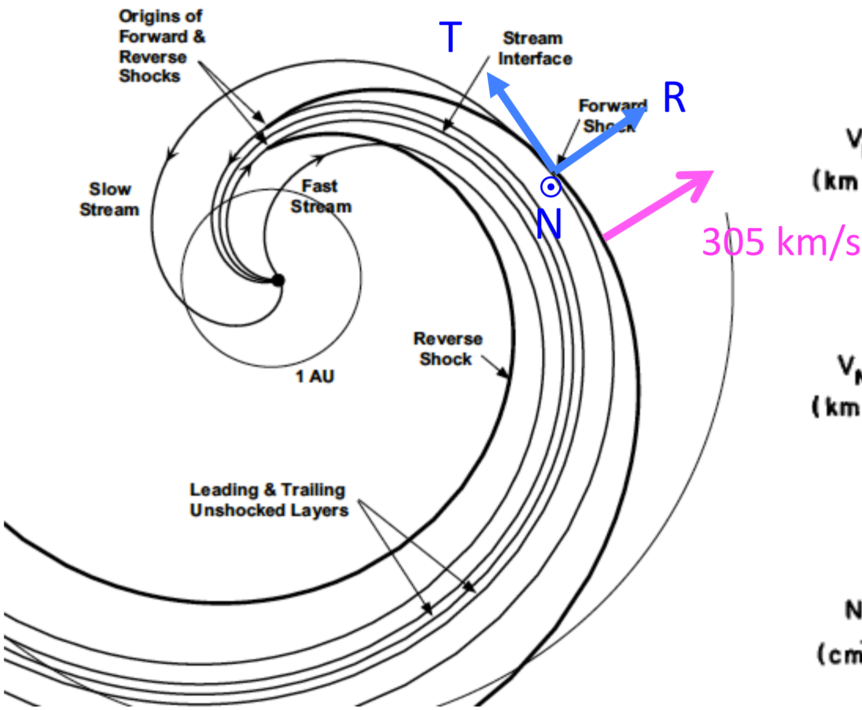
$$p_2 \simeq p_1 + \frac{B_n^2}{4\pi}(M_1^2 - M_2^2) \simeq p_1 + \frac{2}{\gamma+1}\rho_1 u_{n,1}^2$$

Plasma pressure, no limit

Fast Shocks Observed

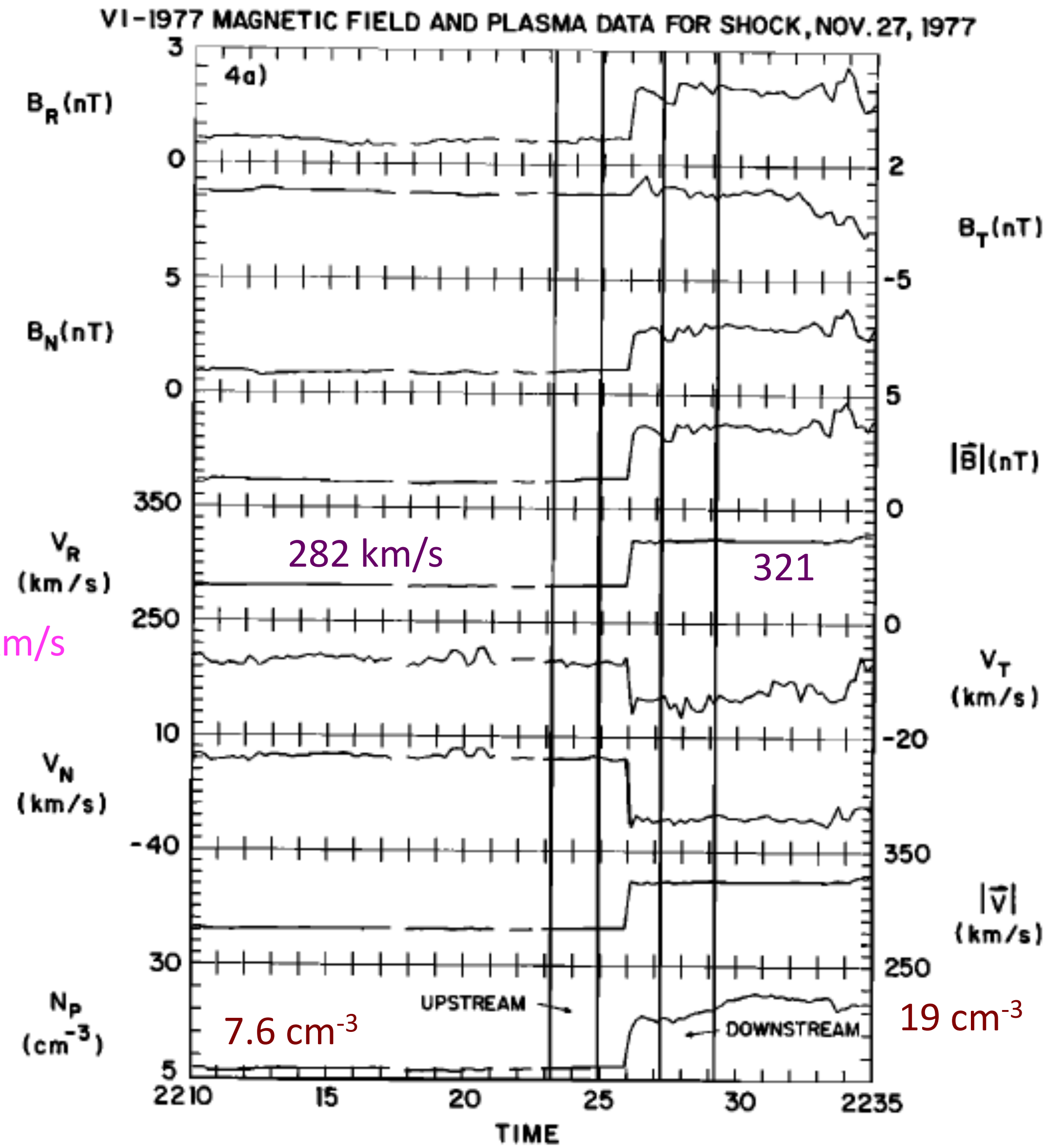
Voyager I @ r=1.6 AU

(v_{esc} = 34 km/s)



$M_{A1} = 8.0/\cos(\theta_1)$

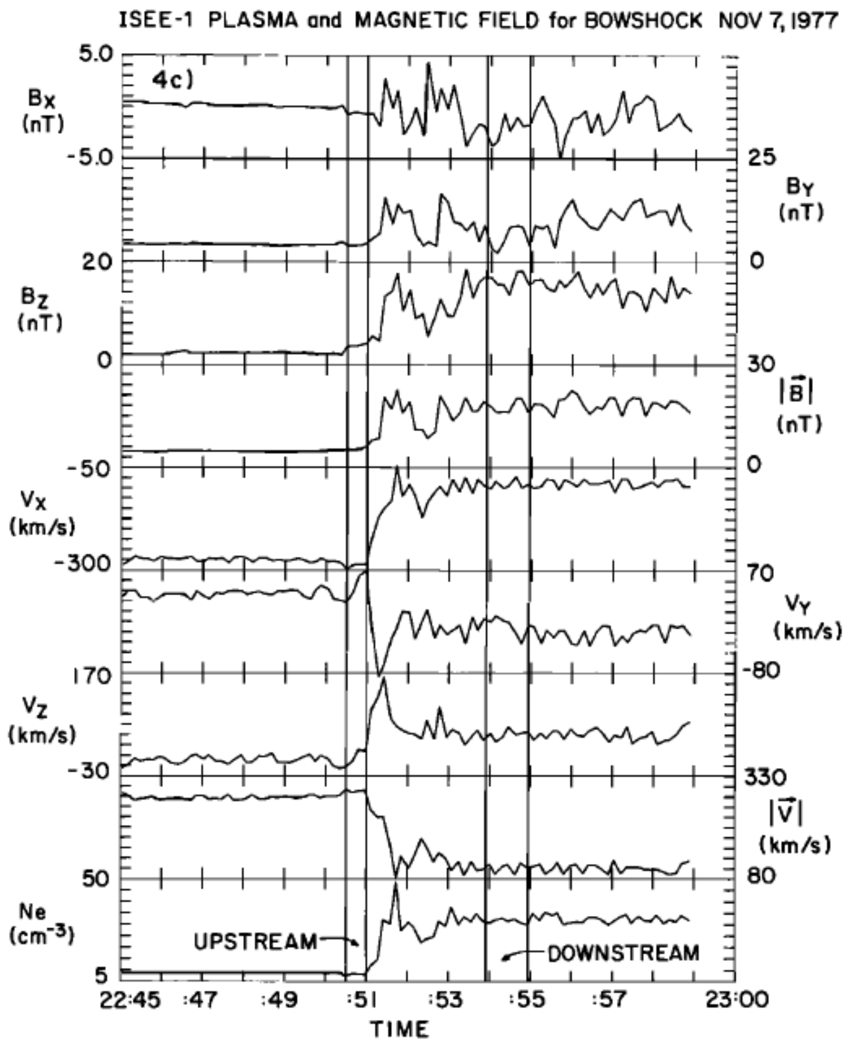
H-sphere: θ_1=84°, β_1=15



Vinas & Scudder 1986

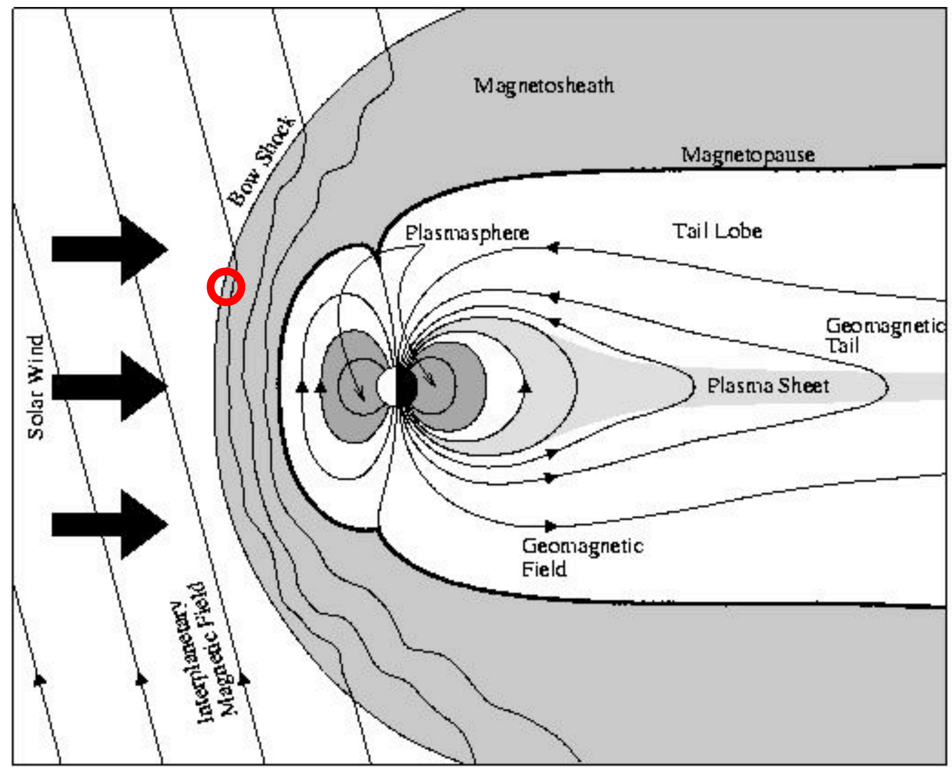
ISEE-1 crosses bow shock

$M_{A1} = 8.1/\cos(\theta_1)$



Vinas & Scudder 1986

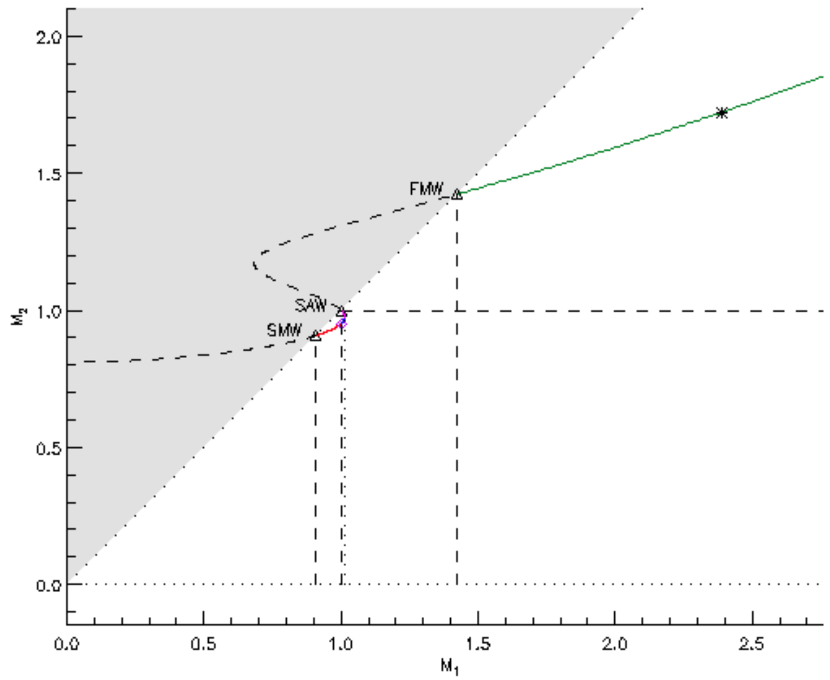
θ_1=74°, β_1=6

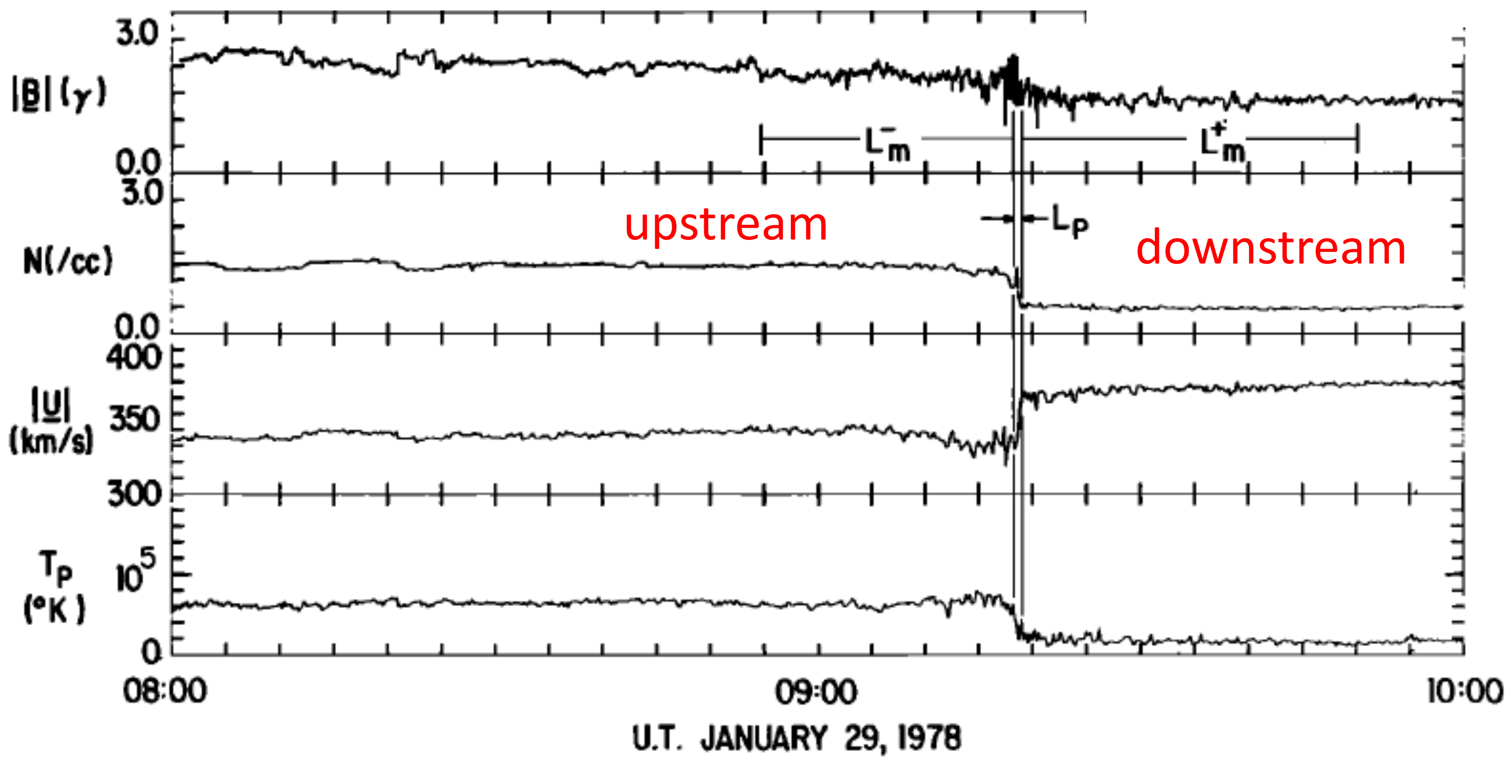


Forward shock

Voyager 2 at r=2.17 AU

$\theta_1=23^o, \ \beta_1=1.7, \ M_{A1}=2.4$

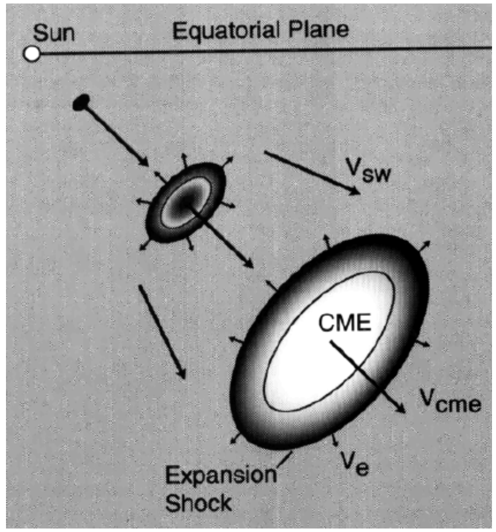


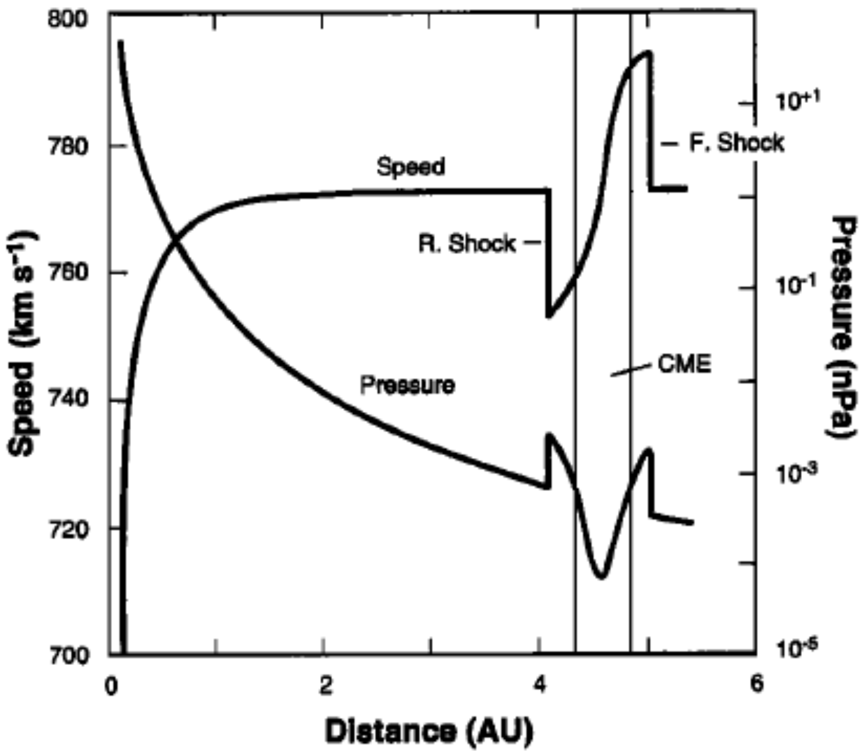


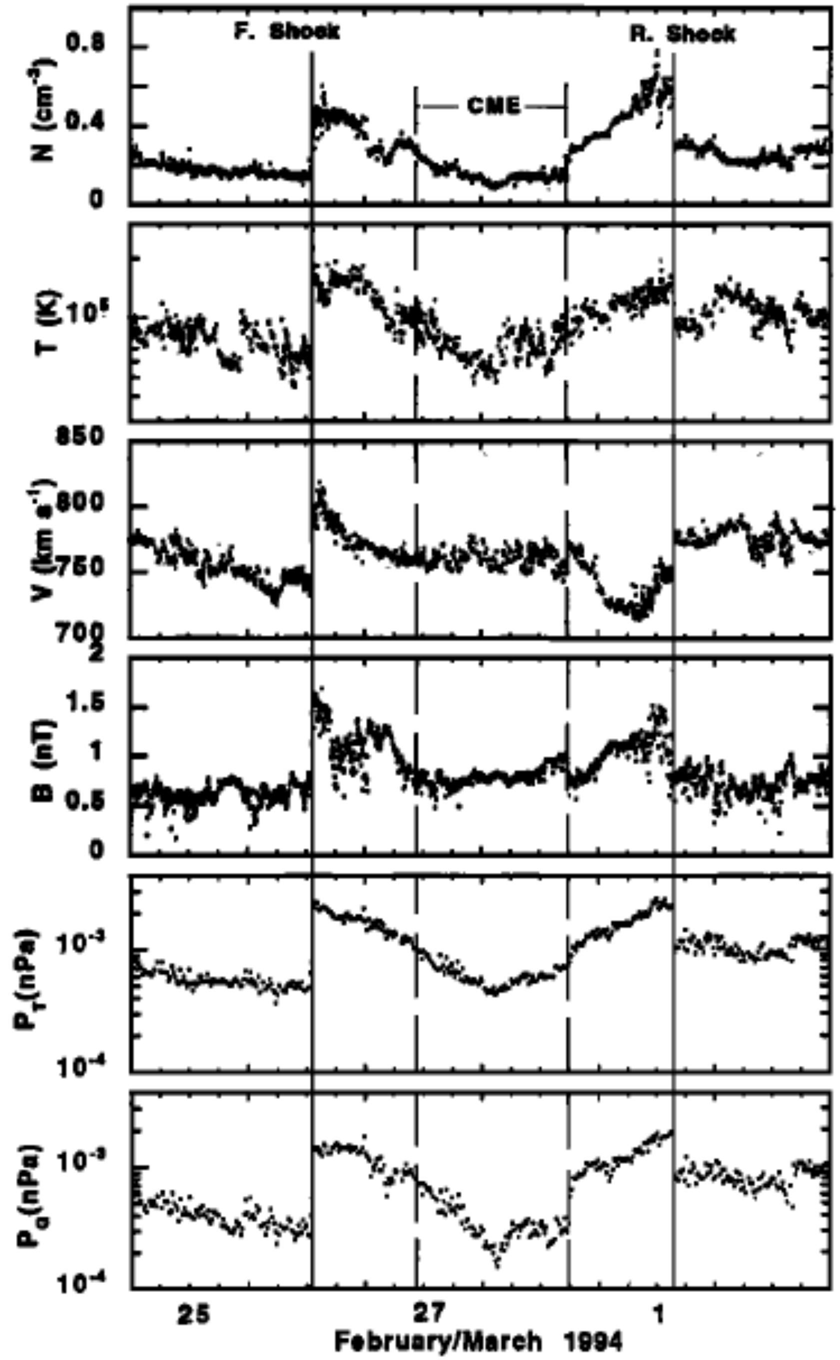
Scudder et al. 1984

2 FM shocks
2 (1?) TDs

Ulysses
@ r=3.5 AU

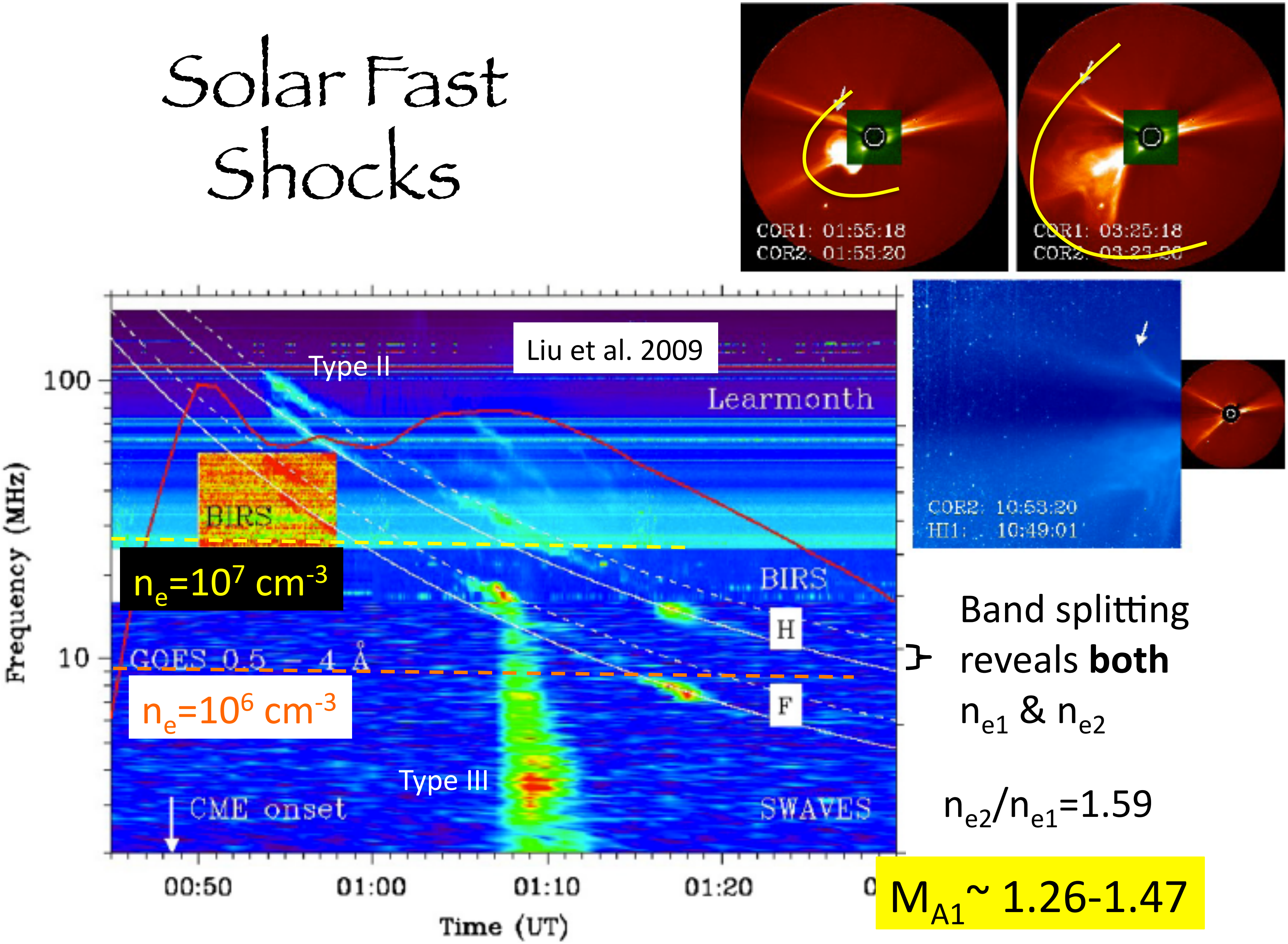






Gosling et al. 1994

Solar Fast Shocks

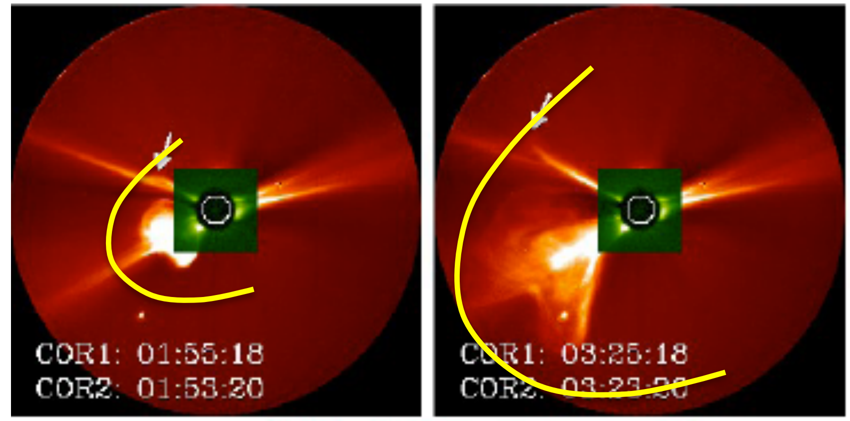


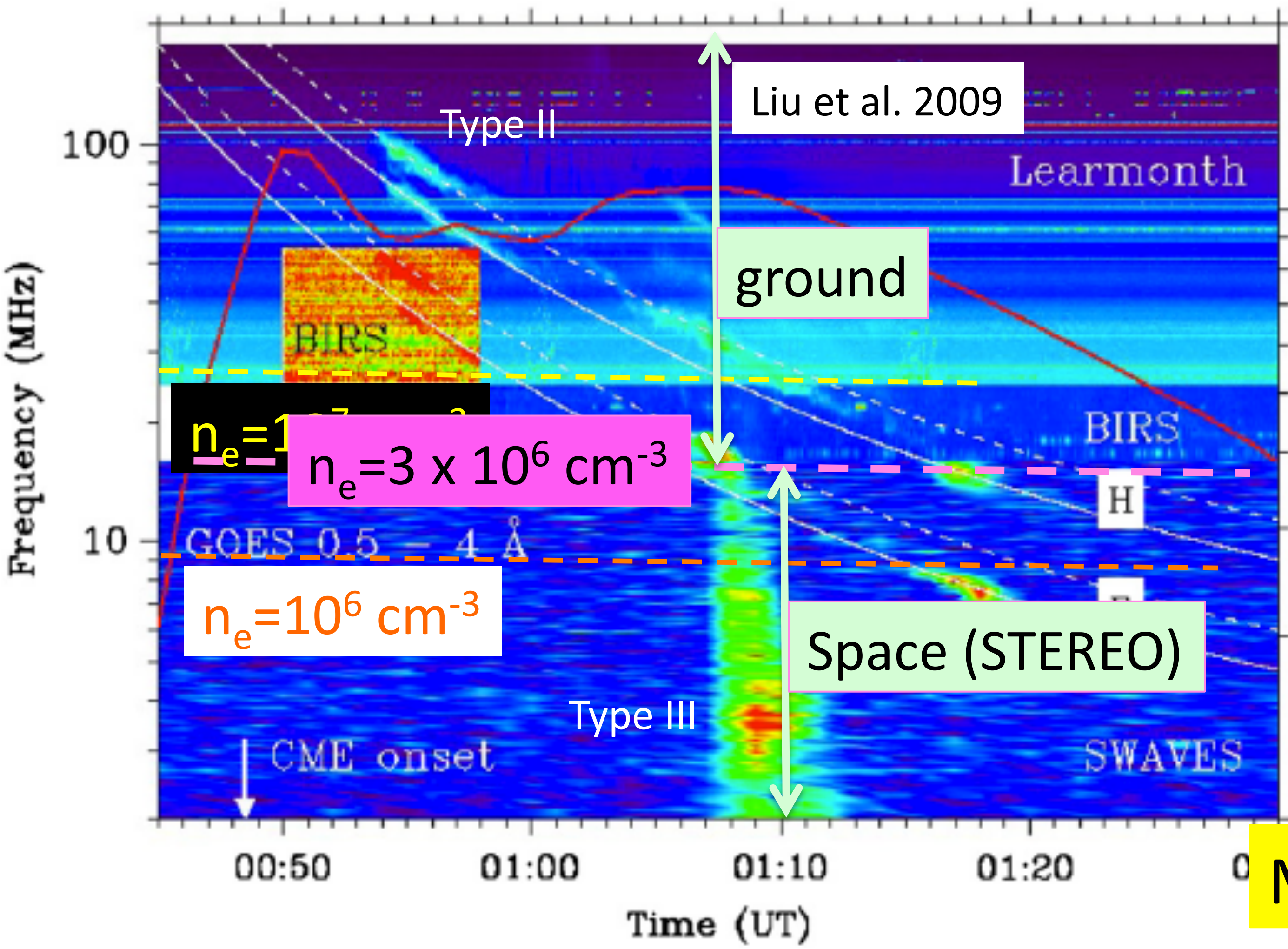
Band splitting reveals **both** n_{e1} & n_{e2}

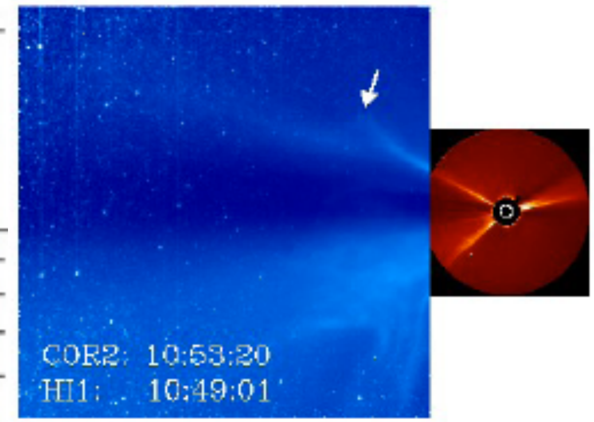
$n_{e2}/n_{e1}=1.59$

$M_{A1}\sim 1.26\text{-}1.47$

Solar Fast Shocks







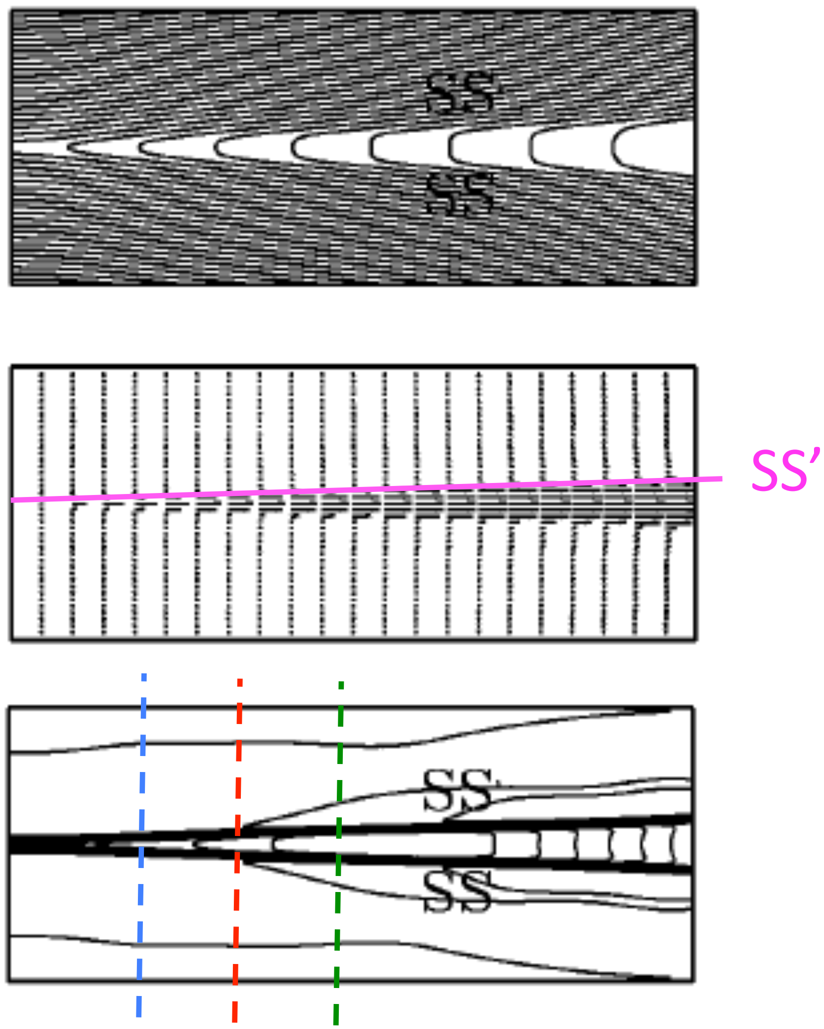
Band splitting reveals **both** n_{e1} & n_{e2}

n_{e2}/n_{e1}=1.59

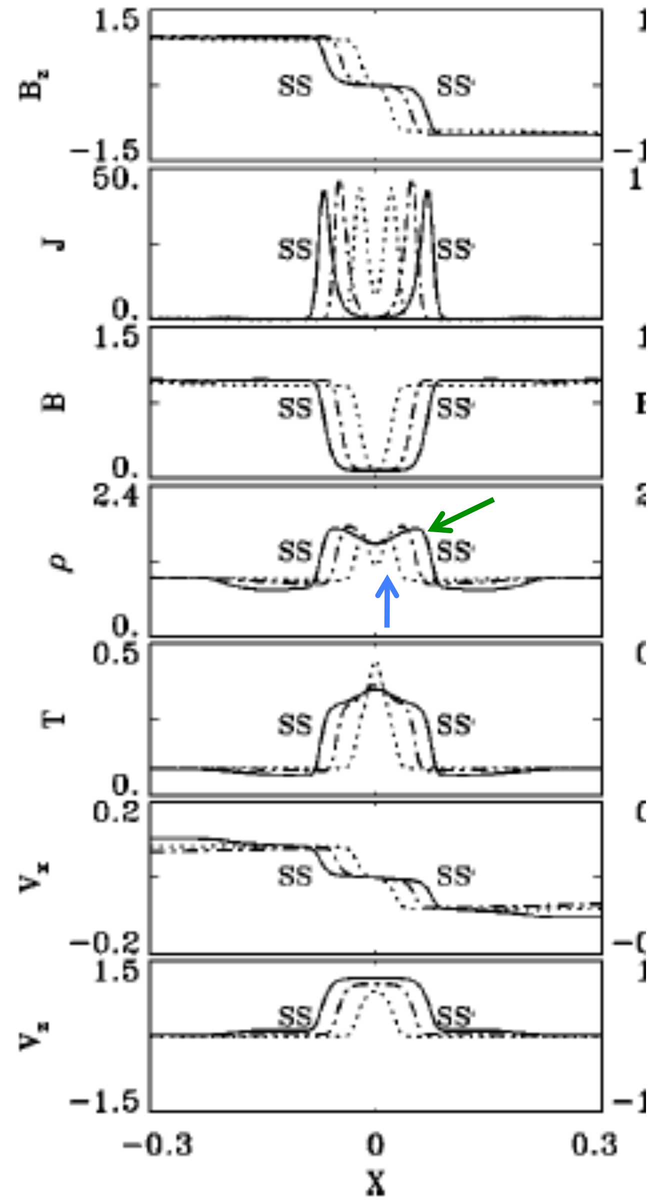
M_{A1}~ 1.26-1.47

Slow shocks: reconnection

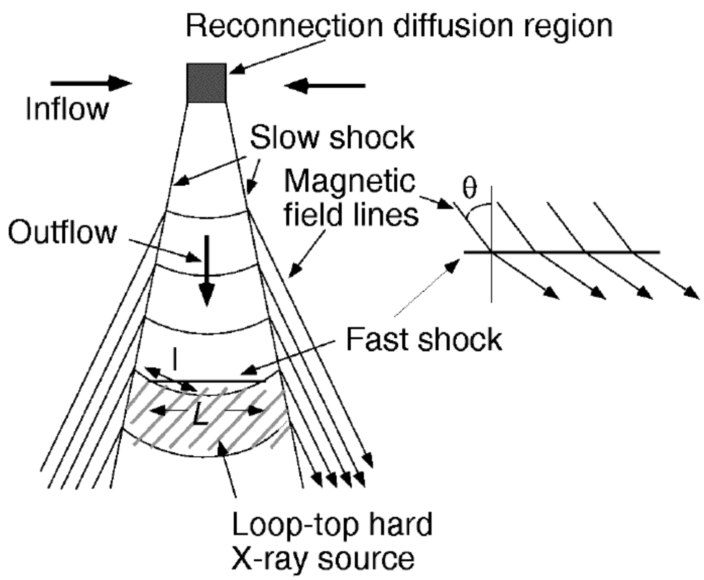
Lin & Lee 1999



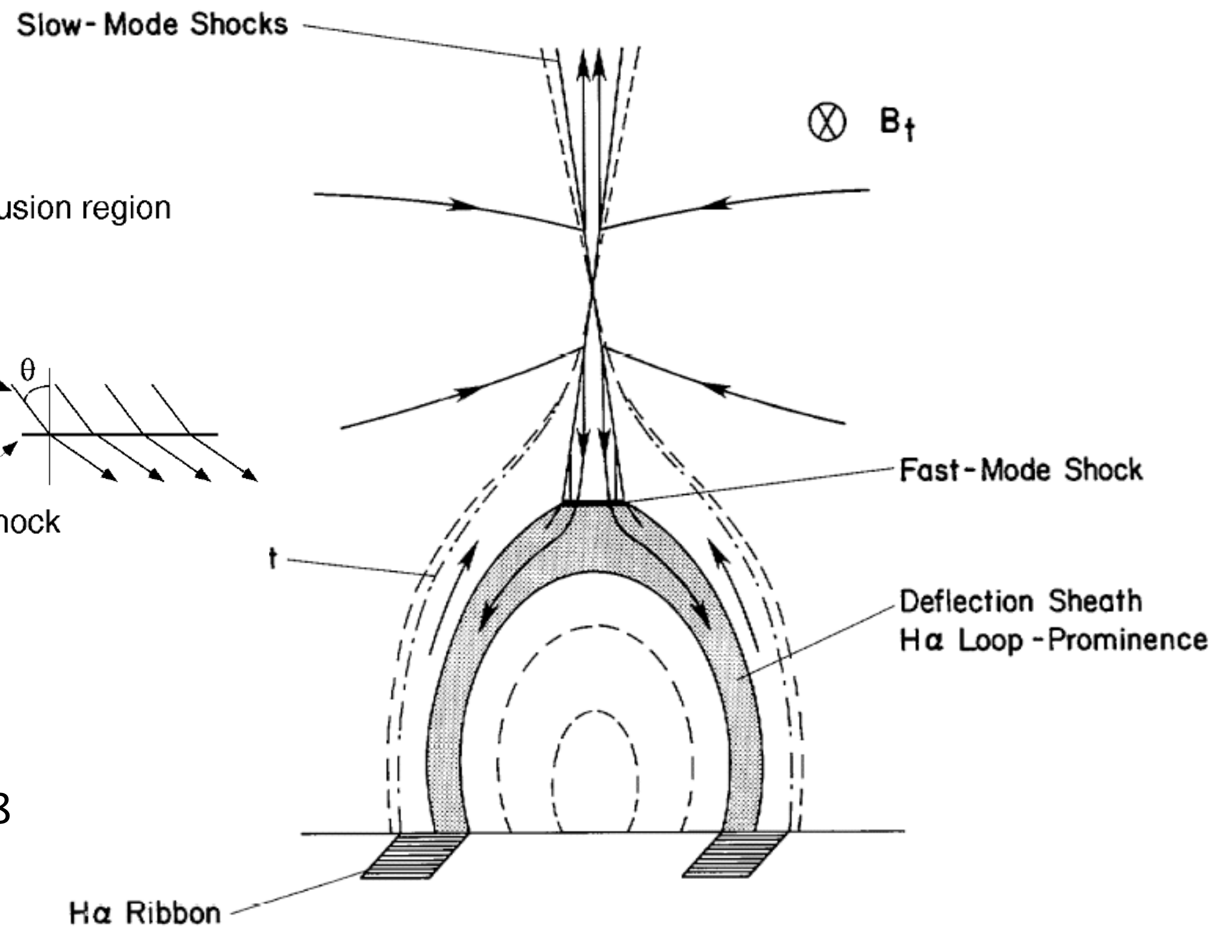
p



Model reconnection



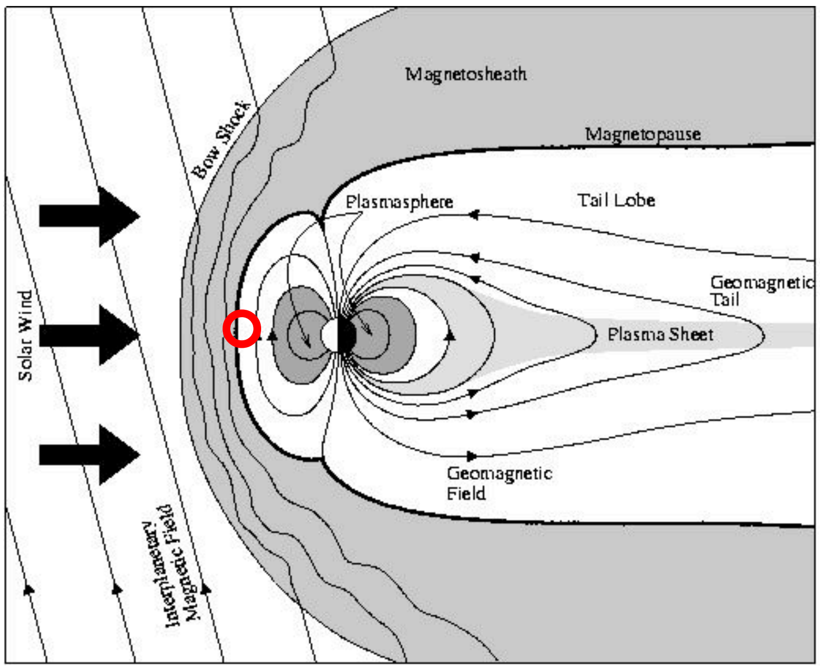
Tsuneta and Naito 1998

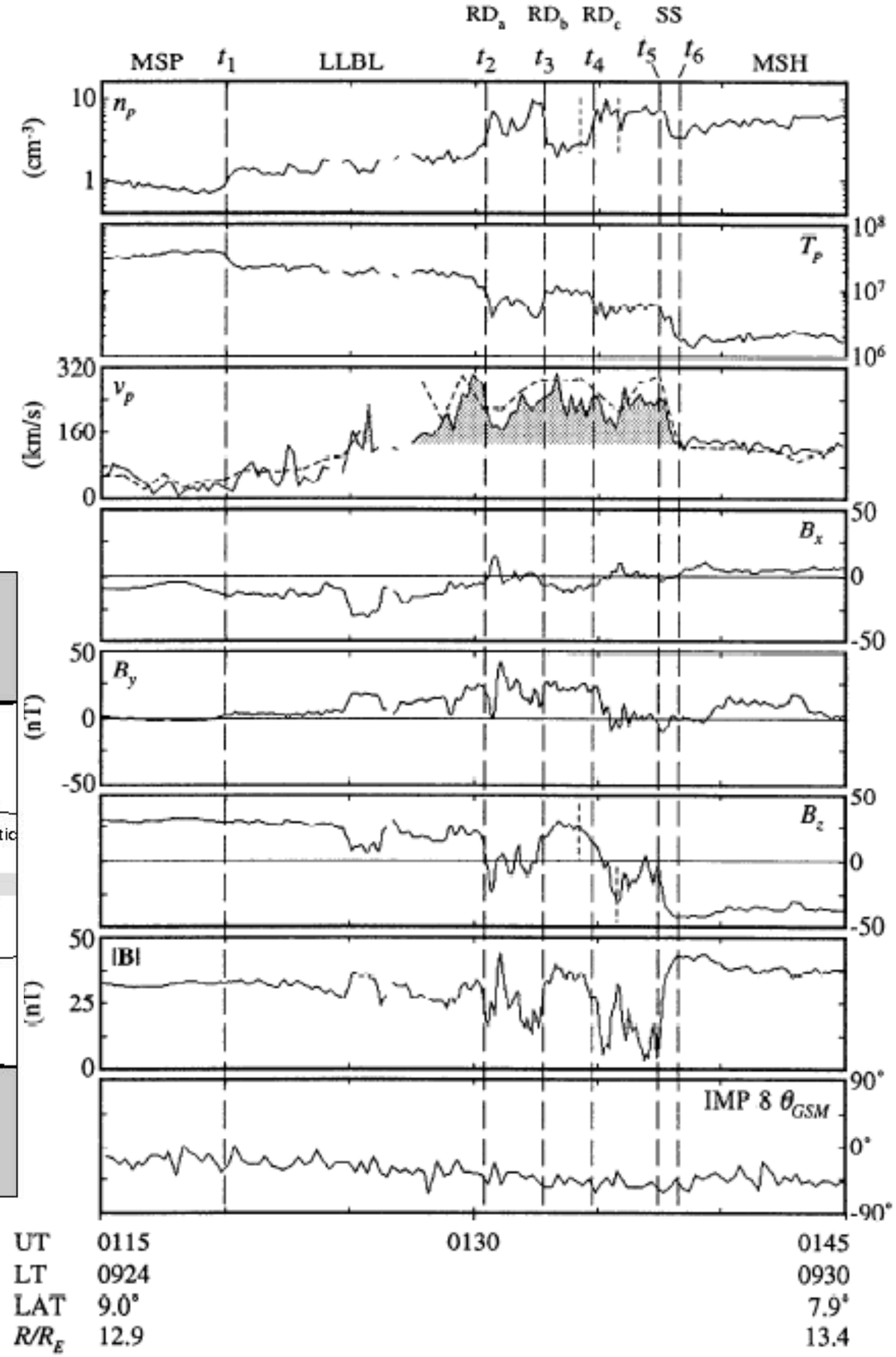


Forbes, T.G., and Malherbe 1986 (v.2 ch. 6)

ISEE-1 at the magnetopause:

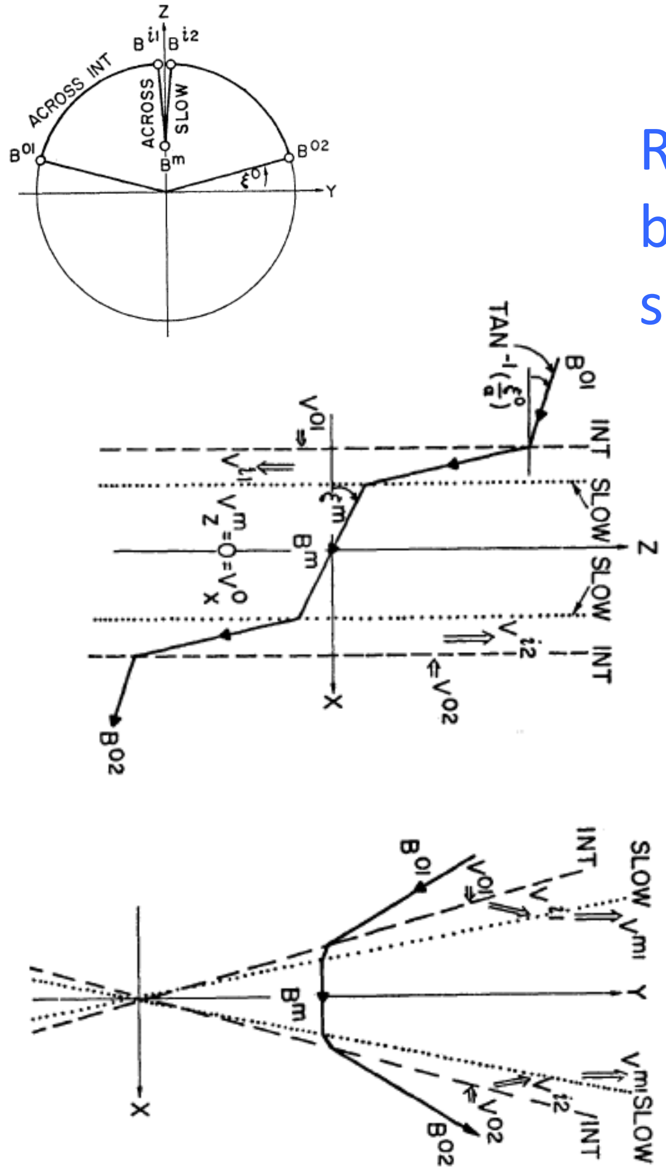
SS from reconnection





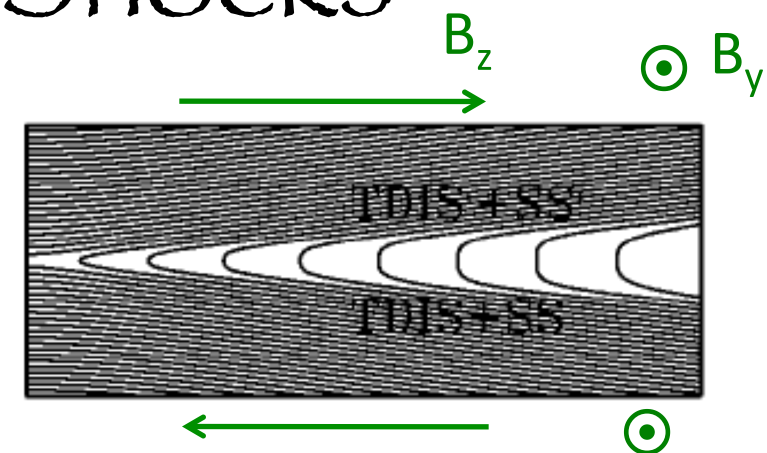
Walthour et al. 1994

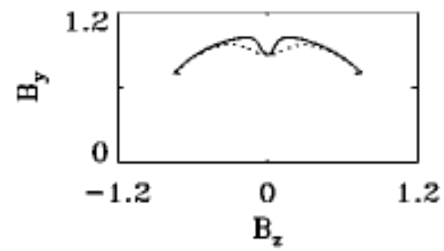
Intermediate Shocks



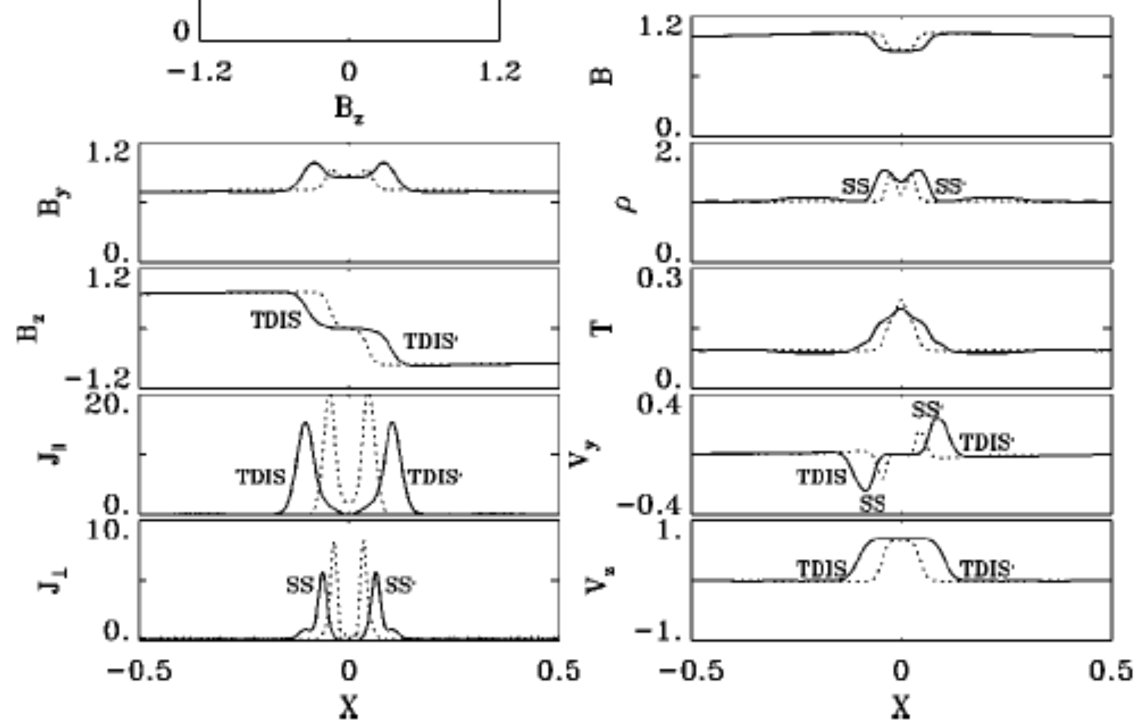
Petschek & Thorne 1967

Reconnection between skewed fields





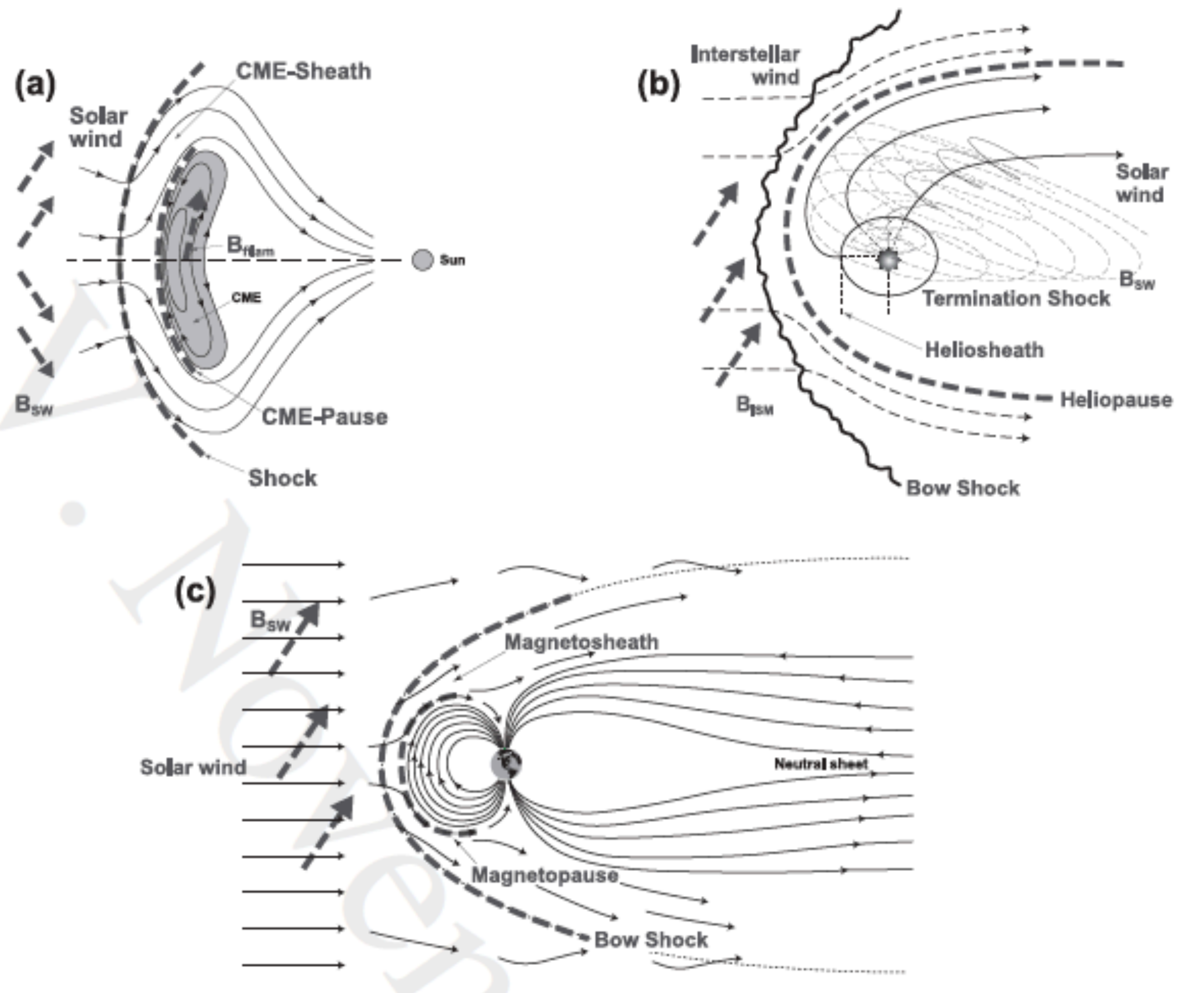
Lin & Lee 1999



Entropy's Tale … the sequel

- A shock converts kinetic to thermal energy
- How? What is the μ-physics?
  - Collisions (i.e. viscosity) – high enough density
  - Otherwise…(story not finished yet)
    - Instabilities, chaotic particle motion, …
    - Related to acceleration of non-thermal particles
    - Easier if particles gyrate near shock: quasi-perp. thermalizes easier than quasi-parallel

Summary


(a)
CME-Sheath
Solar wind
Sun
CME
CME-Pause
Shock
(b)
Interstellar wind
Solar wind
Termination Shock
Heliosheath
Heliopause
Bow Shock
(c)
Magnetosheath
Solar wind
Neutral sheet
Magnetopause
Bow Shock