Equivalence and Emergence within Dualities in Physics

Jeremy Butterfield
jb56@cam.ac.uk
1707.06681 (Sebastian De Haro and JNB) : 1806.01505
'I defy anyone to avoid getting confused by active vs. passive transformations': Graeme Segal

Trinity College, Cambridge

Boston University Colloquium, Friday 19 October 2018

Introduction

I relate De Haro's and my account of dualities to equivalence of theories, and to emergence. We take a duality to be a matter of:

(a): two theories share a common core; (itself a theory, the *bare theory*); and

(b): the theories are isomorphic models of this common core: here, 'model' means a homomorphic copy (cf. representation theory).

About equivalence, my main **Remark** is that dual theories can disagree, either by

(Contr): contradicting each other about a subject-matter; or by

(Diff): describing different (though 'isomorphic') subject-matters .

This **implies** a limitation of proposals to understand theoretical equivalence as logical equivalence or as a weakening of it.

About emergence, I endorse De Haro's account, and set it in the context of dualities. Indeed: I set it **beside** dualities.

Outline

Our usage

A bare theory can be realized (we will say: modelled) in various ways: cf. representation theory. These models are in general *not* isomorphic, and they differ from one another in their specific structure. But we say: *when the models are isomorphic, we have a duality*.

We call the two dual i.e. isomorphic theories, *model triples*, the 'triple' referring to the fact that the theory consists of three items: a state-space, a set of quantities, and a dynamics: $\langle \mathcal{S}, \mathcal{Q}, \mathcal{D} \rangle$.

Beware: the word 'model', as contrasted with 'theory', often connotes:
(i): a specific solution for the physical system concerned, whereas the 'theory' encompasses all solutions—often, for a whole class of systems;
(ii): an approximation, whereas the 'theory' deals with exact solutions;
(iii): being part of the physical world that gives the interpretation, whereas the 'theory' is not part of the world, and so needs interpretation.

Our use of 'model' rejects all three connotations.

Duality as surprising

We usually discover a duality in the context of studying, not a bare theory, but rather: two interpreted models of such a theory.

Usually, we do not initially believe them to be isomorphic in any relevant sense. Or even: to be models of any single relevant theory (even of a bare one).

The surprise is to discover that they are such models—indeed are isomorphic ones. And the surprise is greater, the more detailed is the common structure (like '10-dimensional semisimple Lie group', as against 'group').

Notation for theories and models

A notation for a model M that exhibits how M augments the structure of the theory T with specific structure, $\bar{M}$ say, of its own:—

Do **not** write $M = \langle T, \bar{M} \rangle$, since M uses $\bar{M}$ to build a representation of T's structure. Better to write: $M = \langle T_M, \bar{M} \rangle$. So the subscript M on T reflects that the specific structure $\bar{M}$ is used to build the representation of T. We call T_M, the 'part' of M that represents T, the **model root**.

Thus for a theory as a triple, $T = \langle \mathcal{S}, \mathcal{Q}, \mathcal{D} \rangle$: we write a model as a quadruple:

$$M = \langle \mathcal{S}_M, \mathcal{Q}_M, \mathcal{D}_M, \bar{M} \rangle =: \langle m, \bar{M} \rangle \ , \tag{1}$$

where $m := T_M := \langle \mathcal{S}_M, \mathcal{Q}_M, \mathcal{D}_M \rangle$ is called the **model triple**, as well as *model root*.

A Schema for duality

We propose that a duality is an isomorphism between two model triples. Recall that the model triple is separated from the model's specific structure, and expresses only the model's representing the bare theory. As in Eq. (1): $M = \langle \mathcal{S}_M, \mathcal{Q}_M, \mathcal{D}_M, \bar{M} \rangle =: \langle m, \bar{M} \rangle$, where $m := T_M := \langle \mathcal{S}_M, \mathcal{Q}_M, \mathcal{D}_M \rangle$ is the model triple (model root).

A **duality** between $m_1 = \langle \mathcal{S}_{M_1}, \mathcal{Q}_{M_1}, \mathcal{D}_{M_1} \rangle$ and $m_2 = \langle \mathcal{S}_{M_2}, \mathcal{Q}_{M_2}, \mathcal{D}_{M_2} \rangle$ requires:
an isomorphism between Hilbert spaces (for classical theories: manifolds):

$$d_s : \mathcal{S}_{M_1} \rightarrow \mathcal{S}_{M_2} \text{ using } d \text{ for 'duality' ;} \tag{2}$$

and an isomorphism between the sets (almost always: algebras) of quantities

$$d_q : \mathcal{Q}_{M_1} \rightarrow \mathcal{Q}_{M_2} \text{ using } d \text{ for 'duality' ;} \tag{3}$$

such that:

(i) the values of quantities match:

$$\langle Q_1, s_1 \rangle_1 = \langle d_q(Q_1), d_s(s_1) \rangle_2 \; , \quad \forall Q_1 \in \mathcal{Q}_{M_1}, s_1 \in \mathcal{S}_{M_1} . \qquad (4)$$

(ii) d_s is equivariant for the two triples' dynamics, $D_{S:1}, D_{S:2}$, in the Schrödinger picture; and d_q is equivariant for the two triples' dynamics, $D_{H:1}, D_{H:2}$, in the Heisenberg picture: see Figure 1.

$$\begin{tikzcd} \mathcal{S}_{M_1} \arrow[r, "d_s"] \arrow[d, "D_{S:1}"] & \mathcal{S}_{M_2} \arrow[d, "D_{S:2}"] \\ \mathcal{S}_{M_1} \arrow[r, "d_s"] & \mathcal{S}_{M_2} \end{tikzcd} \qquad \begin{tikzcd} \mathcal{Q}_{M_1} \arrow[r, "d_q"] \arrow[d, "D_{H:1}"] & \mathcal{Q}_{M_2} \arrow[d, "D_{H:2}"] \\ \mathcal{Q}_{M_1} \arrow[r, "d_q"] & \mathcal{Q}_{M_2} \end{tikzcd}$$

Figure : Equivariance of duality and dynamics, for states and quantities.

Interpreting physical theories

I endorse the framework of *intensional semantics*, in the style of Frege, Carnap and Lewis. Words and sentences are assigned intensions: maps from the set of worlds W to extensions.

This framework has the great merit of respecting the meanings of words! That may seem an obviously mandatory feature for any endeavour calling itself 'semantics'. But the 'semantics' in books of logic and model theory investigate the mathematical consequences of assigning arbitrary meanings (specifically, extensions) to words...

It also makes precise the notion of a *subject-matter*, as a partition of the set W of worlds. In a cell of the partition, any two worlds match as regards the subject-matter. Thus a proposition is *entirely about* a subject-matter if the set of worlds at which it is true is a union of cells of the subject-matter.

Subject-matters: (Contr) and (Diff)

I now make precise the **Remark** that dual theories can disagree: either by
(Contr): contradicting each other about a subject-matter; or by
(Diff): describing different (though 'isomorphic') subject-matters.

(Contr): Each of two dual model triples is interpreted as wholly true (its conjunctive proposition is wholly true) at a union of cells of a common subject-matter. But these two unions are disjoint: for the propositions contradict each other.

(Diff): Two dual model triples are interpreted as wholly true (the conjunctive proposition of each is wholly true) at distinct sets of worlds. Each set is a union of cells of the triple's subject-matter, i.e. partition. But the partitions are different, and so are the sets. The sets need not be disjoint: both the model triples could be, both of them, wholly true. But the sets are distinct.

The Remark in classical and quantum physics

(1): Newtonian mechanics with different absolute rests: (Contr)
Two formulations of Newtonian point-particle mechanics (say N particles with gravitation), that differ in what they identify as absolute space: what inertial timelike congruence is 'truly at rest'.
Thus the spacetime is $\mathbb{R}^4$ and the bare theory is a neo-Newtonian (Galilean) formulation of point-particle mechanics.

The specific structure in each dual (model, in our sense) is its specification of absolute rest. The 'left' dual might specify as absolute rest the obvious congruence, i.e. the lines $\langle t, x, y, z \rangle$ (with t varying, and x, y, z fixed, for each line); while on the other hand, the 'right' dual specifies $\langle t, x - vt, y, z \rangle$.

A state is an assignment of (absolute!) position and velocity to each particle: $(x_1, ...z_N; \dot{x}_1, ...\dot{z}_N)$. The duality map d_s on states takes a state of the first ('left') model to the state in the second ('right') model with the same numerical values, with respect to *its* (the right model's) specification of absolute rest. Eq. 4 holds.

Beware: It is tempting to say that the contrary specifications of absolute rest are 'gauge', or 'a distinction without a difference', or 'a sign that we should move to a neo-Newtonian formulation', in which the 'surplus structure of absolute rest is eliminated'.

I agree that it is *tempting* to say these things. But the point is: these temptations are the benefit of hindsight, i.e. of our now knowing the neo-Newtonian formulation. Returning to the earlier epoch of Newton and Clarke: their views are tenable—and the duality illustrates (Contr).

Agreed: these 'temptations' hint at two other important functions of duality. Namely, to prompt us to guess: either
(i): the bare theory, the 'common core', when we have no formulation of it; or
(ii) another theory 'behind the duals', of which the two duals are—not representations (*a la* the Schema) but—approximations.

(2): Position-momentum duality in elementary quantum mechanics: (Diff)

Let the bare theory be $L^2(\mathbb{R})$, equipped with, say, $\mathcal{B}(L^2(\mathbb{R}))$.

The idea will be: the left dual is fixed by the choice of position, the right by the choice of momentum.

It is usual to think of these choices as just 'choices of basis':

A wave function $\psi : \mathbb{R} \to \mathbb{C}$ is mapped by a unitary map F to its Fourier transform $\tilde{\psi} : \mathbb{R} \to \mathbb{C}$. The usual (Schrödinger) representations of position and momentum, $Q : \psi(x) \mapsto x\psi(x)$ and $P : \psi(x) \mapsto -i\frac{d}{dx}\psi(x)$ respectively, are related by: $P = F^{-1}QF$.

This implies the preservation of any expectation value between the 'left' and the 'right': an inner product is the same when calculated in two different orthobases, related by a unitary map.

This is *not* an example of (Contr) or of (Diff). But it is an important prototype of the left and right duals *agreeing*. *Bosonization* is another example: again with a unitary equivalence.

But we *can* get an illustration of (Diff) by thinking 'like Bohr': 'position and momentum 'perspectives' cannot be adopted together'.

The left dual is to be *only* about position. Its states are just the probability distributions for position; i.e. probability densities $p(x) \equiv |\psi(x)|^2$. And the left dual's quantities are to be just position Q and the (Borel) functions of position, $f(Q)$, with expectation values $\langle f(Q) \rangle = \int dx\; f(x)p(x)$.

Similarly, the right dual is only about momentum. Its states are just the probability densities for momentum, $p(k)$, yielding expectation values $\langle f(P) \rangle = \int dk\; f(k)p(k)$.

The duality map d_s maps the state $p(x)$ to the *same* mathematical real function: but now interpreted as a probability density for momentum. So d_s is the identity map on probability distributions: but as interpreted on physical states, it is an active transformation—*not* the identity map.

Similarly for quantities: $Q \mapsto P$.

(3): Kramers-Wannier duality in classical statistical mechanics: (Diff)

The bare theory is the classical equilibrium statistical mechanics of a two-dimensional square lattice with the Ising Hamiltonian: i.e. the canonical ensemble with the Boltzmann probabilistic weights for a configuration s given by $\exp(-\beta H[s])$, where H is the Ising Hamiltonian and $\beta \equiv 1/kT$ is the inverse temperature.

The duals are approximations to the partition function $Z \equiv \Sigma_s \exp(-\beta H[s])$ that are valid at low and high temperatures T, respectively: say, low on the 'left' and high on the 'right'.

We write the partition function using the dimensionless inverse temperature $\nu := J/kT$, and we define ν^* by $\tanh \nu^* := \exp(-2\nu)$. So $\nu^* = 0/\infty$ iff $\nu = \infty/0$ respectively, and low temperature i.e. large ν corresponds to a high conjugate temperature i.e. small ν^*.
Then the expansions for low and high temperatures are related by

$$Z(\nu) = Z(\nu^*) 2^{1-N} (2 \sinh 2\nu)^N \ . \tag{5}$$

where N is the number of lattice sites.

Take the left dual as the family of expansions parameterized by T being in some low range $[T_1, T_2] \subset \mathbb{R}$, i.e. by large $\nu := J/kT$ in the range $[J/kT_2, J/kT_1]$. Then the right dual is the expansions parameterized by small ν^* in the range $[\tanh(\exp(J/kT_1)), \tanh(\exp(J/kT_2))]$.

So the duality map d_s is

$$d_s : Z(\nu) \mapsto Z(\nu^*) := \frac{Z(\nu)}{2^{1-N}(2 \sinh 2\nu)^N} \ . \qquad (6)$$

Like Example (2): this is a case of (Diff). The low temperature regime and high temperature regime are different though isomorphic subject-matters.

And there is *no* temptation—even in hindsight—to say that there is no real difference between the duals, that the contrast between them is 'gauge' etc.

An Implication about ‘theoretical equivalence’

Two dual theories satisfying (Contr) or (Diff) might get formalized so as to be logically equivalent. But obviously such duals are not equivalent—as ‘equivalent’ is normally understood. So logical equivalence is too weak an explication of ‘theoretical equivalence’. And so also, therefore, is any weakening of logical equivalence.

This **Implication** is worth stressing. For the recent philosophical literature on theoretical equivalence has focussed on logical equivalence being too strong. Think of *synonymy*, e.g. classical electromagnetism written in English and French.

So it is worth seeing that there is also a problem ‘in the other direction’. In short: dualities illustrating (Contr) or (Diff) show how an appropriate *homonymy* can render logical equivalence too weak.

In terms of the proverbial Alice advocating the left dual, and Bob advocating the right dual:—
If they are 'unwise enough' to use the same words in their advocacies, and an appropriate subset of their words have appropriately 'inverted' meanings, then: what they *say* can be the same, despite their theories disagreeing. So their theories might well get formalized so as to be logically equivalent.

Thus in Example (3), Kramers-Wannier duality: suppose Alice speaks standard English, so that in advocating her expansion for the Ising lattice at low temperatures, she says 'low temperature expansion' etc. And suppose the semantics (intended interpretation) of Bob's language is 'high-low' inverted with respect to Alice's. So Bob, in advocating the right dual, i.e. describing the lattice at high temperature, says 'low temperature expansion' etc.

Thanks to the duality: Alice and Bob, in advocating their different theories, might say the very same set of sentences.

In terms of the duality map d_s: Alice describes a state $Z(\nu)$, i.e. an expansion of the partition function at low temperature, by saying the lattice has a free energy $F = F(Z(\nu))$ for a temperature $T = T(\nu)$.

Bob describes the transformed state $d_s(Z(\nu)) := Z(\nu^*)$ (eq. 6), i.e. the expansion at the conjugate high temperature, by the same words.

Their assertions differ, but are compatible: (Diff). For 'low' in Bob's mouth means high (in Alice's mouth, and ours). More exactly: the notation ('numeral') ν in Bob's mouth means the real number $\nu^* := \nu^*(\nu) := \tanh^{-1}(\exp(-2\nu))$.

Similarly in other Examples. In Example (2): we can suppose Alice speaks standard English, so that her word 'position' means position; while Bob uses the word 'position' to mean momentum. So when Alice describes a state s by saying 'the particle has a Gaussian distribution for position centred at $x = 5$', Bob describes the transformed state $d_s(s)$ by saying the same words. Their assertions differ, but are compatible: (Diff).

This **Implication** also holds good for other proposed notions of 'theoretical equivalence' that are weakenings of logical equivalence: for example, *definitional equivalence*.

The idea of definitional equivalence is that starting from one theory, one can introduce definitions of the other theory's notions and then rigorously deduce all the claims, i.e. theorems, of the other theory; and vice versa, starting with the other theory.

T_1 and T_2 are *definitionally equivalent* iff: (i) one can add to T_1 a definition of each vocabulary item of T_2, in such a way that within the resulting augmentation of T_1 one can deduce all of T_2; and of course (ii) vice versa.

(i) is called: making a *definitional extension* of T_1, and showing T_2 to be (a sub theory of) a definitional extension of T_1.

So: definitional equivalence is a matter of the two theories having a *common definitional extension*.

This notion makes no difference to the **Remark** and **Implication**—for the now-familiar reason, that the logical framework deliberately sets aside intended meanings.

Thus even if the advocates, Alice and Bob, of such a pair of dual theories are ‘wise enough’ to use disjoint vocabularies, their theories might nevertheless, once formalized, be definitionally equivalent.

The same moral applies beyond the case of definitional equivalence, to other weakenings of logical equivalence that have been proposed (Barrett, Halvorson, Hudetz and Weatherall). (Agreed: the proposals have evident merits as explications of ‘theoretical equivalence’). Namely: *generalized definitional equivalence* and *categorical equivalence*.

Emergence beside dualities

Cf. De Haro's talk! We take ontological emergence as:

(i) a relation between theories, given by a linkage map from the "bottom" theory T_{b} to the "top" theory T_{t}, written as $link : T_{\mathrm{b}} \to T_{\mathrm{t}}$;

(ii) the domains of interpretation are distinct: $D_{\mathrm{t}} \neq D_{\mathrm{b}}$.

So the set-up is:

$$\begin{tikzcd} T_{L,t} \arrow[r, "d_t"] & T_{R,t} \\ T_{L,b} \arrow[u, "link_L"'] \arrow[r, "d_b"] & T_{R,b} \arrow[u, "link_R"'] \end{tikzcd}$$

Figure : The linkage and duality maps: drawn as commuting

Our examples, both of (Contr) and of (Diff), illustrate this:—

both "at top" and "at bottom"; and

with ontological emergence, and without it.

(1): Newtonian mechanics: (Contr)—revisited

(At the top): Take the two formulations of Newtonian mechanics, with rival absolute rests, as $T_{\mathrm{L,t}}$, $T_{\mathrm{R,t}}$. They are dual by the "boost" map on states and quantities discussed above, now written $d_{\mathrm{t}} : T_{\mathrm{L,t}} \rightarrow T_{\mathrm{R,t}}$.

So take as $T_{\mathrm{L},b}$ and $T_{\mathrm{R},b}$, two formulations of *non-relativistic quantum theory*, set in Newtonian spacetime, with rival absolute rests. They are dual by a "boost" map on quantum states and quantities that is the analogue of d_{t}: call it $d_{\mathrm{b}} : T_{\mathrm{L,b}} \rightarrow T_{\mathrm{R,b}}$.

It is the analogue because the boost concept is regardless of the quantum-classical transition.

And $T_{\mathrm{L},b}$ and $T_{\mathrm{R},b}$ illustrate (Contr), just like $T_{\mathrm{L,t}}$, $T_{\mathrm{R,t}}$.

Define *link* as "$\hbar \rightarrow 0$": whatever you believe defines the emergence of classical from quantum! So: $link_{\mathrm{L}} : T_{\mathrm{L,b}} \rightarrow T_{\mathrm{L,t}}$, and $link_{\mathrm{R}} : T_{\mathrm{R,b}} \rightarrow T_{\mathrm{R,t}}$.

Most would say both *link* maps give ontological emergence. Certainly, that issue has no dependence on a boost. So either both $link_{\mathrm{L}}$, $link_{\mathrm{R}}$ give ontological emergence, or both do not.

(At the bottom): Take the two given formulations of Newtonian mechanics, with rival absolute rests, as $T_{\mathrm{L,b}}$, $T_{\mathrm{R,b}}$. They are dual by the "boost" map on states and quantities, now written $d_{\mathrm{b}} : T_{\mathrm{L,b}} \to T_{\mathrm{R,b}}$.

Let *link* merely define *the motions of the centre-of-mass* (or of the c.o.m. and some few other collective variables) of the N particles. So $T_{\mathrm{L,t}}$, $T_{\mathrm{R,t}}$ are two "mini-theories".

They are also dual by the "boost" map on states and quantities, $d_{\mathrm{t}} : T_{\mathrm{L,t}} \to T_{\mathrm{R,t}}$.

Most would say both *link* maps do not give ontological emergence, but "at best, epistemic emergence".

Certainly, that issue has no dependence on a boost. So again: either both $link_{\mathrm{L}}$, $link_{\mathrm{R}}$ give ontological emergence, or both do not.

And $T_{\mathrm{L},t}$ and $T_{\mathrm{R},t}$ illustrate (Contr), just like $T_{\mathrm{L,b}}$, $T_{\mathrm{R,b}}$.

(2): Position-momentum duality: (Diff)—revisited

(At the top): Take the "monomaniac" position and momentum theories as $T_{\mathrm{L},t}$, $T_{\mathrm{R},t}$.

They are dual by the duality maps on states (viz. identity on probability density functions $p(x)$) and on quantities ($Q \mapsto P$) discussed above, now written $d_{\mathrm{t}} : T_{\mathrm{L,t}} \rightarrow T_{\mathrm{R,t}}$.

So take as $T_{\mathrm{L},b}$ and $T_{\mathrm{R},b}$, the "monomaniac-position" and "monomaniac-momentum" sectors of *pilot-wave theory*.

Thus $T_{\mathrm{L},b}$ uses a state $\langle\psi, \mathbf{x}_1, ..., \mathbf{x}_N\rangle$; but only so as to extract from it the (orthodox) probability density function for position $p(x) = |\psi|^2$. And $T_{\mathrm{L},t}$ extracts $|\psi|^2$, taken as probability density for momentum.

$T_{\mathrm{L},b}$ and $T_{\mathrm{R},b}$ are dual by the obvious duality maps: viz. on states, identity on both probability density functions $p(x)$, and on particles' positions $\langle\mathbf{x}_1, ..., \mathbf{x}_N\rangle$; and on quantities, $Q \mapsto P$. Write this analogue of d_{t} as: $d_{\mathrm{b}} : T_{\mathrm{L,b}} \rightarrow T_{\mathrm{R,b}}$.

d_{b} is the analogue of d_{t} because the position-to-momentum "interpretative flip" is regardless of believing in particles' positions $\langle\mathbf{x}_1, ..., \mathbf{x}_N\rangle$.

$T_{\mathrm{L},b}$ and $T_{\mathrm{R},b}$ illustrate (Diff), just like $T_{\mathrm{L,t}}$, $T_{\mathrm{R,t}}$.

Define *link* as usual in pilot-wave theory: as dropping the particles' positions $\langle \mathbf{x}_1, ..., \mathbf{x}_N \rangle$. So: $link_{\mathrm{L}} : T_{\mathrm{L,b}} \to T_{\mathrm{L,t}}$, and $link_{\mathrm{R}} : T_{\mathrm{R,b}} \to T_{\mathrm{R,t}}$.

Most would say both *link* maps do not give ontological emergence, but "at best, epistemic emergence". For from a pilot-wave perspective, orthodox quantum theory cannot claim such novelty—and this applies on both the Left and the Right.

(At the bottom): Take the "monomaniac" position and momentum theories as $T_{\mathrm{L},b}$, $T_{\mathrm{R},b}$.They are dual by the identity map on probability density functions $p(x)$ etc., now written $d_{\mathrm{b}} : T_{\mathrm{L,b}} \rightarrow T_{\mathrm{R,b}}$.

Let *link* merely define *the motions of the mean of the distribution*; or of the mean and some other statistic, i.e. functional of the distribution.

So $T_{\mathrm{L,t}}$ is a mini-theory about the mean etc. of position probabilities; and $T_{\mathrm{R,t}}$ is a mini-theory about the mean etc. of momentum probabilities.

They are also dual by the the position-to-momentum "interpretative flip", $d_{\mathrm{t}} : T_{\mathrm{L,t}} \rightarrow T_{\mathrm{R,t}}$.

Again: most would say both *link* maps do not give ontological emergence, but "at best, epistemic emergence".

Certainly, the issue has no dependence on the contrast between position and momentum. So again: either both *link*$_{\mathrm{L}}$, *link*$_{\mathrm{R}}$ give ontological emergence, or both do not.

And $T_{\mathrm{L},t}$ and $T_{\mathrm{R},t}$ illustrate (Diff), just like $T_{\mathrm{L,b}}$, $T_{\mathrm{R,b}}$.

Thank you!

References

Butterfield, J. (2018). "Dualities and Equivalences Between Physical Theories", in: *Philosophy Beyond Spacetime*, B. Le Bihan, et al. (Eds.) (OUP): 1806.01505.

De Haro, S. and Butterfield, J.N. (2018). "A Schema for Duality, Illustrated by Bosonization", in: *Foundations of Mathematics and Physics, one century later: new perspectives*, Kouneiher, J. (Ed.). Springer. 1707.06681

De Haro, S. (2016). "Spacetime and Physical Equivalence". In *Space and Time after Quantum Gravity*, Huggett, N., Wüthrich, C. (Eds.), (CUP) to appear. An expansion, "Duality and Physical Equivalence", is at: http://philsci-archive.pitt.edu/id/eprint/12279

De Haro, S. (2017). "Dualities and emergent gravity: gauge/gravity duality". *Studies in History and Philosophy of Modern Physics*, 59, 109-125.

The Remark in string theory

The Remark is not an artefact of our having chosen elementary Examples. Nor of our logically weak definition of the Schema for duality. For requiring a duality to be non-obvious, scientifically important ... will retain as Examples, e.g. Kramers-Wannier and related dualities in statistical mechanics.

And in string theory too—tentatively ... Gauge/gravity duality, and T-duality, provide Examples of (Contr).

(4): Gauge-gravity duality: (Contr)

'Gauge/gravity duality' is the umbrella term for dualities between a string theory (hence including a description of gravity) on a D-dimensional spacetime (the 'bulk') and a quantum field theory (a gauge theory, with no description of gravity) on a $(D-1)$-dimensional space or spacetime that forms the bulk's boundary.

De Haro (2016, 2016a) argues that in terms of our Schema, the common core, i.e. bare theory, of which the bulk and boundary theories are models (in our representation-theory sense) has as its spacetime:
the $(D-1)$-dimensional boundary manifold equipped—not with a metric, but merely—with an equivalence class of them, under local conformal transformations.

Suppose the bulk theory (the 'left dual') says spacetime is five-dimensional ($D = 5$); so the boundary theory, the right dual, says it is is four-dimensional. But both theories are putative 'theories of everything', 'toy cosmologies'. They are both about a single topic, the cosmos; in philosophers' jargon, the actual world. So the theories make contrary assertions about that single topic, the actual world, namely about the dimension of its spacetime. So this is a case of (Contr). In terms of the simple logic—or rhetoric!—of the situation, we have come full circle, back to Example (1).

I agree that there is a temptation to say: 'the real truth lies in what is in common, or what is behind, the two duals'. That is: either

(i) to formulate the duals' common core/bare theory (if we have no formulation or a defective one), and-or

(ii) to formulate another theory 'behind the duals', of which they are approximations, not representations.

This is indeed the heuristic function of dualities. Recall the analogous temptation for Example (1): either

(i) to move to Galilean (neoNewtonian) spacetime, or

(ii) to move to geometrized gravity, such as in general relativity.

I of course agree that this temptation is worthy: scientifically, heuristically, valuable—and accordingly stressed by physicists' discussions. But do not let this temptation, oriented to the further development of our theories, distort the activity of interpreting them *as now formulated*.

(5): T-duality: (Contr)

'T-duality' is the umbrella term for two dualities between two pairs of string theories (as currently formulated). Both dualities involve inverting the radius of one of the compact ('curled up' like a circle) dimensions of space. Thus a type IIA theory postulating that a certain dimension of space has radius R is dual to a type IIB theory where the dimension is $1/R$.

Objection! If one theory, say a type IIA theory, postulates a radius R so small that it could not be empirically detected, $1/R$ may well be so large that it *could* be detected—if it was real.
Reply! Measuring the radius of a putative compact dimension—say by sending off a particle and timing how long it takes to return to you—can be naturally accommodated by *both* dual theories.
For what one dual describes as a journey through physical space, is described by the other dual as a journey through an internal space.

I interpret the duals as both being about a single topic: the cosmos, the actual world. They make contrary assertions about this topic. So they disagree: a case of (Contr).

Just like Example 4: except that disagreement over a spatial radius replaces disagreement over spatial dimensionality.

Again: we here set aside the heuristic function of dualities.

I agree that maybe one could treat the two string theories, not as theories of everything (TOEs, 'toy cosmologies'), but as both true in a single cosmos/possible world with, say a 10-dimensional space.

Namely: the type IIA describes one compact dimension as radius R, and the type IIB describes another compact dimension as radius $1/R$.

This turns the duals' disagreement into a case of (Diff), not (Contr).

Forgive me, O guru from Illinois...

Huggett (2017) takes the two duals to *agree*. He writes:

> *[He concedes that it] would not be a logical fallacy, nor [contravene] unavoidable semantic or ontological principles, [to deny that] the duals describe the same physical possibility. [But ...] from a practical scientific point of view, it makes sense to treat those differences as non-physical . . . long established well-motivated scientific reasoning should lead us to think that dual total theories represent the same physical situation (2017: 86).*

He goes on to address the resulting question: how can we make sense of the 'appearance' that the dual theories contradict each other about the radius of space?

He distinguishes two answers, called 'interpretation one' (p. 84) and 'interpretation two' (p. 85), and argues in favour of the second ...