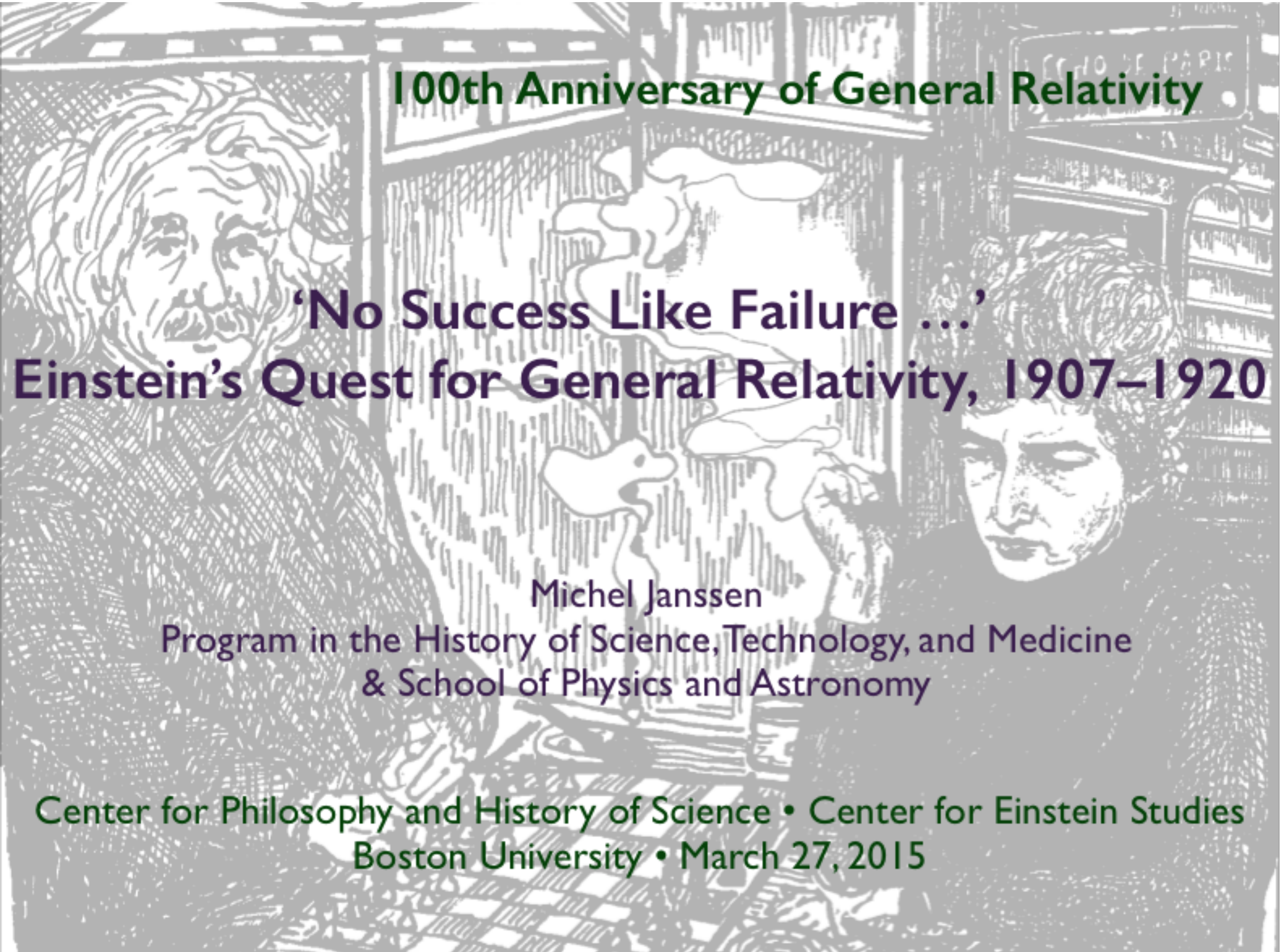

100th Anniversary of General Relativity

‘No Success Like Failure …’ Einstein’s Quest for General Relativity, 1907–1920

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The Cambridge Companion to
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Edited by
MICHEL JANSSEN
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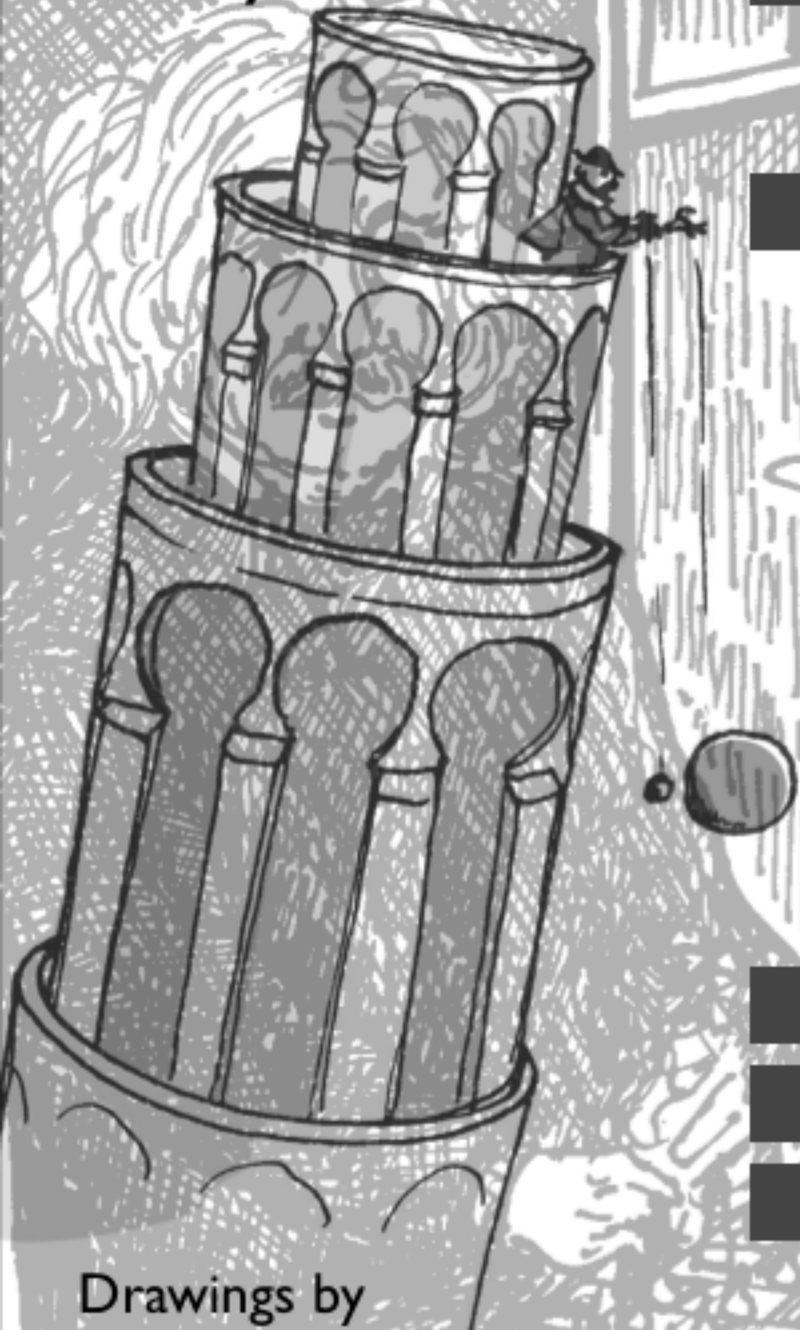
6 "No Success Like Failure …"
Einstein's Quest for General Relativity,
1907–1920
1. INTRODUCTION
THEMES
Field

Gravity and inertia



- Uniqueness of free fall (Galileo, early 1600s): all objects fall with the same acceleration [as long as air resistance can be neglected].
- Uniqueness of free fall in Newton's theory (*Principia*, 1687)

(*i*) law of motion	(*ii*) law of gravity
$F = m_i a$	$F = m_g g$
m_i = inertial mass	m_g = gravitational mass
resistance to acceleration	susceptibility to gravity

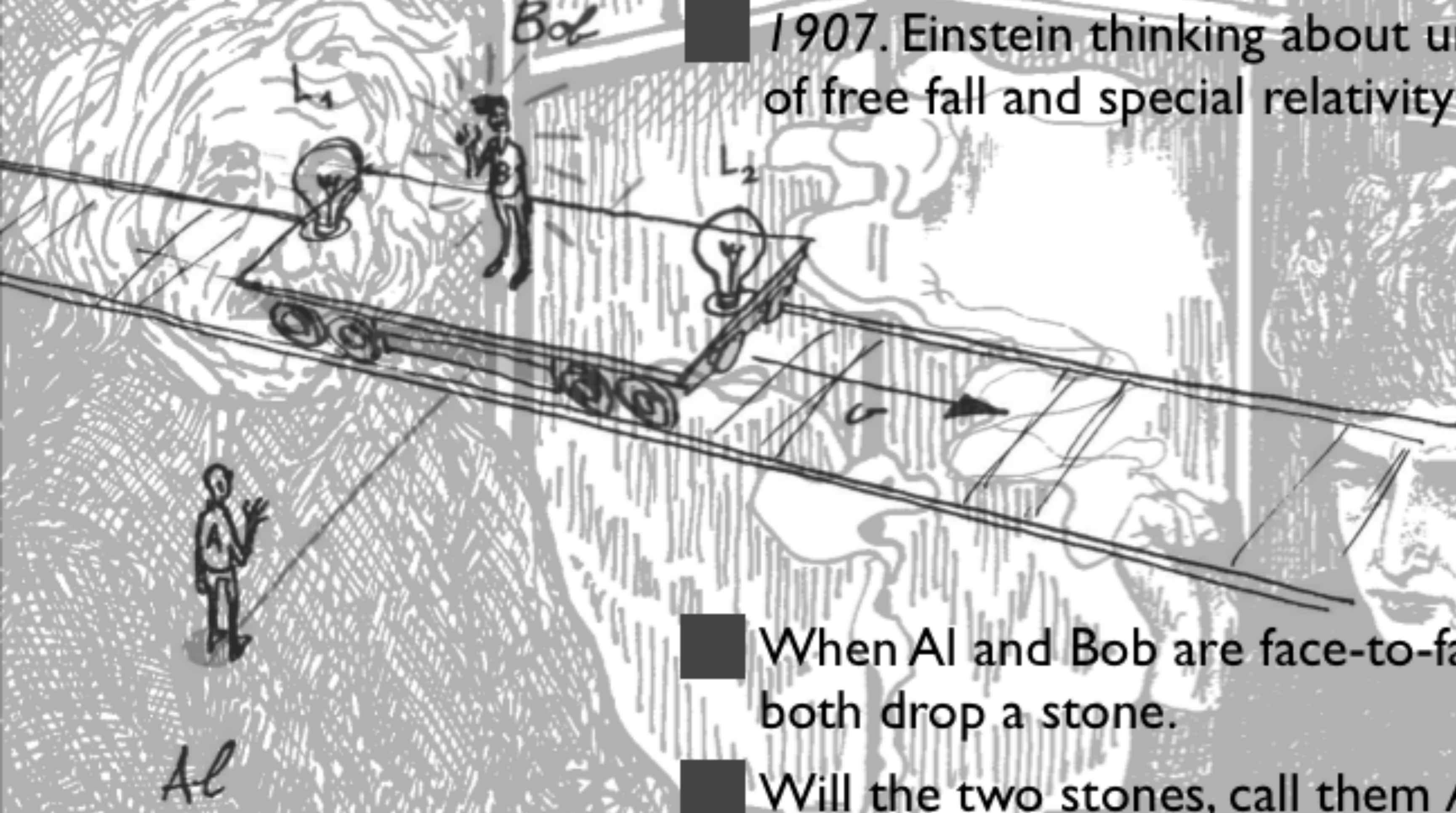
- From (*i*) and (*ii*): $a = (m_g/m_i)g$.
- Uniqueness of free fall requires: $m_i = m_g$
- Equality of inertial and gravitational mass is an unexplained coincidence in Newtonian theory

Drawings by Laurent Taudin

Michel Janssen, 'No Success Like Failure …': Einstein's Quest for General Relativity, 1907–1920

Gravity and inertia

- Fast forward two and a half centuries:
- *1907*. Einstein thinking about uniqueness of free fall and special relativity

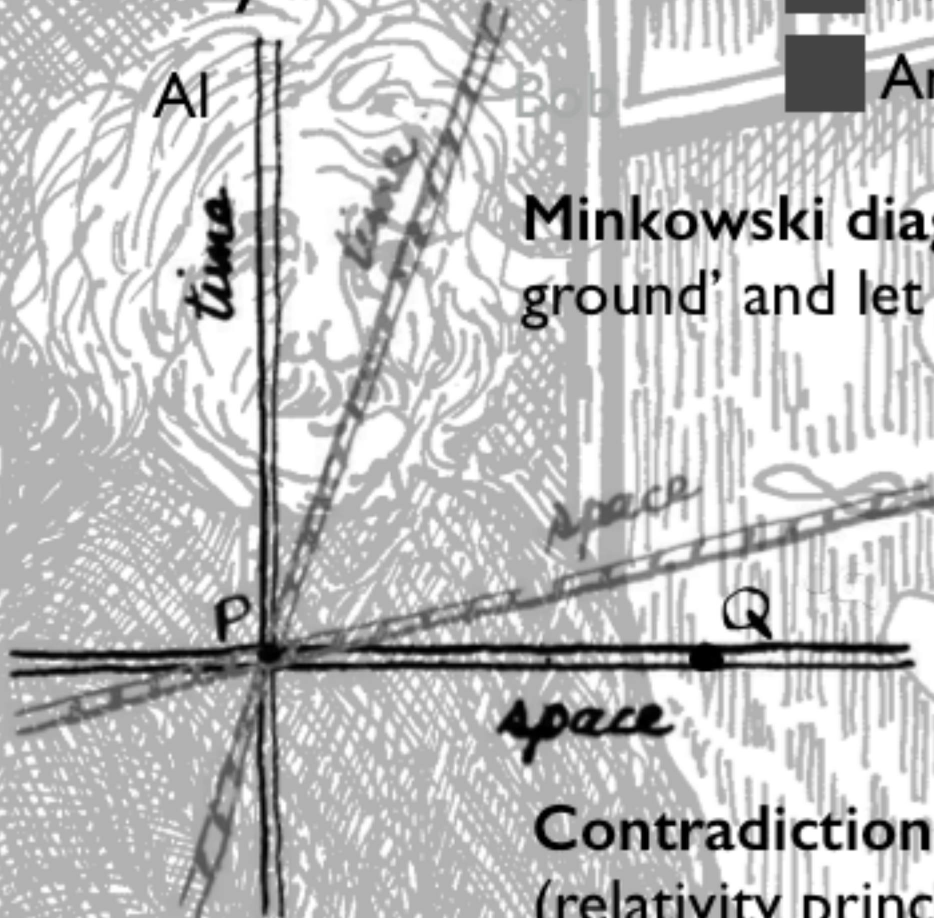


- When Al and Bob are face-to-face, they both drop a stone.
- Will the two stones, call them *A* and *B*, hit the ground simultaneously?
- Answer in Newtonian theory: YES
- Answer in special relativity: NO!

Michel Janssen, 'No Success Like Failure …': Einstein's Quest for General Relativity, 1907–1920

Gravity and inertia

- Will *A* and *B* hit the ground simultaneously?
- Answer in special relativity: NO



Minkowski diagram. Let *P* be the event '*A* hits the ground' and let *Q* be the event '*B* hits the ground'

P and *Q* are simultaneous for Al
→ *P* happens after *Q* for Bob

P and *Q* are simultaneous for Bob
→ *Q* happens after *P* for Al

Contradiction: Al and Bob are equivalent observers (relativity principle). It cannot be that horizontal velocity reduces vertical acceleration in a gravitational field for one of them but not for the other.

Solution: *horizontal motion reduces vertical acceleration in a gravitational field for both Al and Bob* → *Q* happens after *P* for Al; *P* happens after *Q* for Bob

Gravity and inertia

Einstein (reminiscing about 1907 in 1933):

"the equality of inertial and gravitational mass was now brought home to me in all its significance. I was in the highest degree amazed … and guessed that in it must lie the key to a deeper understanding of inertia and gravitation"

—George A. Gibson Foundation Lecture, University of Glasgow, June 20, 1933.

Albert Einstein (1879–1955)

… the key to a deeper understanding of inertia and gravitation …

- Particles get their marching orders from two departments:
  - **Space-time structure**: keep moving in a straight line at constant speed, unless forces tell you otherwise (law of inertia) (law of inertia).
  - **Assorted forces**: deviate from your straight line following the specific instructions given to you.

    E.g., electric forces have jurisdiction only over charged particles.

… the key to a deeper understanding of inertia and gravitation …

- The gravitational force gives marching orders as indiscriminate and universal as those issued by the space-time structure.
- Newton accounted for this universality by setting inertial mass equal to gravitational mass.
- Einstein accounted for this universality by moving gravity from the department of assorted forces to the department of space-time structure. Thou shalt not tear asunder what God has united!
- Einstein combined space-time structure and gravitational field into one **inertio-gravitational field**, which determines the trajectories of particles on which no additional forces are acting.
- This removes the mystery of equality of inertial and gravitational mass, making inertia and gravity two sides of the same coin (mature 1918 version of the **equivalence principle**).

How the metric tensor $g_{\mu\nu}$ characterizes the geometry of space-time

Describing 3+1D curved space-time analogous to describing 2D curved surface. Example: making a map of the globe

- At the equator, map and globe coincide and all conversion factors are equal to 1.
- Away from the equator, the distance between lines of equal *longitude* is *larger* on the map than on the globe. Hence, the 'east-west' conversion factor gets *smaller* as you move away from the equator.
- At the poles, the 'east-west' conversion factor is zero.
- Away from the equator, the distance between lines of equal *latitude* is *smaller* on the map than on the globe. Hence, the 'north-south' conversion factor gets *larger* as you move away from the equator.

How the metric tensor $g_{\mu\nu}$ characterizes the geometry of space-time

Describing 3+1D curved space-time analogous to describing 2D curved surface. Example: making a map of the globe

- Conversion factors depend on position and direction.
- For an 3+1D space-time we need 10 independent conversion factors at every point.
- These conversion factor are given by the symmetric metric tensor field $g_{\mu\nu}(x^{\alpha})$, with $\mu, \nu, \alpha = 0, 1, 2, 3$.
- This description of the geometry of curved surfaces works exactly the same for any map—any coordinate system—we choose (general covariance).

Some milestones in the development of Einstein's new theory of gravity

- *1912*. Concrete theory making gravity part of fabric of space-time.
  - Use the *metric tensor field*, $g_{\mu\nu}(x^\rho)$, to represent (what we now call) the inertio-gravitational field.
  - Free particles (i.e., <u>no</u> non-gravitational forces) follow *geodesics* in (curved) space-time [Misner, Thorne, and Wheeler, 1970: "Space[-time] acts on matter, telling it how to move"]

- *November 1915*. Four short communications to the Berlin Academy:
  - Generally covariant *Einstein field equations* for the metric tensor field, setting the *Ricci tensor* equal to the *energy-momentum tensor* for matter plus a trace term: $R_{\mu\nu} = -\kappa(T_{\mu\nu} - 1/2\, g_{\mu\nu}\, T)$ [Misner, Thorne, and Wheeler, 1970: "matter reacts back on space[-time], telling it how to curve"].
  - Explanation of the anomalous 43″ per century in the perihelion motion of Mercury.

Some milestones in the development of Einstein's new theory

Some milestones in the development of Einstein's new theory of gravity (cont'd)

- *November 1915*. Four short communications to the Berlin Academy:
  - Generally covariant *Einstein field equations* for the metric tensor field, setting the *Ricci tensor* equal to the *energy-momentum tensor* for matter plus a trace term: $R_{\mu\nu} = -\kappa(T_{\mu\nu} - 1/2\, g_{\mu\nu} T)$ [Misner, Thorne, and Wheeler, 1970: "matter reacts back on space[-time], telling it how to curve"].
  - Explanation of the anomalous 43″ per century in the perihelion motion of Mercury.

- *February 1917*. "Cosmological Considerations ..."
  - Applying the new theory to the universe at large.
  - Adding a term with the cosmological constant λ to the field equations: $R_{\mu\nu} - 1/2\, \lambda\, g_{\mu\nu} = -\kappa(T_{\mu\nu} - 1/2\, g_{\mu\nu} T)$.
  - The birth of relativistic cosmology.

The good, the bad, and the ugly

- How did this **good** new theory of gravity get mixed up with the **bad** idea that all motion, uniform or accelerated, would be relative?
- How did such a good theory get such an **ugly** name?

Sir Hermann Bondi (1919–2005): "It is rather the Principle of ... the name of Einstein's ... general relativity ... physically meaningless ... can only be viewed as a historical memento of a curious ... observation ... One may ... and embrace Einstein's theory of gravitation while rejecting the ... however heuristically useful ... himself found it."

J. L. Synge (1897–1995): "The Principle of Equivalence performed the essential office of midwife at the birth of general relativity ... suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced."

General Relativity

What was Einstein thinking?

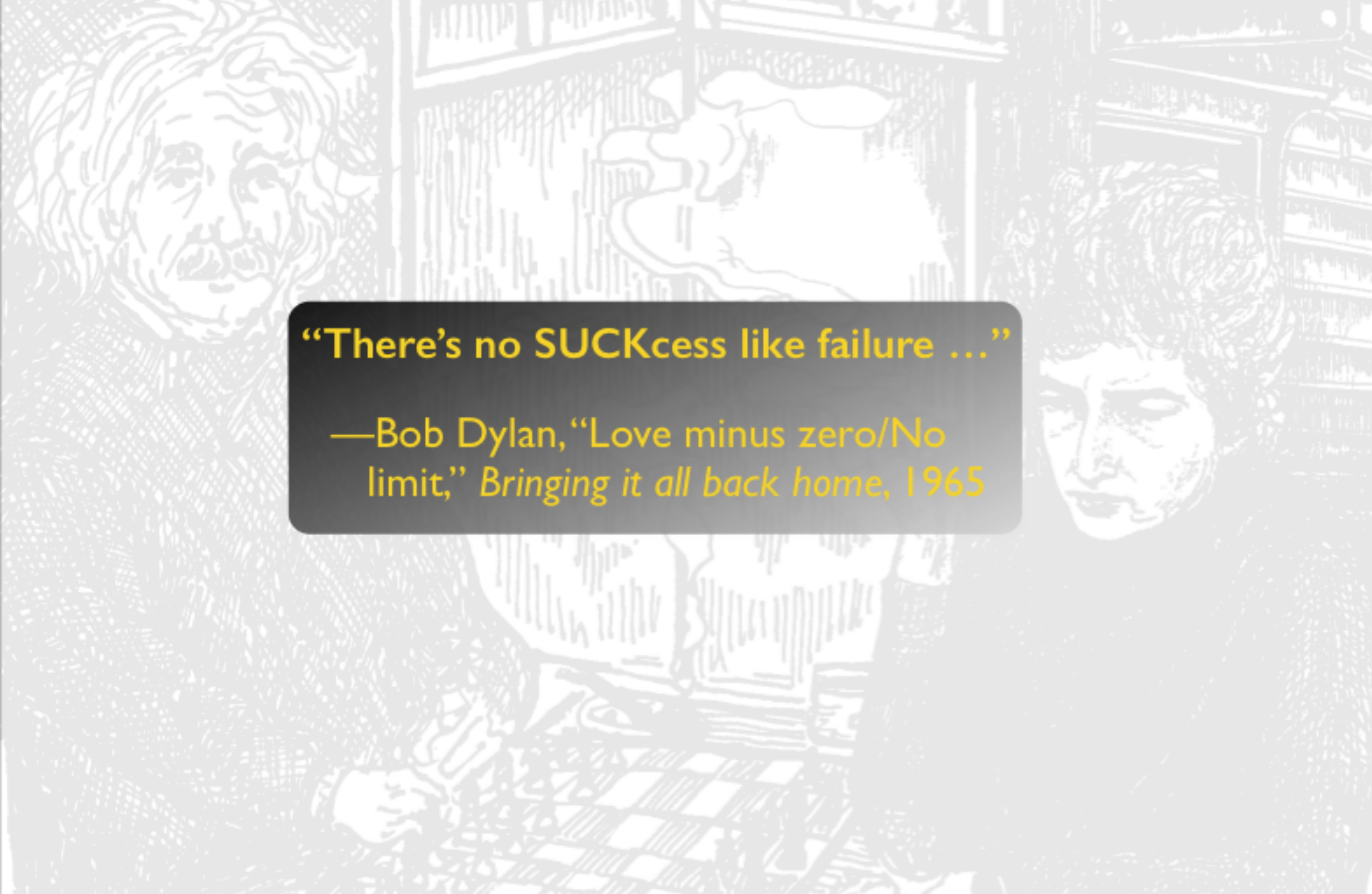
- Imagine two passengers in two trains in non-uniform motion with respect to one another. The accelerated observer can maintain to be the one at rest if she is prepared to ascribe the inertial effects she experiences to some gravitational field.
  - **Equivalence principle**. At least locally, the effects of acceleration are indistinguishable from those of gravity.
  - **Mach's principle**. The gravitational field substituted for an object's acceleration on the basis of the equivalence principle can be ascribed to a material source—anything from the object's immediate surroundings to the distant stars.
  - **General covariance**. All physical laws have the same form for all observers, regardless of their state of motion.
- This characterization of Einstein's project in terms of these three principles follows a short 1918 paper in which Einstein identifies them as the cornerstones of his new theory

Einstein's four attempts to make all motion relative

Einstein's four attempts to make all motion relative

1. *1907–12.* Reduce acceleration to gravity (crude equivalence principle)
2. *1912–13, 1915–16.* General covariance = general relativity.
3. *1913–15.* Vindicate Mach's account of Newton's bucket.
4. *1917–20.* Add cosmological constant to enforce Mach's principle.

- All four attempts failed …
- The uplifting moral: Although he never reached his original destination, the bounty of Einstein's 13-year odyssey was rich by any measure.



“There’s no SUCKcess like failure …”

—Bob Dylan, “Love minus zero/No limit,” *Bringing it all back home*, 1965

Michel Janssen, ‘No Success Like Failure …’: Einstein’s Quest for General Relativity, 1907–1920

Attempt #3: Vindicate Mach’s account of

Attempt #3: Vindicate Mach's account of Newton's rotating bucket (1913–15)

Newton's rotating-bucket experiment

Mach, Einstein, and Newton's bucket

bucket at rest rotating rotating at rest

- **Conclusion**: the shape of the water—flat or concave—is not determined by the relative rotation of the water and the bucket.

water at rest at rest rotating rotating

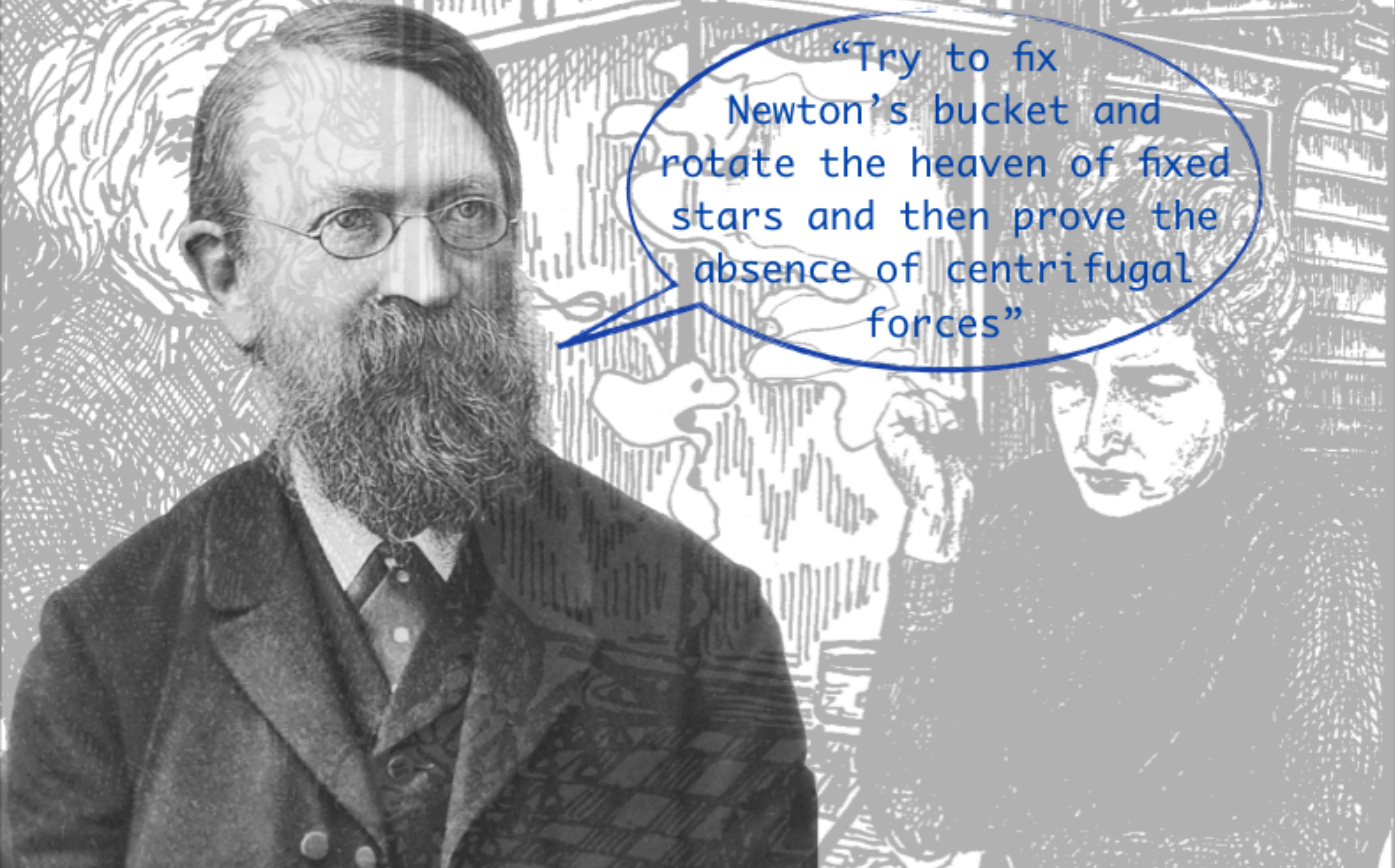
- **Question**: What does determine the shape of the water?

relative rotation NO YES NO YES

- **Newton's answer**: absolute rotation of bucket + water with respect to absolute space.
- **Mach's answer** [as interpreted by Einstein]: relative rotation of bucket + water with respect to the distant stars.

Michel Janssen, 'No Success Like Failure …': Einstein's Quest for General Relativity, 1907–1920

Mach, Einstein, and Newton's bucket



Mach, Einstein, and Newton's bucket

bucket and water rotating
shell and earth at rest

bucket and water at rest
shell and earth rotating

- Einstein's implementation of Mach's idea: represent distant stars by large hollow shell.
- What Mach suggests is that situations I and II are equivalent.

Mach, Einstein, and Newton's bucket

bucket and water rotating
shell and earth at rest

bucket and water at rest
shell and earth rotating

- Problem with Einstein-Mach idea: Newton's theory predicts that the surface is concave in I but flat in II [surface in II should be drawn flat]
- Einstein's comeback: Too bad for Newton's theory! My theory predicts that the surface is concave in both cases! [Not true.]

Mach, Einstein, and Newton's bucket

- For situations I and II to be the same situation viewed by two different observers in Einstein's theory, two conditions must be satisfied:
  - (1) The 'rotation field'* is a vacuum solution of the gravitational field equations

    bucket and water rotating   bucket and water at rest

    shell and earth at rest   shell and earth rotating
  - (2) The inertio-gravitational field of a rotating shell near its center is the same as this 'rotation field.'

*The Coriolis and centrifugal forces resulting from rotation in Minkowski space-time reinterpreted on the basis of the equivalence principle as forces coming from a gravitational field.

Condition (1): is the 'rotation field' a vacuum solution of the gravitational field equations?

the gravitational field equations?

- For 1915 generally covariant field equations: **YES**.

  Follows directly from their general covariance: Minkowski space-time is a solution in standard (non-rotating) coordinates → it is a solution in all coordinates including rotating ones.

- For 1913 non-covariant field equations: **NO**.

  Since the equations are not generally covariant, this needs to be explicitly checked. Einstein flip-flops for two years:

  - ✓ June 1913.
  - ✗ August 1913.
  - ✓ October 1914.
  - ✗ September 1915.

Condition (1): is the 'rotation field' a vacuum solution of the gravitational field equations? Einstein flip-flopping

Einstein-Besso Manuscript , ± June 1913

Result found by Einstein: g_{44} = constant – $\alpha^2\rho^2$

Result after correction of errors: g_{44} = constant – 3/4 $\alpha^2\rho^2$

Condition (1): is the 'rotation field' a vacuum solution of the gravitational field equations? Einstein flip-flopping

“It would also be nice at some point to check the calculation of the gravitational effects connected to rotational motion … Five or six times Einstein has done this now: calculational errors have produced a different result almost every time”

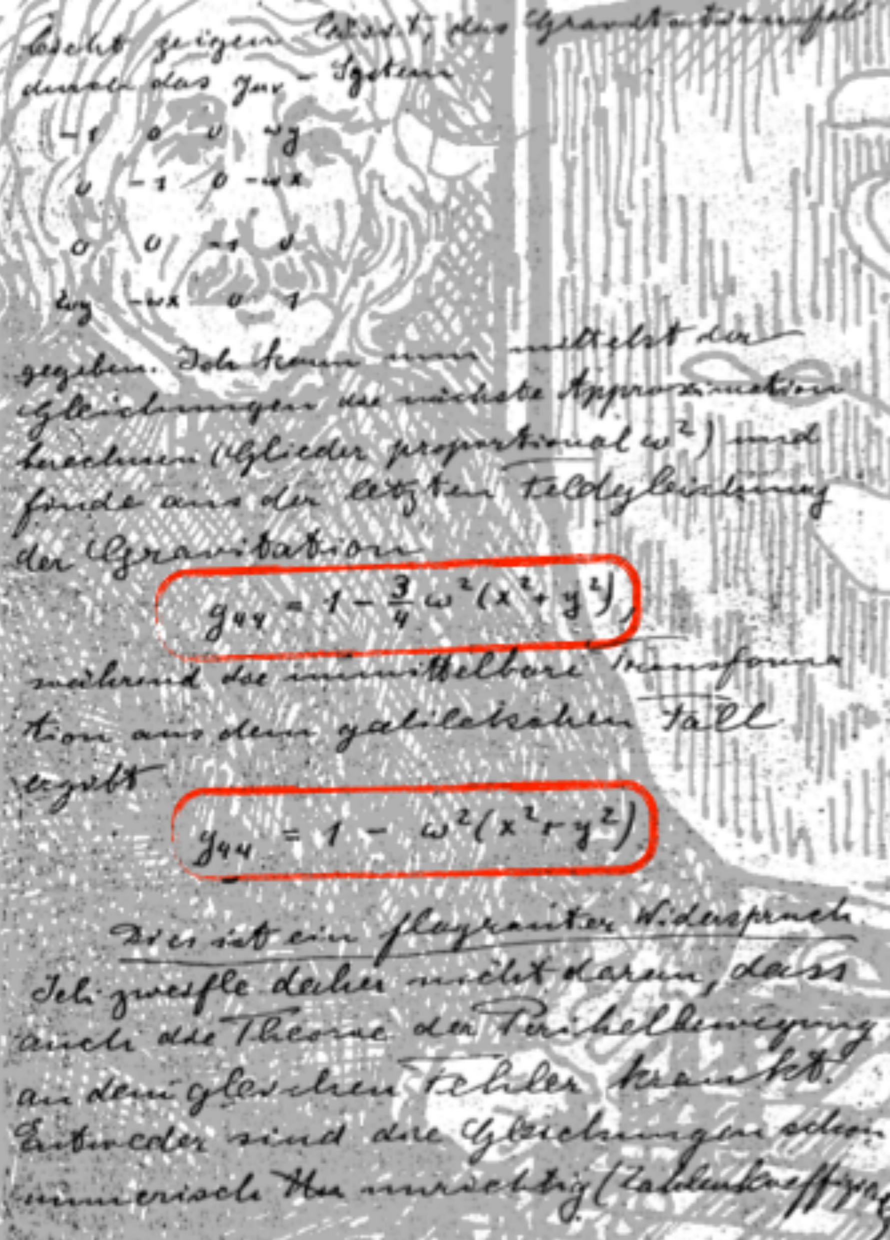
Ehrenfest to Lorentz, August 1913

Paul Ehrenfest (1880–1933)

Condition (1): is the ‘rotation field’ a vacuum solution of the gravitational field equations? Einstein flip-flopping

- Calculation done on a sheet subsequently cannibalized for a letter to Otto Naumann, late September 1915.
- Einstein finally established once and for all that the rotation field is **not** a solution of his non-covariant field equations.

Condition (1): is the 'rotation field' a vacuum solution of the gravitational field equations? Einstein flip-flopping



- Einstein to Erwin Freundlich, September 30, 1915.
- "*This is a blatant contradiction* … I do not believe I am able to find the mistake myself, for in this matter my mind is set in a deep rut. More likely, I will have to rely on some fellow human being with unspoiled brain matter finding the mistake."

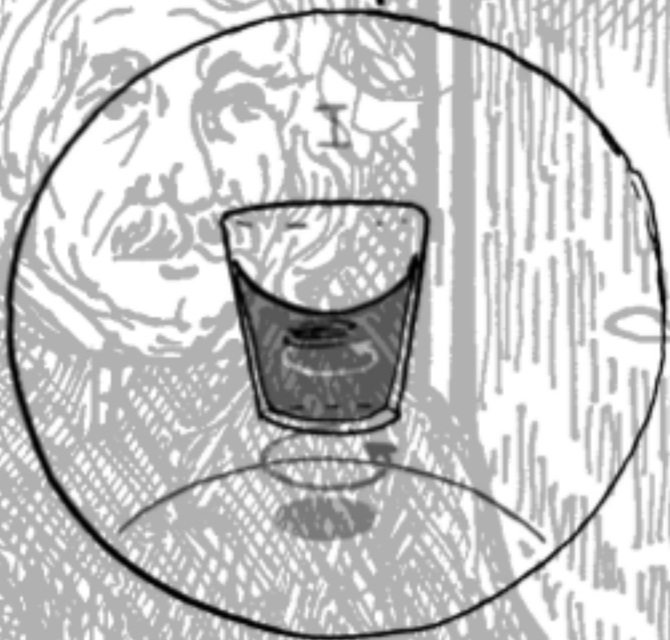
The non-covariant theory gives: $g_{44} = 1 - 3/4\,\omega^2(x^2 + y^2)$

It should be: $g_{44} = 1 - \omega^2(x^2 + y^2)$

- Because of this problem, Einstein gave up these non-covariant equations and not long thereafter found the generally covariant ones.

Condition (2): is the inertio-gravitational field of a rotating shell near its center the same as the 'rotation field'? NO for both 1913 and 1915 field equations

and 1915 field equations



Situation I viewed from the bucket:

- Minkowski metric in rotating coordinates;
- Stars form rotating spherical shell: no forces between them; kept in orbit by Coriolis & centrifugal forces.

$\neq$

Situation II viewed from the bucket:

- small perturbation of Minkowski metric in non-rotating coordinates;
- stars form rotating spherical shell; strong forces needed to prevent shell from flying apart (water surface should be drawn flat).

Late 1915–early 1916: Happy interlude

- Einstein has the illusion that

general covariance suffices

1. to extend the relativity principle from uniform to arbitrary motion.
2. to vindicate Mach's account of Newton's bucket experiment.

- Erich Kretschmann (1887–1973) disabuses him of illusion #1 in 1917.
- Willem de Sitter (1872–1934) disabuses him of illusion #2 in the fall of 1916.

The problem of boundary conditions

- De Sitter points out to Einstein [during a visit by the latter to Leyden in 1916]: you still

visit by the latter to Leyden in 1916]: you still have a remnant of absolute space if you assume Minkowskian boundary values for the metric at spatial infinity.

- Einstein's escape: assume degenerate values for the metric at spatial infinity + masses outside the visible part of the universe ensuring that those degenerate values turn into the Minkowskian boundary values at the edge of the visible universe.
- De Sitter's response: cure worse than the disease. What if we get better telescopes and the visible universe gets bigger? Is Einstein then going to move his masses further out?

Einstein comes up with clever response to De Sitter: Boundary values at infinity a problem? abolish infinity!

- Einstein to De Sitter, February 2, 1917: "I have completely abandoned

- Einstein to De Sitter, February 2, 1917: "I have completely abandoned my views, rightfully contested by you, on the degeneration of the $g_{\mu\nu}$. I am curious to hear what you will have to say about the somewhat crazy idea I am considering now."

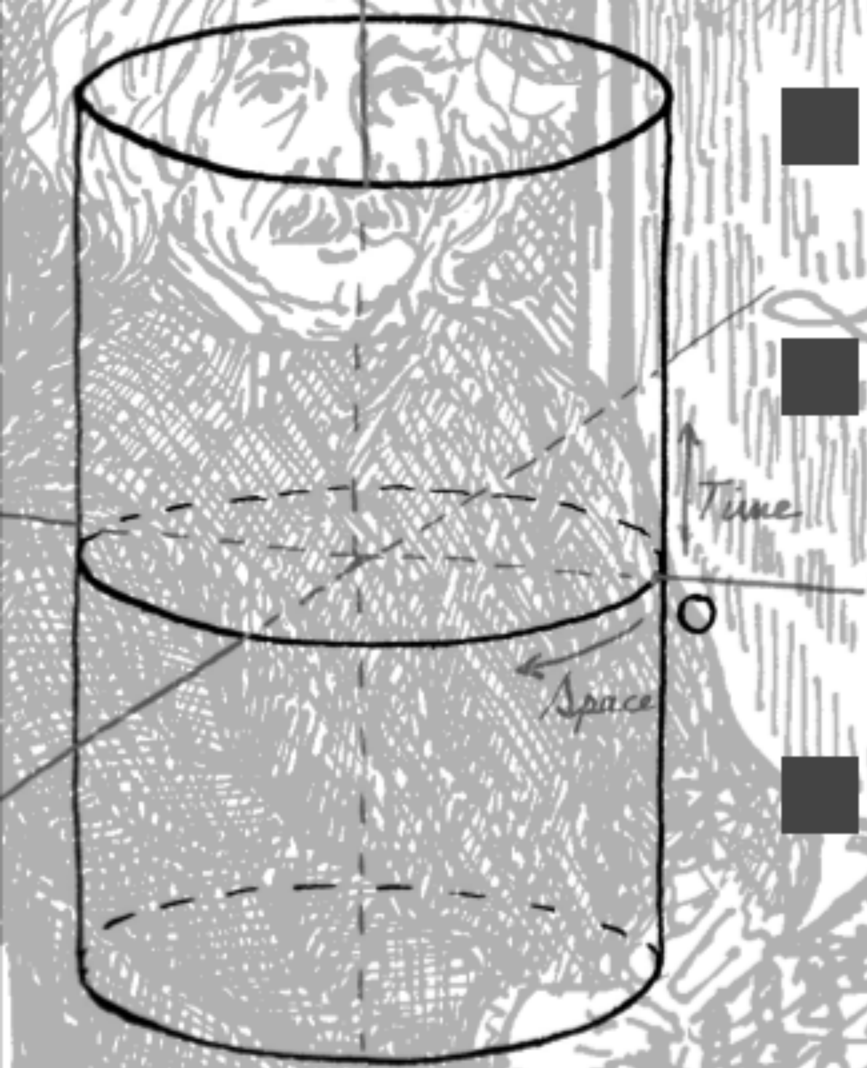
Ich habe ganz
von meiner von Ihnen mit Recht
bekämpften Ansicht vom Ausarten
der $g_{\mu\nu}$ abgekommen. Ich bin neugierig,
was Sie zu der etwas phantastischen
Auffassung sagen werden, die ich
jetzt ins Auge gefasst habe.
Mit herzlichem Gruss
Ihr A. Einstein

Einstein's crazy idea: assume that the universe is spatially closed

Einstein's 'cylinder world' and the cosmological constant

- Simplification: consider 1+1D space-time embedded in 2+1D space-time.

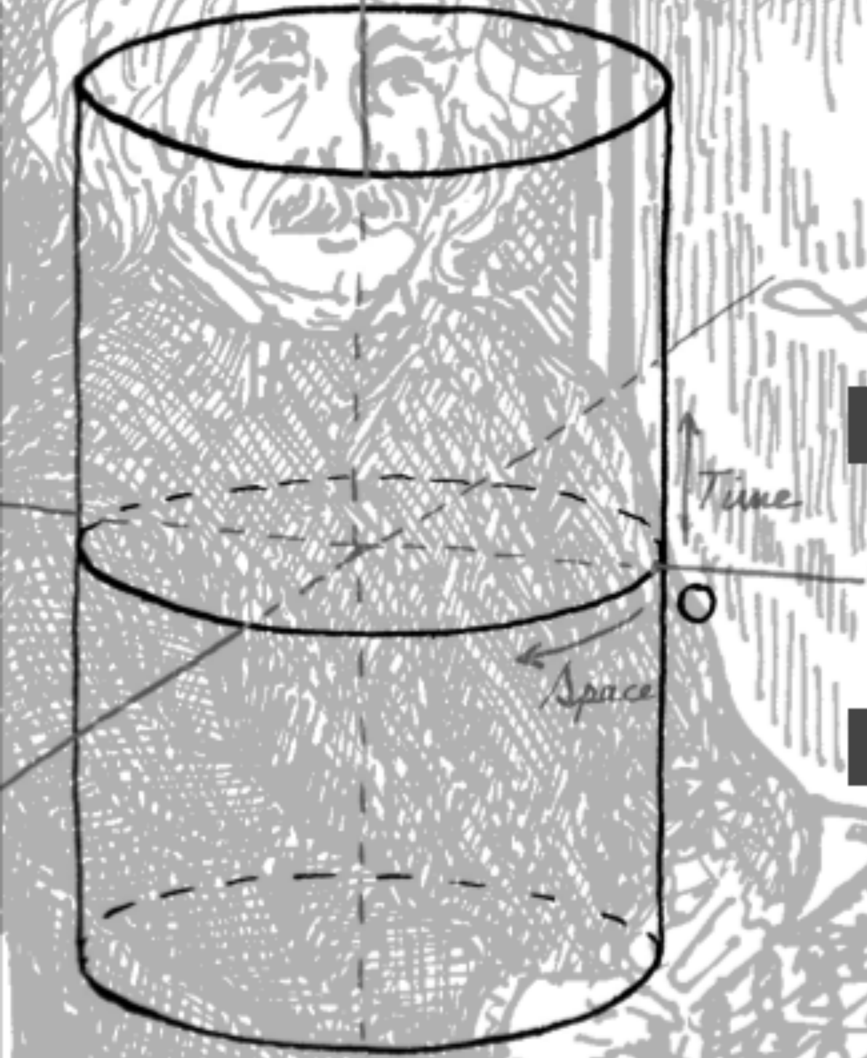
embedded in 2+1D space-time.



- **Question**: is the metric field describing the space-time geometry of the cylinder universe a solution of the 1915 field equations?
- **Answer**: NO! It is only a solution if (*i*) a cosmological term, $-1/2\ \lambda\ g_{\mu\nu}$, is added and (*ii*) the relation $\lambda = 1/R^2 = \kappa\rho/2$ holds [R = radius of the universe, ρ = mass density, κ = Einstein's gravitational constant].
- It follows that there is mass everywhere in the cylinder world. Without the anti-gravity provided by the cosmological term the cylinder world would end in a big crunch.

How to sell the cosmological constant to De Sitter?

- **Problem**: How to preempt the predictable criticism that the



predictable criticism that the cosmological term is there only because of Einstein's Machian hobby horse? (Just like the masses right outside the visible universe earlier.)

- **Solution**: Argue that even Newtonian cosmology calls for a cosmological constant to get the anti-gravity needed to get a stable static universe
- Never mind that astronomers already know, as De Sitter would tell Einstein repeatedly, that there is no reason to think that the universe is static.

How to sell the cosmological constant to De Sitter?

Clever introduction of the constant in print: "Cosmological Considerations in the General Theory of Relativity" (1917).

Considerations in the General Theory of Relativity" (1917).

- **Context of discovery**
  - Dream up closed static model: cylinder world;
  - Add cosmological constant to field equations to allow this model;
  - Give plausibility argument for adding the cosmological constant.
- **Context of justification** (just the other way around)
  - Give plausibility argument for adding the cosmological constant;
  - Look for closed static model allowed by field equations with cosmological constant;
  - Show—*surprise, surprise*—that the cylinder world is such a model.

The paper that launched relativistic cosmology: "Cosmological Considerations in the General Theory of Relativity" (1917)

Kosmologische Betrachtungen zur allgemeinen

144 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917

der an sich nicht beansprucht, ernst genommen zu werden; er dient nur dazu, das Folgende besser hervortreten zu lassen. An die Stelle der Poissonschen Gleichung setzen wir

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen hinzutreten für das räumlich Unendliche, falls man die Welt wirklich als räumlich unendlich ausgedehnt anzusehen hat.

Bei der Behandlung des Planetenproblems habe ich diese Grenzbedingungen in Gestalt folgender Annahme gewählt: Es ist möglich, ein Bezugssystem so zu wählen, daß sämtliche Gravitationspotentiale $g_{\mu\nu}$ im räumlich Unendlichen konstant werden. Es ist aber a priori durchaus nicht evident, daß man dieselben Grenzbedingungen ansetzen darf, wenn man größere Partien der Körperwelt ins Auge fassen will. Im folgenden sollen die Überlegungen angegeben werden, welche ich bisher über diese prinzipiell wichtige Frage angestellt habe.

§ 1. Die Newtonsche Theorie.

Es ist wohlbekannt, daß die Newtonsche Grenzbedingung des konstanten Limes für ϕ im räumlich Unendlichen zu der Auffassung hinführt, daß die Dichte der Materie im Unendlichen zu null wird. Wir denken uns nämlich, es lasse sich ein Ort im Weltraum finden, um den herum das Gravitationsfeld der Materie, im großen betrachtet, Kugelsymmetrie besitzt (Mittelpunkt). Dann folgt aus der Poissonschen Gleichung, daß die mittlere Dichte ρ rascher als $\frac{1}{r^2}$ mit wachsender Entfernung r vom Mittelpunkt zu null herabsinken muß, damit ϕ im

nur dann, das Folgende besser hervortreten zu lassen. An die Stelle der Poissonschen Gleichung setzen wir

$$\Delta\phi - \lambda\phi = 4\pi K\rho, \qquad (2)$$

wobei λ eine universelle Konstante bedeutet. Ist ρ_0 die (gleichmäßige) Dichte einer Massenverteilung, so ist

$$\phi = -\frac{4\pi K}{\lambda}\rho_0 \qquad (3)$$

eine Lösung der Gleichung (2). Diese Lösung entspräche dem Falle, daß die Materie der Fixsterne gleichmäßig über den Raum verteilt wäre, wobei die Dichte ρ_0 gleich der tatsächlichen mittleren Dichte der Materie des Weltraumes sein möge. Die Lösung entspricht einer unendlichen Ausdehnung des im Mittel gleichmäßig mit Materie erfüllten Raumes. Denkt man sich, ohne an der mittleren Verteilungsdichte etwas zu ändern, die Materie örtlich ungleichmäßig verteilt, so wird sich über den konstanten ϕ-Wert der Gleichung (3) ein zusätzliches ϕ überlagern, welches in der Nähe dichterer Massen einem Newtonschen Felde um so ähnlicher ist, je kleiner λ, gegenüber $4\pi K\rho$ ist.

Eine so beschaffene Welt hätte bezüglich des Gravitationsfeldes keinen Mittelpunkt. Ein Abnehmen der Dichte im räumlich Unendlichen müßte nicht angenommen werden, sondern es wäre sowohl das mittlere Potential als auch die mittlere Dichte bis ins Unendliche konstant. Der bei der Newtonschen Theorie konstatierte Konflikt mit der statistischen Mechanik ist hier nicht vorhanden. Die Materie ist bei einer bestimmten (äußerst kleinen) Dichte im Gleichgewicht, ohne daß für dies Gleichgewicht innere Kräfte der Materie (Druck) nötig wären.

§ 2. Die Grenzbedingungen gemäß der allgemeinen Relativitätstheorie.

Im folgenden führe ich den Leser auf dem von mir selbst zurückgelegten, etwas indirekten und holperigen Wege, weil ich nur so hoffen kann, daß er dem Endergebnis Interesse entgegenbringe. Ich komme nämlich zu der Meinung, daß die von mir bisher vertretenen Feldgleichungen der Gravitation noch einer kleinen Modifikation bedürfen, um auf der Basis der allgemeinen Relativitätstheorie jene prinzipiellen Schwierigkeiten zu vermeiden, die wir im vorigen Paragraphen für die Newtonsche Theorie dargelegt haben. Diese Modifikation entspricht vollkommen dem Übergang von der Poissonschen Gleichung (1) zur Gleichung (2) des vorigen Paragraphen. Es ergibt sich dann

The paper that launched relativistic cosmology: "Cosmological Considerations in the General Theory of Relativity" (1917)

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müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$\boxed{G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right).} \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle Konstante, in das Hamiltonsche Prinzip einführt, welches Prinzip ja die Giltigkeit von Erhaltungssätzen gewährleistet. Daß die Feldgleichung (13a) mit unseren Ansätzen über Feld und Materie vereinbar ist, wird im folgenden gezeigt.

Einstein sends the paper to De Sitter

Nun zu unserer Sache! Vom Standpunkte der Astronomie
ist es natürlich ein geräumiges Luftschloss, das sich aber
... Aber für mich war die Frage brennend, ob sich der

- Einstein to De Sitter, early March 1917: "From the standpoint of astronomy, I have, of course, just built a spacious castle in the air. It was a burning question for me, however, whether the relativity thought can be followed through to its conclusion, or whether one runs into contradictions ... Whether the model I constructed corresponds to reality is another matter ..."

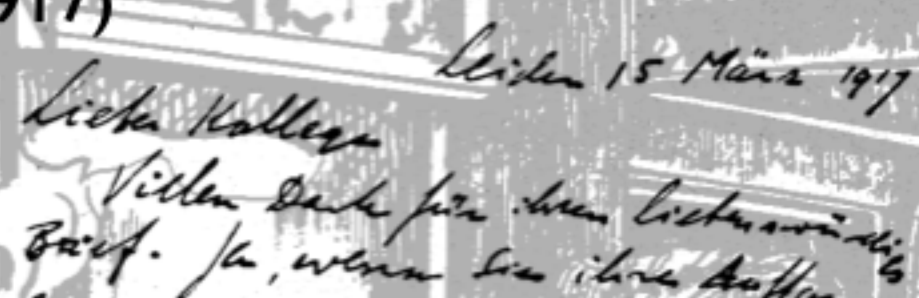
'No success like failure': Einstein's failed crusade against absolute motion (project in philosophy) launched relativistic cosmology (project in physics)

- Einstein wrote his cosmology paper because of his philosophical predilections, **not** because he wanted to apply general relativity to cosmology.
- But what started as part of a philosophical debate about the relativity of motion ended up launching relativistic cosmology.

- Einstein to De Sitter, early March 1917: "From the standpoint of astronomy, I have, of course, just built a spacious castle in the air. It was a burning question for me, however, whether the relativity thought can be followed through to its conclusion, or whether one runs into contradictions… Whether the model I constructed corresponds to reality is another matter."

Michel Janssen & Charles Midwinter, '*Castles in the Air* …': *The Einstein-De Sitter Debate, 1916–1918*. In preparation

De Sitter's response (March 15, 1917)



... ja, wenn Sie Ihre Auffassung nur der Wirklichkeit nicht aufzwingen wollen, dann sind wir einig. Als widerspruchsfreie Gedankenkette habe ich nichts dagegen, und bewundere ich sie. Ganz und gar zustimmen kann ich nicht, bevor ich damit gerechnet habe, was mir jetzt nicht möglich ist.

- "Well, as long as you don't want to force your conception on reality, we are in agreement. As a consistent train of thought, I have nothing against it and I admire it. I cannot give you my final approval before I have had a chance to calculate with it"

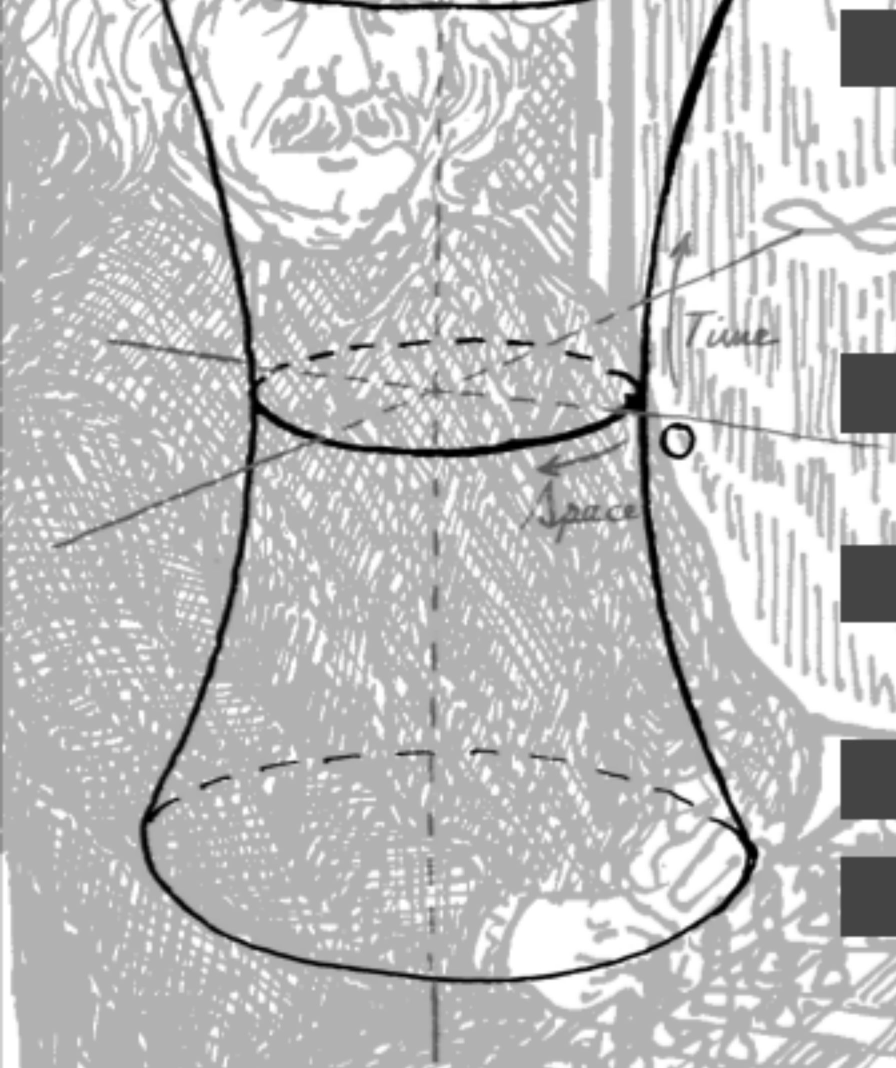
Five days later (March 20, 1917), De Sitter has come up with an alternative to Einstein's cylinder world

Dreidimensional
Mit übernatürlichen Massen.
$\lambda = \frac{1}{R^2}$

Vierdimensional
Ohne jede Masse.
$\lambda = \frac{3}{R^2}$

De Sitter's 'hyperboloid world'

- Recall simplification: consider 1+1D space-time embedded in 2+1D space-time.



- Consider 1+1D surface of an hyperboloid in 2+1D Minkowski spacetime (analogue of 2D surface of a sphere in 3D Euclidean space) [Model suggested by Ehrenfest]
- **Question**: is this model allowed by the field equations with cosmological term?
- **Answer**: YES, as long as (*i*) $\lambda = 3/R^2$ and (*ii*) $\rho = 0$.
- **The hyperboloid world is empty!**
- Bad news for Einstein: λ was supposed to rule out space-times without matter.

Einstein's reaction

- Einstein to De Sitter, March 24, 1917: "It would be unsatisfactory, in my opinion, if a world without matter were possible. Rather, it should be the case that the $g^{\mu\nu}$-field is fully determined by matter and cannot exist the case that the g field is fully determined by matter and cannot exist without the latter. This is the core of what I mean by the requirement of the relativity of inertia" [called **Mach's principle** in 1918].

- Quoted in postscript to March 1917 paper in which De Sitter reported his result to the Amsterdam Academy

Naschrift.

De heer EINSTEIN, wien ik den inhoud van het bovenstaande in het kort had medegedeeld, schrijft mij: „Es wäre nach meiner Meinung unbefriedigend, wenn es eine denkbare Welt ohne Materie gäbe. Das $g_{\mu\nu}$-Feld soll vielmehr *durch die Materie bedingt sein, ohne dieselbe nicht bestehen können.* Das ist der Kern dessen, was ich unter der Forderung von der Relativität der Trägheit verstehe".

Einstein tried to show that the De Sitter hyperboloid world is not empty and hence not a counter-example to Mach's principle after all.

- De Sitter produced an alternative map for his hyperboloid world that allows for easy comparison to Einstein's cylinder world.

Einstein tried to show that the De Sitter hyperboloid world is not empty and hence not a counter-example to Mach's principle after all.

Einstein's reasoning about static representation of hyperboloid world:

- Line segments representing one time unit in the hyperboloid world get longer and longer as we go from O to P*

get longer and longer as we go from O to P.*

- This suggested to Einstein that clocks slow to a crawl as we go from O to P.
- Clocks slow to a crawl when lowered into a strong gravitational field, e.g., when approaching some massive object (*gravitational redshift*).
- Hence, Einstein concluded, there must be an enormous amount of mass tucked away at P in the supposedly empty hyperboloid world!

Hyperboloid world not empty! Mach's Principle saved! Cosmological constant does away with absolute motion after all!

*Note (for future reference): conversion factor from coordinate time to proper time [= square root of the 00-component of the metric $g_{\mu\nu}$] goes from 1 at O to 0 at P.

March 1918. Einstein Triumphant

Einstein publishes two papers in response to De Sitter's challenge to his latest attempt to abolish absolute motion.

1. "On the Foundations of the General Theory of Relativity."

1. On the Foundations of the General Theory of Relativity.

Einstein officially introduces: "Mach's Principle: The *G*-field is fully determined by the masses of bodies."

2. "Critical Comment on the Solution of the Gravitational Field Equations Given by Mr. De Sitter."

"The De Sitter solution might not correspond to the case of a matter-free world at all, but rather to that of a world, in which all matter is concentrated on the surface $r = (\pi/2)R$" [= the point P in our 1+1D version].

Einstein's conjecture: "this could well be proven by considering the limit of a spatial matter distribution turning into a surface distribution."

May 1918. Two famous mathematicians get in on the action

- Hermann Weyl (1885–1955) writes to Einstein that he has found the proof Einstein suggested that there is mass is tucked away at the equator of the hyperboloid world.
- Felix Klein (1849–1925) writes to Einstein explaining that the odd

- Felix Klein (1849–1925) writes to Einstein explaining that the odd behavior of the time conversion factor in the hyperboloid world has nothing to do with the presence of mass.
- Einstein doesn't get Klein's point and in his response explains Weyl's result to Klein.

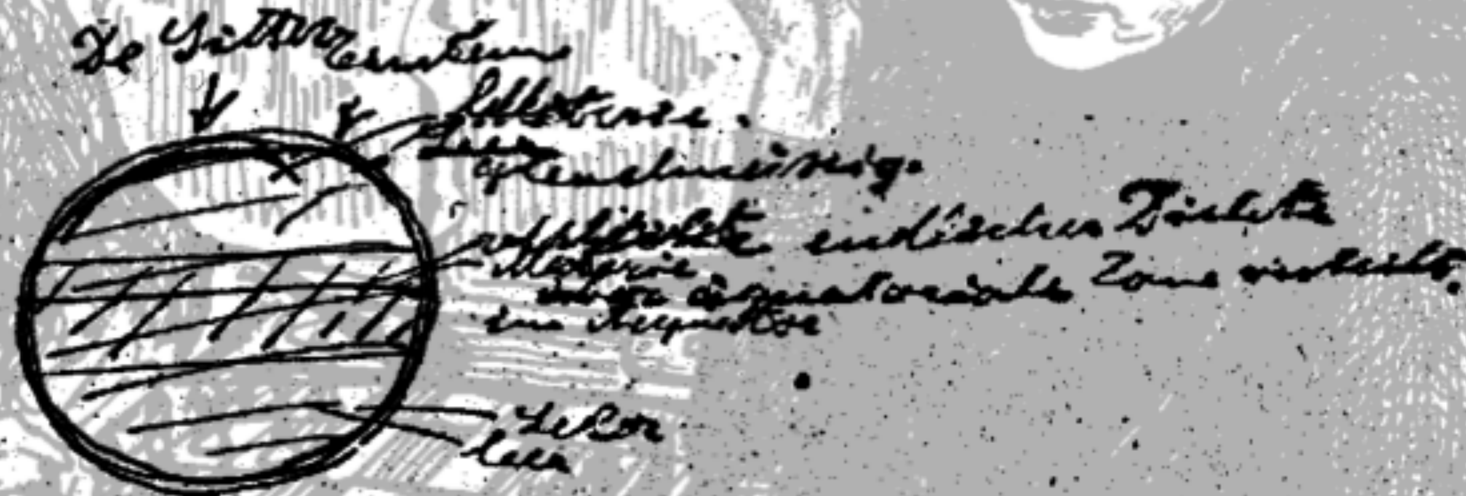
empty

De Sitter: matter of finite density distributed over equator zone

Einstein: empty matter uniformly distributed

empty

empty



June 1918. Einstein bites the dust

- Klein explains it again in simpler terms and this time Einstein gets it.
- Einstein to Felix Klein, June 1918: "You are completely right. The De Sitter world in and of itself is free of singularities and

all its points are equivalent.”

Klein showed Einstein that the singularity in the static form of the De Sitter solution is a coordinate singularity and has nothing to do with mass being present there.

- Recall: conversion factor from coordinate time to proper time = square root of the time-time component (g_{00}) of the metric.
- Conversion factor is zero at *P* because one and the same point *P* on the hyperboloid is mapped onto a whole line of points P_1, P_2, P_3, … (cf. poles on standard map of the earth)

Out with Mach's principle … and the cosmological constant!

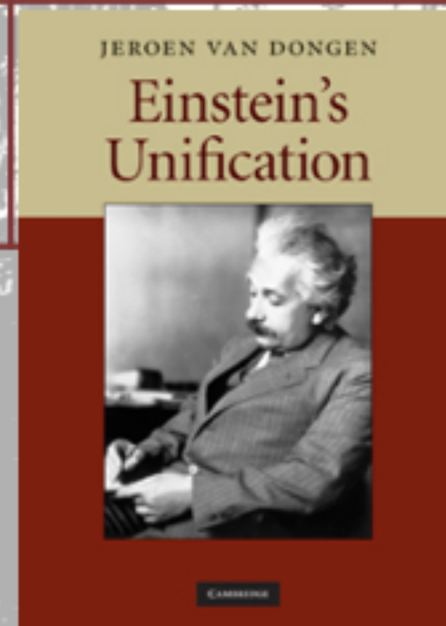
- If Mach's principle holds, absolute motion is eradicated: if the metric field can be fully reduced to matter, then '*motion with respect to the metric field*' is just a *façon de parler* about '*motion with respect to matter generating the metric field*.' Alas, Mach's principle does not hold.

- But Mach's principle is anachronistic by the 1910s anyways: it is predicated on an antiquated billiard-ball ontology:
  - 19th century: matter consists of particles
  - 20th century: matter consists of fields (for Einstein: electromagnetic fields)
- Einstein to Felix Pirani, 2 February 1954: "In my view one should no longer speak of Mach's principle at all. It dates back to the time in which one thought that the "ponderable bodies" are the only physically real entities and that all elements of the theory which are not completely determined by them should be avoided. (I am well aware of the fact that I myself was long influenced by this *idée fixe*)."

Einstein's new project (ca. 1920–1955): don't reduce one field to another: unify the two!

Special relativity	electric field	magnetic field	electromagnetic field

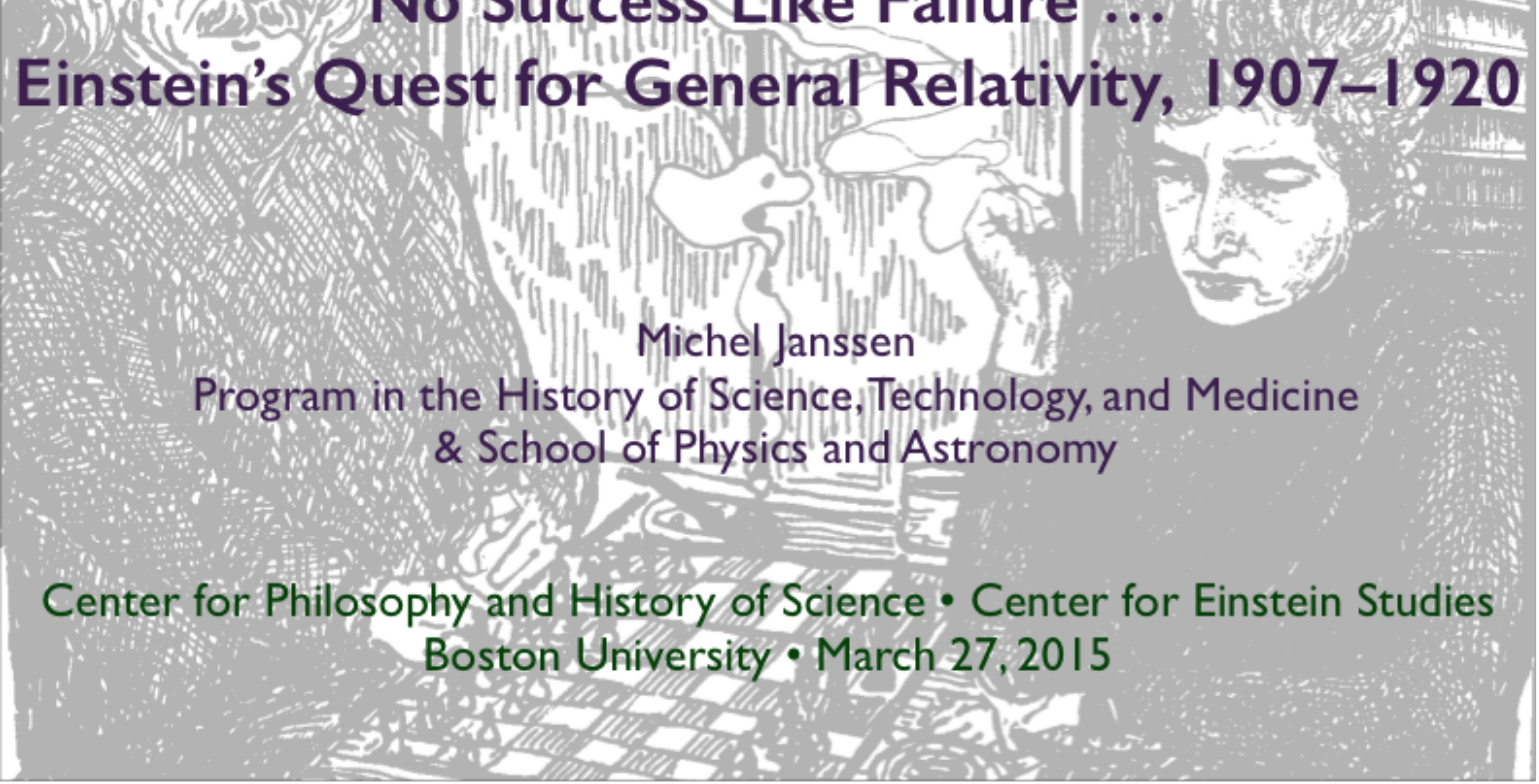
relativity			field
General relativity	inertia	gravity	inertio-gravitational field
Unified field theory	electromagnetic field	inertio-gravitational field	



Jeroen van Dongen, *Einstein's Unification* (Cambridge University Press, 2010).

100th Anniversary of General Relativity

'No Success Like Failure ...'


No Success Like Failure …
Einstein's Quest for General Relativity, 1907–1920
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