

# Is there a Church-Turing thesis for social algorithms?

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Some of the work shown here was joint with Walter Dean,  
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Wiitzel

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Social algorithms deal with much more complex 'inputs' and the output may be something more complex as well.

And yet the project of developing a taxonomy of social algorithms seems both important and interesting.

## Two levels of social algorithms

- ▶ Individual level
- ▶ Societal level



## Individual level

Amartya Sen points out in *Commodities and Capabilities* that our reliance on utility theory is a bit naive, for people want many things which may not be comparable or compatible.

*I argue in favour of focusing on the capability to function, i.e., what a person can do or can be and argue against the more standard concentration on opulence (as in 'real income' estimates) or on utility (as in traditional 'welfare economic' formulations). Insofar as opulence and utility have roles (and they certainly do), these can be seen in terms of their indirect connections with well-being and advantage, in particular, (1) the causal importance of opulence, and (2) the evidential importance of utility (in its various forms, such as happiness, desire-fulfilment and choice)*

## Society as operating system

Suppose that I am going from my apartment to a hotel in Chicago. Then my schedule (or program) will consist of several steps. Taking a cab to Laganardia airport, checking in, boarding the plane, and then in Chicago, taking a cab to the hotel. Each step needs to be tested for correctness. For instance, if I do not have a picture ID, I cannot board the plane, and it is irrelevant that the other steps would have gone well.

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The schedule I described is a simple *straight line* program. There could also be decision points like, *take a taxi if available, otherwise take a bus*. A plan with many such decision points would look like a tree rather than a line. But the entire schedule does have to be checked for correctness – something we do informally.

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One role which society can play in our lives is to serve as an [operating system](#) within which we can write our individual programs. When society builds a bridge or starts a bus line it increases our capabilities and thereby increases our wellbeing.

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There were no drive by shootings before there were motorcycles and cars (and guns).

## The Social Level

Sometimes a social planner wants to achieve some social goal. He needs to be aware that the plan can only succeed if the various individuals act in a manner which is consonant with the plan; but also that they themselves will act in a certain way depending on their beliefs, on their desires and on their possibilities of action. The trick is to see that what they do on their own coincides with what the plan requires.

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Their actions can be influenced, either by influencing their beliefs or their possibilities of action, or their desires (the last is harder but can be achieved via advertising or brain washing).

## Inducing Beliefs: Shakespeare's Much ado about Nothing

At Messina, a messenger brings news that Don Pedro, a Spanish prince from Aragon, and his officers, Claudio and Benedick, have returned from a successful battle. Leonato, the governor of Messina, welcomes the messenger and announces that Don Pedro and his men will stay for a month.

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Various events take place and Claudio wins the hand in marriage of Hero, Leonato's only daughter and the wedding is to take place in a week.

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According to this strategem, the men led by Don Pedro proclaim Beatrice's love for Benedick while knowing he is eavesdropping on their conversation. Thus we have, using  $b$  for Benedick,  $d$  for Don Pedro and  $E$  for the event of eavesdropping,

$$K_b(E), K_d(E) \text{ and } \neg K_b(K_d(E))$$



All these conditions are essential and of course the plot would be spoiled if we had  $K_b(K_d(E))$  instead of  $\neg K_b(K_d(E))$ . Benedick would be suspicious and would not credit the conversation.

The women led by Hero carry on a similar charade for Beatrice. Beatrice and Benedick, are now convinced that their own love is returned, and hence decide to requite the love of the other.

*The play ends with all four lovers getting married.*

## Benedick's Decision problem

	love	nolove
propose	100	-20
nopropose	-10	0

Here *love* means “Beatrice loves me” and *nolove* the other possibility.

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Justified true belief is not knowledge.

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But it is a little out of the way and I will say no more about it.

## Inducing false beliefs in the tigers of the Sundarbans

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In 1987 no one wearing a mask was killed by a tiger, but 29 people without masks were killed.

Unfortunately the tigers eventually realized it was a hoax, and the attacks resumed.

## Creating knowledge states

Let a finite group of agents know nothing about  $p$ . It is common knowledge that no one knows if  $p$  is true and no one knows that  $p$  is false. Let  $S$  be a state of knowledge among these agents which can be described by a finite Kripke structure.

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Then the state  $S$  can be created among these agents by a single  $n$ -tuple of signals sent to them by an outside agent.

## Example

$$1 : p \leftarrow A \longrightarrow 2 : p \leftarrow B \longrightarrow 3 : \neg p$$

Here there are three worlds. Agent A knows  $p$  in 1 and 2. Agent B knows  $p$  in 1 but not in 2 or 3. The actual world, say is 2. Then agent B does not know  $p$  but A does not know that B does not know  $p$ .

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We can achieve this by sending a pair of signals to both A and B. Each signal contains a description of the Kripke structure. And as a second part of each signal, A is told that he real world is one of 1 and 2 and B is told that the real world is one of 2 and 3.

Of course tigers do not understand Kripke structures, so some other method must be used for them!! Anyway they cannot achieve arbitrary states of knowledge.

It is probably impossible to convince tiger A that tiger B does not know that tiger C knows that there is a hunter with a gun behind the rock. Jim Corbett might disagree.



## The El Farol problem

This problem, also known as the Santa Fe Bar problem, is due to W. Brian Arthur.

The problem is as follows: There is a particular, finite population of people. Every Thursday night, all of these people want to go to the El Farol Bar. However, the El Farol is quite small, and it's no fun to go there if it's too crowded. So much so, in fact, that the preferences of the population can be described as follows:

- ▶ If less than 60% of the population go to the bar, they'll all have a better time than if they stayed at home.
- ▶ If more than 60% of the population go to the bar, they'll all have a worse time than if they stayed at home.

Unfortunately, it is necessary for everyone to decide at the same time whether they will go to the bar or not. They cannot wait and see how many others go on a particular Thursday before deciding to go themselves on that Thursday.

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This problem is clearly a version of the Russell paradox if we just assume that there is only one person involved and 'crowded' means, there is at least one person in the bar.

# Theory of Mind

A group of children are told the following story:

*Maxi goes out shopping with his mother and when they come back, Maxi helps mother put away the groceries, which include chocolate. There are two cupboards, red and blue. Maxi puts the chocolate in the red cupboard and goes out to play.*

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*While Maxi is gone, mother takes the chocolate out of the red cupboard, uses some of it to bake a cake, and then puts the rest in the blue cupboard.*

*Now Maxi comes back from play and wants the chocolate*

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Children at the age of five or more say, *In the red cupboard.*

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Children at the age of five or more say, *In the red cupboard.*

But children up to the age of three or four say, *Maxi will look in the blue cupboard.*

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Children at the age of five or more say, *In the red cupboard.*

But children up to the age of three or four say, *Maxi will look in the blue cupboard.*

What three year old children lack, according to psychologists Premack and Woodruff is a **Theory of Mind**

# Animal Cognition

Do animals have a Theory of Mind?



BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB



Food 1



Food 2



## What chimps think about other chimps

In the last slide, the chimp at the bottom is subservient to the dominant chimp at the top and has to decide which group of bananas to go for.

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In the last slide, the chimp at the bottom is subservient to the dominant chimp at the top and has to decide which group of bananas to go for. In experiments, the sub-chimp tends to go for Food 1 which the dom-chimp cannot see. Is there use of epistemic logic by the sub-chimp? This is an issue of some controversy

The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate “given” resources—if “given” is taken to mean given to a single mind which deliberately solves the problem set by these “data.” It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know.

**F. Hayek, 1945**



# Applications of Epistemic Reasoning to Society

## Learning from Communication

**Observation** (Lewis, Aumann): Suppose a group of people are commonly aware of a number of possibilities (states) among which they are uncertain. They commonly know some fact  $\psi$  if  $\psi$  is true of all these possibilities.

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What are the consequences for the candidate of this fact?

## Brief survey of 2008 US election

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Important political liability for Obama: His association with **Jeremiah Wright**, a fiery black preacher who had made anti-America comments.

**Our concern in this talk will be to study the way in which communication is used to change voter views**

## An illustrative example

- ▶ Hillary Clinton (while campaigning in Indiana):

*D* = As a child, I shot a duck.

- ▶ Why would she say that?

- ▶ Indiana is a conservative state. So most of her immediate audience  $V_1$  will be conservatives.
- ▶ Conservatives tend to **dis**favor gun control.
- ▶ Hearing  $D$  is likely to improve HC in the eyes of  $V_1$ 
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- ▶ Hearing  $D$  is likely to improve HC in the eyes of  $V_1$ 
  - say by amount  $u_1$ .
- ▶ But (virtually) all statements a candidate makes are **public announcements**.
- ▶ So another group of voters  $V_2$  (say liberals in Massachusetts) also hear HC say  $D$ .
- ▶ This is likely to make her go down for  $V_2$ 
  - say by amount  $u_2$ .
- ▶ But we likely have  $|u_1| > |u_2|$  since
  - i)  $|V_1| > |V_2|$ , or at least  $V_1$  cares more passionately about the issue than  $V_2$ .
  - ii)  $D$  merely **implicates** that HC will not impose gun control.



## Towards a formal model: languages and theories

- ▶ We begin by considering a single candidate  $C$ .
- ▶  $C$ 's views about the issues are formulated in a proposition language  $\mathcal{L}$  containing **finitely many** atomic propositions  $At = \{P_1, \dots, P_n\}$ .
- ▶ For instance:
  - ▶  $P_1 =$  We should withdraw from Iraq.
  - ▶  $P_2 =$  I will impose no new taxes.
  - ▶ ...
  - ▶  $P_n =$  We should bail out the banks.
- ▶  $T_a = C$ 's **actual theory** (i.e. the entirety of her views)
- ▶  $T_c = C$ 's **current theory** (i.e. what's she's said thus far)
- ▶ Typically (but not always)  $T_c \subseteq T_a$ .

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Let us call that set  $X_c$ . Then  $X_c = \{w | w \models T_c\}$

$X_c$  is the set of those worlds which are compatible with what the candidate has said.

## Worlds and preferences

- ▶ We conflate propositional valuations and worlds  $w \in 2^{At}$ .
- ▶ We also define  $w[i] = \begin{cases} 1 & \text{if } w \models P_i \\ -1 & \text{if } w \not\models P_i \end{cases}$
- ▶ We initially consider a single group of voters  $V$  (think of this as a constituency).
- ▶ The voters in  $V$  are characterized by their preference for an **ideal world**.
- ▶ This is formalized via two functions  $p_v, x_v$ :
  - ▶  $p_v(i) = \begin{cases} 1 & V \text{ would prefer } P_i \text{ to be true} \\ 0 & V \text{ is neutral about } P_i \\ -1 & V \text{ would prefer } P_i \text{ to be false} \end{cases}$
  - ▶  $x_v : At \rightarrow [0, 1]$  the **weight** which  $V$  assigns to  $P_i$  s.t.  $\sum w_v(i) \leq 1$ .

## Utilities of worlds and theories

- ▶ The utility of a world for  $V$  is defined as

$$u(w) = \sum_{1 \leq i \leq n} p_v(i) \cdot x_v(i) \cdot w[i]$$

- ▶ Note that a candidate's current theory  $T_c$  is likely to be **incomplete** – i.e. she may not express a view on some  $P_j$ .
- ▶ To calculate the utility of an arbitrary  $T$  we need to know how  $V$  will “fill in the blanks.”
- ▶ That is, extend the evaluation from a single world to a **set** of worlds.

## Voter types

- ▶ We postulate that there are three types of voters:
  - ▶ **Optimistic voters** (assume the best about  $C$  given  $T_c$ )
  - ▶ **Pessimistic voters** (assume the worst about  $C$  given  $T_c$ )
  - ▶ **Expected value voters** (average across possibilities compatible with  $T_c$ ).
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How will the different kinds of voters evaluate the candidate's theory  $T$ ?

## Voter types

- ▶ optimistic voters:  $ut^o(T) = \max\{u(w) : w \models T\}$
- ▶ pessimistic voters:  $ut^p(T) = \min\{u(w) : w \models T\}$
- ▶ expected value voters:  $ut^e(T) = \frac{\sum_{w \models T} u(w)}{|\{w : w \models T\}|}$



## Voter types

- ▶ optimistic voters:  $ut^o(X) = \max\{u(w) : w \in X\}$
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Above, if we assume the probability  $p$  to be uniform, then  $p(w)$  will be just  $\frac{1}{|X|}$

Note that with  $ut^e$  we will have a convexity property. If  $X, Y$  are disjoint, then  $ut^e(X \cup Y)$  will be in the closed interval whose endpoints are  $ut^e(X)$  and  $ut^e(Y)$

## The value of a message

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- ▶ Suppose  $T$  is the logical closure  $C$  of  $T_c$ .
- ▶ What's the best thing for her to say next?
- ▶ Roughly:  $val(A, T) = ut(T \circ A) - ut(T)$
- ▶  $T \circ A$  is what  $T$  becomes after  $A$  is added,
- ▶ and  $val(A, T)$  is the value of uttering  $A$  when her current theory (as seen by voters) is  $T$ .

- ▶ But the precise definition will depend on
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  - ▶ the set from which  $A$  is selected
- ▶ Wrt the latter, consider  $A$  from
  - ▶  $\mathcal{X}_a = T_a$  (i.e. only “true convictions”)
  - ▶  $\mathcal{X}_t = \mathcal{L} - \{\neg A : T_a \vdash A\}$  (i.e. anything consistent with “true convictions” = **tactical**)
  - ▶  $\mathcal{X}_m = \mathcal{L} - \{\neg A : T_c \vdash A\}$  (i.e. anything consistent with the current theory = **Machiavellian**)
  - ▶  $\mathcal{X}_\ell = \mathcal{L}$  (i.e. any sentence in the language, allowing for contradictions and lying)
- ▶ Note:  $\mathcal{X}_a \subseteq \mathcal{X}_t \subseteq \mathcal{X}_m \subseteq \mathcal{X}_\ell$

## The value of a message (cont.)

- ▶ If we have  $\mathcal{X} = \mathcal{X}_\ell$  then  $T_c$  may become **inconsistent**.
- ▶ In this case,  $\circ = *$  (i.e. an AGM-like update operation).
- ▶ In the other cases,  $\circ = \dot{+}$   
addition of  $A$  followed by logical closure.
- ▶ If  $\mathcal{X} = \mathcal{X}_a, \mathcal{X}_t$  or  $\mathcal{X}_m$ , then we let

$$val(A, T) = ut(T \dot{+} A) - ut(T)$$

where  $ut$  is one of  $ut^o$ ,  $ut^p$  or  $ut^e$ .

- ▶ We can now define **best statements** for  $C$  given  $T$  from  $\mathcal{X}$  as follows:

$$best(T, \mathcal{X}) = argmax_A val(A, T) : A \in \mathcal{X}$$



# Complex statements

## Proposition (1)

Assume **e**-voters. For all  $A, B$  s.t.  $A, B, A \wedge B \in \mathcal{X}_m$ , (i.e.,  $A, B, A \wedge B$  consistent with  $T_c$ ) there exist  $a, \dots, f \in [0, 1]$  s.t.

- 1)  $a \cdot \text{val}(A, T) + b \cdot \text{val}(\neg A, T) = 0$
- 2)  $\text{val}(A \wedge B, T) = \text{val}(A, T) + \text{val}(B, T \dot{+} A) = \text{val}(B, T) + \text{val}(A, T \dot{+} B)$
- 3)  $c \cdot \text{val}(A \vee B) + d \cdot \text{val}(A \wedge B, T) = e \cdot \text{val}(A, T) + f \cdot \text{val}(B, T)$

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Proof: For 1),  $ut(T) = a \cdot ut(T + A) + (1 - a) \cdot ut(T + \neg A)$

where  $a = \frac{|\{w \mid w \models T \dot{+} A\}|}{|\{w \mid w \models T\}|}$ .

## Moving to complete theories

### Corollary

There is a **complete**  $T \supseteq T_c$  s.t.  $ut^e(T) \geq ut^e(T_c)$ .

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Proof: From the above above, we must have exactly one of

- i)  $val(P_i, T) = val(\neg P_i, T) = 0$
- ii)  $val(P_i, T) > 0$  and  $val(\neg P_i, T) < 0$
- iii)  $val(P_i, T) < 0$  and  $val(\neg P_i, T) > 0$

Suppose  $Q_i, \dots, Q_k$  ( $k \leq n$ ) are all the atoms not in  $T_c$ .

Let  $T_0 = T_c$  and  $T_{i+1} = \begin{cases} T_i \cup Q_i & val(Q_i, T_i) \geq 0 \\ T_i \cup \neg Q_i & \text{else} \end{cases}$

Let  $T = Cn(T_k)$ .

## Moving to complete theories (cont.)

### Corollary

*One of the best extensions of  $T_c$  is a complete theory  $T \supseteq T_c$*

## Moving to complete theories (cont.)

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Proof:

- ▶ Suppose  $T'$  is a best extension of  $T_c$  and  $T'$  is incomplete.
- ▶ By the previous corollary, there is  $T'' \supseteq T'$  which is a complete extension of  $T'$  (and thus of  $T_c$ ) such that  $ut^e(T'') \geq ut^e(T')$ .
- ▶  $T''$  is complete and among the best extensions.

## Moving to complete theories (cont.)

- ▶ The previous result suggests that if  $C$  assumes **e**-voters, then it will never be to  $C$ 's disadvantage to move towards a complete theory.
- ▶ This will also be the case if the voters are either **e**-voters or pessimistic voters.
- ▶ But why then do we have the *Onion* phenomenon?
- ▶ I.e. why do candidates state vacuities like “God bless America” or “9/11 was a tragedy.”

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- ▶ This will also be the case if the voters are either **e**-voters or pessimistic voters.
- ▶ But why then do we have the *Onion* phenomenon?
- ▶ I.e. why do candidates state vacuities like “God bless America” or “9/11 was a tragedy.”
- ▶ Conjecture: They must be assuming that there are at least some **o**-voters (who ‘always assume the best’).
- ▶  $T \supseteq T' \implies \max\{u(w) \mid w \models T'\} \leq \max\{u(w) \mid w \models T\}$
- ▶ I.e.  $T \supseteq T' \implies ut^o(T') \leq ut^o(T)$



## Knowledge leads to action

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But we do not acquire knowledge, or transmit it to others just to pass the time.

Knowledge means influence and power.

It is a commonplace that what we do depends on what we know. And given that most of us have at least the rudiments of a *theory of mind* (cf. Premack and Woodruff) we also know that what others do will depend on what *they* know.

## Is Knowledge always beneficial?

Kamien, Tauman and Zamir consider the following example. A black or white card is chosen from a deck and player 1 is invited to guess its color. After 1 makes her choice, which is announced, player 2 is invited to make a choice. The payoffs are as follows:

- ▶ If both players guess correctly, then both get 2.
- ▶ If neither player guesses correctly, then both get 0.
- ▶ If only one player guesses correctly, then the correct player gets 5 and the other player gets 0.

- ▶ Suppose neither player knows the color then player 1 should choose randomly, player 2 should choose a different color and the expected payoff for both is 2.5 (half of  $5+0$ )
- ▶ If player 1 is allowed to see the card, then the dominant strategy for her is to announce the correct color, player 2 should choose the same color and the expected payoff for both (a certain payoff in fact) is 2.

So the knowledge of player 1 makes her worse off.

However, player 1 is not harmed by the fact that she knows the color but by the fact that player 2 knows that 1 knows the color. Neyman shows that if we can make one player know more but prevent other players from having more knowledge, then the player who knows more cannot lose.

We now consider how the same game may be played differently depending on the information available to the agents and their temperaments.



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If there is a **knowledge manipulator - KM** who can control how much information the various agents can have then that agent can influence the way the game is played and the outcome.

## Wife and husband

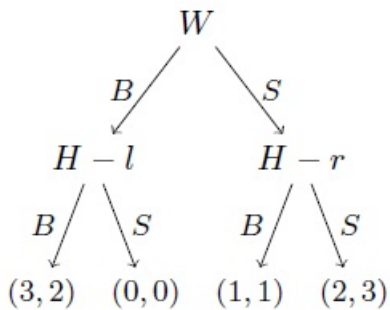


Fig. 1

In the last figure we assume that the wife moves first and the husband after.

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We consider various scenarios involving the husband's knowledge and temperament. We assume that the wife knows the husband's payoffs and temperament and he does not know hers.

Case 1) Husband does not know wife's move (and she knows this).  
a) He is **aggressive**. Then being aggressive, he will choose  $S$  (Stravinsky) for his move since the highest possible payoff is 3. Anticipating his move, she will also choose  $S$ , and they will end up with payoffs of  $(2,3)$ .

b) If the husband is **conservative**, then not knowing what his wife chose, he will choose  $B$  since the minimum payoff of 1 is better than the minimum payoff of 0. Anticipating this, the wife will also choose  $B$  and they will end up with  $(3,2)$ .

2) Finally if the husband **will** know what node he will be at, then the wife will choose  $B$ , the husband will also choose  $B$  and they will end up at  $(3,2)$ .

We consider now the question of how KM can create these various knowledge scenarios of the last example.

KM is capable of creating all these three situations by means of signals, as well as the one we did not mention where the husband does not know but the wife does not know that he will not.

For case 1a),  $s(H - l) = (l, a)$  and  $s(H - r) = (r, a)$ . The wife knows (if she did not already) which node they are at, but the husband will not.



For case 2,  $s(H - l) = (l, l)$  and  $s(H - r) = (r, r)$ . Both will know which node they are at.

Finally if KM wants the wife to be in doubt whether the husband knows, he could make  $s(H - l) = \{(l, l), (l, a)\}$  and  $s(H - r) = \{(r, r), (r, a)\}$ . Then if the wife chose left and receives an  $l$ , she will not know if the husband got an  $l$  or the neutral  $a$ . If KM does send  $(l, l)$  then the husband will know, but will also know that his wife did not know whether he *would* know.

# Conclusions

When we are programming people, the task is much more complex than it is with computers.

- ▶ People have their own motivations.
- ▶ Information which people need in order to act properly must be made available to them and sometimes, some information may need to be hidden.
- ▶ When different people have opposing motives, then conflicts can arise.

Nonetheless, issues arise in **Social Software** which are similar to the issues which arise in programming.

- ▶ We need to be clear about the desired post conditions of the procedure we propose.
- ▶ We need to make sure what are the preconditions needed for the procedure to work.
- ▶ Exchange of information, and preserving the proper order of actions must be attended to just as it is in Distributed Computing.

Some of my recent papers can be downloaded from the site

<http://cuny.academia.edu/RohitParikh>

See also

<http://www.sci.brooklyn.cuny.edu/cis/parikh/>