Turing and Wittgenstein

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Turing and Wittgenstein: Some Approaches

1. Turing a computational reductionist about mind.
2. Wittgenstein an anti-reductionist.

Wittgenstein essentially hostile to science and mathematical logic:

overly negative about idealization, ignorant, sloppy, insignificant, obsessed with style, dyslexic, propagandistic, social constructivist

(discussions on contradictions with Turing, 1939 Cambridge lectures)

My View

- Philosophy of mind was not the main issue between Wittgenstein and Turing; the foundations of mathematics and logic were. This has philosophical, and not only mathematical and/or scientific (engineering) significance.

- With respect to classical foundational arguments, Turing and Wittgenstein shared an attitude: we *prescind* from ideological positions; we show what logic *is* by focusing on what it is *for*; and we look to the user end, in particular, *what the user does*.

- The philosophical significance of this is easy to miss. *It looks like a nothing*.

- Yet they influenced one another.

- Their disputes were civil, not metaphysical or principled: how best to understand the idea of a "common sense" or "naïve" standpoint on mathematics, in particular Turing's 1936 analysis of the general notion of a formal system.

Turing to Moral Sciences Club, 1933:

The purely logistic view of mathematics is inadequate; mathematical propositions possess a variety of interpretations, of which the logistic is merely one.

Turing (1937): “On computable numbers, with an application to the *Entscheidungsproblem*”

Turing resolved Hilbert’s decision problem (1928):

Find a definite method by which every statement of mathematics expressed formally in an axiomatic system can be determined to be true or false based on the axioms.

(The method need not generate a proof; it had only to be always correct. Mainly a logical problem.)

Turing (1937): There can be no such method.

Davis (1982): A remarkable piece of applied philosophy.

Floyd (2012): *A language game*.

Turing had his mother send Wittgenstein an offprint of his paper in February 1937.

That summer, Alister Watson, who had introduced Wittgenstein to Turing, arranged a discussion group at Cambridge when Turing returned from Princeton.

That fall Wittgenstein tried to write up a discussion of Gödel's theorem and submitted the first version of the *Philosophical Investigations* to the Cambridge Press. Developed his later remarks on rule-following.

Gödel August 1963

The precise and unquestionably adequate definition of the general concept of formal system [made possible by Turing's work allows the incompleteness theorems to be] proved rigorously for every consistent formal system containing a certain amount of finitary number theory.

1. Turing gives an analysis of what a formal system *is*, by looking at what a formal system *does* or *is for*.

2. Turing's result is absolute. It does not depend upon which foundational system one espouses. Nor on any theory of mind. His analysis is not entangled in any particular formal system. It is not axiomatic in style.("Logic free", "formalism freeness"): but it answers to our conception of the logical.

Wittgenstein's 1939 Cambridge Lectures on the Foundations of Mathematics, ed. C. Diamond

Wittgenstein mentions incompleteness, and truth, and Turing doesn't respond. There is little discussion of philosophy of psychology proper. Focus instead is on the status of contradictions, and the notion of a paradigm (a calculation, as opposed to an experiment).

Turing 1944-45:
"The Reform of Mathematical Notation"

It is not difficult to put the theory of types into a form in which it can be used by the mathematician-in-the-street without having to study symbolic logic, much less use it. The statement of the type principle given below was suggested by lectures of Wittgenstein, but its shortcomings should not be laid at his door.

(p. 6, AMT/C12; highlighted by Gandy)

Turing 1944-45:
"The Reform of Mathematical Notation"

We should conduct an extensive examination of current mathematical, physical and engineering books and papers with a view toward listing all commonly used forms of notation and examine them to see what they really mean.

(p.2, AMT/C12)

Turing 1940 (44-5)

This will usually involve statements of various implicit understandings as between writer and reader. But the laying down of a code of minimum requirements for possible notations should be exceedingly mild, avoiding the straightjacket of a logical notation.

Wittgenstein, RPP I §1096 [Z §695](1947)

Turing's 'Machines'. These machines are humans who calculate. And one might express what he says also in the form of games. And the interesting games would be such as brought one via certain rules to nonsensical instructions (*unsinnigen Anweisungen)*. I am thinking of games like the “racing game”. One has received the order "Go on in the same way" when this makes no sense, say because one has got into a circle. For that order makes sense only in certain positions. (Watson.)

Wittgenstein on Turing (1950)

Wittgenstein to Malcolm 1950 on the Turing paper on "Computing Machinery and Intelligence":

I haven't read it but it is probably no leg pull.

The passage after: RPP I §1097

A variant of Cantor's diagonal proof:

Let N = F (K, *n*) be the form of the law for the development of decimal fractions (*Entwicklung von Dezimalbrüchen*).

N is the *n*th decimal place of the Kth development.

The diagonal law of N = F (*n*,*n*) = Def F'(*n*).

To prove that F’ (*n*) cannot be one of the rules F (K, *n*).

Assume it is the 100th. Then the formation rule of F' (1) runs F (1, 1), of F' (2) F (2, 2) etc.

But the rule for the formation of the 100th place of F' (*n*) will run F (100, 100); that is, it tells us only that the hundredth place is supposed to be equal to itself, and so for *n* = 100 it is not a rule. The rule of the game runs "Do the same as..."—and in the special case it becomes "Do the same as you are doing".

N = $F\,(K, n)$, the nth decimal place of the Kth development

n →

K ↓

	1	2	3	4	5	...
r_1	0	0	1	1	0	...
r_2	1	1	0	0	1	...
r_3	1	1	1	0	0	...
r_4	0	0	0	0	1	...
r_5	1	0	1	0	1	...
...						

$$F\,(n, n) = \mathrm{F}'\,(n)$$

Which is K (100,100)? 0 or 1?

n →

K ↓

	1	2	3	4	5	...
r_1	0	0	1	1	0	...
r_2	1	1	0	0	1	...
r_3	1	1	1	0	0	...
r_4	0	0	0	0	1	...
r_5	1	0	1	0	1	...
...						

$$F'\ (100) = K\,(100{,}100) = ??$$

The Positive Russell Paradox

$x_i \in x_j$?	1	2	3	4	...	
1	1	0	0	1	1	...
2	0	1	0	1	1	...
3	1	1	1	0	1	...
4	0	0	0	0	1	...
...						
						???

Take $S = \{x_i \mid x_i \in x_i\}$, the diagonal subset.

Is $S = x_j$ for some j?

$x_j \in x_j$ iff $x_j \in S$

Wittgenstein’s argument:

1. Is intensional, not extensional.
2. Is free of any tie to a particular formalism or picture or diagramming method.
3. Proves that there is a new rule (or command) that is not like the other rules on the list, in that it cannot be followed, because it is tautologous.
4. Depends upon our ability to *see* that a rule cannot be followed, but not upon the law of the excluded middle or negation or seeing a contradiction.
5. Depends upon an everyday idea about rules and following, but not upon community wide agreement. Yet it is a proof.

Turing 1937, §8

- Does not use the “Halting” Argument to show that the class of computable numbers is closed under diagonalization.
- Mentions a general way of applying Cantor’s diagonal argumentation, but expresses concern that this kind of proof may not convince the reader. (It may "leave the reader with a feeling that 'there must be something wrong’”). He then frames a different, more direct diagonal argument, “to give a certain insight into the significance” of the notion of a computable sequence. *This* is the argument to which Wittgenstein is alluding in 1947.

The Argument from the Pointerless Machine

Construct a “pointerless” machine, which reaches a command line that is tautologous or circular, of the form “do what you do” (“write what you write”).

This machine does not have negation “built into” it (is not a Contrary machine). Instead, it is told to do what it does.

Wittgenstein's 1947 is a revisiting of Turing's On Computable Numbers (1937)

A "variant" of Cantor's diagonal argument.

Reformulates Turing's Argument from the Pointerless Machine in terms of language games. The idea will be to establish the absence of uniformity in conceptual tools.

Turing (1954), p. 23:

These [limitative] results, and some other results of mathematical logic, may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of ‘reason’ unsupported by common sense.

- “Wittgenstein's Diagonal Argument: A Variation on Cantor and Turing”, in *Epistemology versus Ontology: Essays on the Philosophy and Foundations of Mathematics in Honour of Per Martin-Löf*, eds. Peter Dybjer, Sten Lindström, Erik Palmgren and Göran Sundholm (Springer Verlag, 2012), 25-44.

- "Turing, Wittgenstein and Types: Philosophical Aspects of Turing’s ‘The Reform of Mathematical Notation’ (1944-5)", in *Alan Turing – His Work and Impact*, eds. S. Barry Cooper and Jan van Leeuwen (Elsevier, 2012), 12-15.

- "Wittgenstein, Carnap and Turing: Contrasting Notions of Analysis", *Carnap's Ideal of Explication*, ed. Pierre Wagner (Palgrave Macmillan 2012), 34-46.