Dual Grid-Forming Converter

Federico Milano

University College Dublin

November 2025

Table of Contents

Research Questions

Power Balance

Is frequency the only metric to define power balance in power systems?

Synchronous Machine

Is the synchronous machine the “best” possible way to generate power?

Table of Contents

Frequency and Power Variations

This section defines the link between complex power and *complex frequency* in ac power systems.

Let us consider the power injection at network buses:

$$\bar{\boldsymbol{s}}(t) = \boldsymbol{p}(t) + \jmath \boldsymbol{q}(t) = \bar{\boldsymbol{v}}(t) \circ \bar{\boldsymbol{\imath}}^*(t)\,,$$

where voltages and currents are Park's vectors (or analytic signals), i.e., are valid in transient conditions:

$$\bar{\boldsymbol{v}}(t) = \boldsymbol{v}_{\mathrm{d}}(t) + \jmath \boldsymbol{v}_{\mathrm{q}}(t)\,.$$

Assumption

Let us assume that transmission line dynamics are fast, hence:

$$\bar{\boldsymbol{\imath}}(t) \approx \bar{\mathbf{Y}} \, \bar{\boldsymbol{v}}(t) \,,$$

where $\bar{\mathbf{Y}}$ is the conventional admittance matrix of the grid.

Hence the power injections into the grid nodes can be rewritten as:

$$\bar{\boldsymbol{s}}(t) = \bar{\boldsymbol{v}}(t) \circ [\bar{\mathbf{Y}} \, \bar{\boldsymbol{v}}(t)]^* \,.$$

Complex Frequency

Let us rewrite the Park vector of the voltage in polar coordinates:

$$\bar{v} = v \, e^{\jmath \theta} = e^{(u + \jmath \theta)}$$

where $u = \ln(v)$.

Then, the *complex frequency* is defined as follows:

$$\bar{\eta} = \frac{d}{dt}\big(u + \jmath \, \theta\big) = u' + \jmath \, \theta' = \rho + \jmath \, \omega \,,$$

It is possible to show that the complex frequency is a special case of *geometric frequency*.

Link of the Complex Frequency with the Current

From the previous definition, the following identity holds:

$$\bar{\boldsymbol{v}}' = \frac{d}{dt}\bar{\boldsymbol{v}} = \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\eta}}\,.$$

Then, from $\bar{\boldsymbol{\imath}} \approx \bar{\mathbf{Y}}\,\bar{\boldsymbol{v}}$, one obtains:

$$\bar{\boldsymbol{\imath}}' = \bar{\mathbf{Y}}\,\bar{\boldsymbol{v}}' = \bar{\mathbf{Y}}\,(\bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\eta}}) = \bar{\mathbf{Y}}\,\mathrm{diag}(\bar{\boldsymbol{v}})\,\bar{\boldsymbol{\eta}} = \bar{\mathbf{I}}\,\bar{\boldsymbol{\eta}}\,.$$

Link of the Complex Frequency with the Complex Power

Then taking the conjugate and multiplying by the voltage

$$\bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}'^{*} = \bar{\mathbf{S}}\, \bar{\boldsymbol{\eta}}^{*} \,.$$

Where $\bar{\mathbf{S}}$ is a matrix whose elements are the complex power flow in the branches of the grid.

Rate of Change of Power (RoCoP) [grid side]

And finally, we note that:

$$\begin{aligned}\bar{\boldsymbol{s}}' &= \frac{d}{dt}(\bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}^*) \\ &= \bar{\boldsymbol{v}}' \circ \bar{\boldsymbol{\imath}}^* + \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}'^* \\ &= \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\eta}} \circ \bar{\boldsymbol{\imath}}^* + \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}'^* \\ &= \bar{\boldsymbol{s}} \circ \bar{\boldsymbol{\eta}} + \bar{\boldsymbol{v}} \circ \bar{\boldsymbol{\imath}}'^*\end{aligned}$$

So we obtain the expression:

$$\boxed{\bar{\boldsymbol{s}}' - \bar{\boldsymbol{s}} \circ \bar{\boldsymbol{\eta}} = \bar{\mathrm{S}}\, \bar{\boldsymbol{\eta}}^*}$$

We need now an expression for $\bar{\boldsymbol{s}}'$ from the device side . . .

Alternative Expression of the RoCoP

Note that, in general, the complex frequency of the voltage is not equal to the complex frequency of the current, hence:

$$\bar{v}' = \bar{\eta}_v \bar{v}$$

$$\bar{\imath}' = \bar{\eta}_\imath \bar{\imath}$$

Then, one obtains:

$$p' = (\rho_v + \rho_\imath)p - (\omega_v - \omega_\imath)q$$

and

$$q' = (\omega_v - \omega_\imath)p - (\rho_v + \rho_\imath)q$$

System Model

Let consider the conventional DAE model for transient stability analysis:

$$\begin{aligned} \boldsymbol{z}' &= \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{y}) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{z}, \boldsymbol{y}) \end{aligned}$$

Under usual assumptions, we can write:

$$\begin{aligned} \boldsymbol{y}' = \frac{\partial \boldsymbol{\phi}}{\partial \boldsymbol{z}} \boldsymbol{z}' &= \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}} \right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \boldsymbol{z}' \\ &= \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}} \right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{\phi}(\boldsymbol{z})) \, . \end{aligned}$$

Rate of Change of Power (RoCoP) [device side]

In the conventional DAE model of power systems, voltages and powers are algebraic variables.

Let assume we can write the expression of the power injections of each device connected to the grid as:

$$\bar{\boldsymbol{s}}' = \bar{\boldsymbol{s}}'(\bar{\boldsymbol{v}}, \boldsymbol{z}, \boldsymbol{y})$$

Then, the time derivatives of $\bar{\boldsymbol{s}}'$ can be written as:

$$\bar{\boldsymbol{s}}' = \frac{\partial \bar{\boldsymbol{s}}}{\partial \bar{\boldsymbol{v}}}\, \bar{\boldsymbol{v}}' + \left[\frac{\partial \bar{\boldsymbol{s}}}{\partial \boldsymbol{z}} + \frac{\partial \bar{\boldsymbol{s}}}{\partial \boldsymbol{y}} \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}} \right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \right] \boldsymbol{z}'$$

where we already know that $\bar{\boldsymbol{v}}' = (\rho + \jmath\omega) \circ \bar{\boldsymbol{v}} = \bar{\boldsymbol{\eta}} \circ \bar{\boldsymbol{v}}$, hence:

$$\bar{\boldsymbol{s}}' = \frac{\partial \bar{\boldsymbol{s}}}{\partial \bar{\boldsymbol{v}}}\, \bar{\boldsymbol{\eta}} \circ \bar{\boldsymbol{v}} + \left[\frac{\partial \bar{\boldsymbol{s}}}{\partial \boldsymbol{z}} + \frac{\partial \bar{\boldsymbol{s}}}{\partial \boldsymbol{y}} \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}} \right)^{-1} \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \right] \boldsymbol{z}'$$

Component of the RoCoP

From the definition of complex frequency we can define the following components of the RoCoP:

$$\bar{\boldsymbol{s}}'_1 = \jmath \bar{\boldsymbol{s}} \circ \boldsymbol{\omega} - \jmath \bar{\mathrm{S}} \, \boldsymbol{\omega} \, ,$$
$$\bar{\boldsymbol{s}}'_2 = \bar{\boldsymbol{s}} \circ \boldsymbol{\varrho} + \bar{\mathrm{S}} \, \boldsymbol{\varrho} \, .$$

where

$$\bar{\boldsymbol{s}}' = \bar{\boldsymbol{s}}'_1 + \bar{\boldsymbol{s}}'_2 \, .$$

Special Cases: Constant Power Injection

The constraint is $\bar{s} = \mathrm{const.}$

Then, we obtain:

$$\bar{s}' = 0 \quad \Rightarrow \quad \bar{s}_1' = -\bar{s}_2'$$

This is a quite interesting result as it indicates that, during a transient, a constant power device (even a constant power load) affects the frequency at a bus if the voltage magnitude changes, and *vice versa*!

Special Cases: Constant Admittance

The constraint is $\bar{\imath} = \bar{Y}_o \bar{v}$

Then (after some tedious algebra), we obtain:

$$\bar{s}'_1 = 0 \quad \text{and} \quad \bar{s}' = \bar{s}'_2$$

This is another interesting result as it indicates that a constant admittance cannot impact the frequency. It only impacts the voltage magnitude.

Special Cases: Constant Current and Power Factor

The constraint is $|\bar{\imath}| = \text{const.}$ and $\phi = \text{const.}$

Then (after some tedious algebra), we obtain:

$$\bar{s}_2' = 0 \quad \text{and} \quad \bar{s}' = \bar{s}_1'$$

Yet another interesting result. This tells us that a constant current device cannot impact the voltage. It only impacts the frequency.

Approximated Expressions

Then, one can define some approximated expressions:

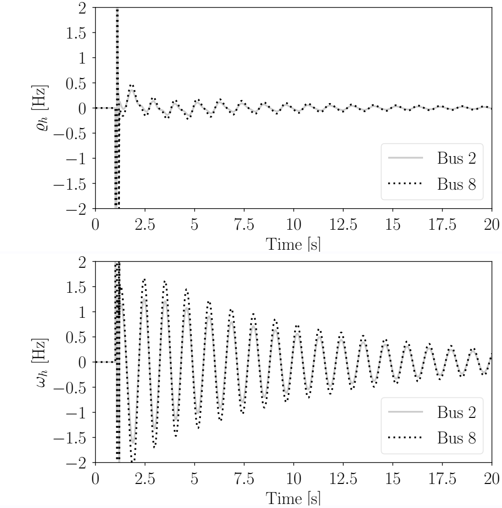
$$\boldsymbol{p}_1' \approx \mathbf{B}_1 \boldsymbol{\omega} \,,$$
$$\boldsymbol{q}_1' \approx \mathbf{G}_1 \boldsymbol{\omega} \,,$$

and

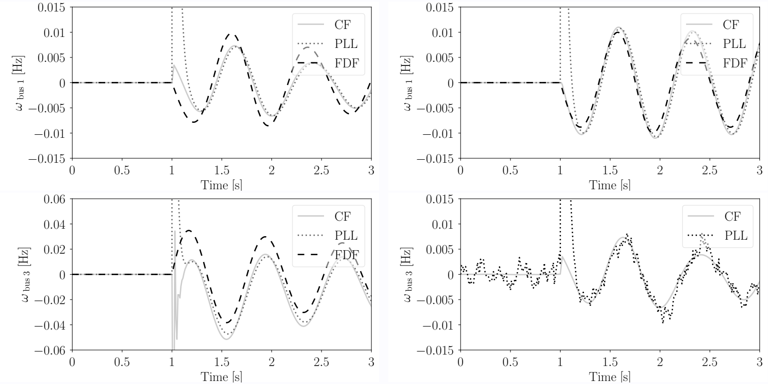
$$\boldsymbol{p}_2' \approx \mathbf{G}_2 \boldsymbol{\varrho} \,,$$
$$\boldsymbol{q}_2' \approx \mathbf{B}_2 \boldsymbol{\varrho} \,,$$

where $\mathbf{B}_1$, $\mathbf{G}_1$, $\mathbf{B}_2$ and $\mathbf{G}_2$ are approximated susceptance and conductance matrices obtained from $\bar{\mathbf{Y}}$.

Example: ρ and ω

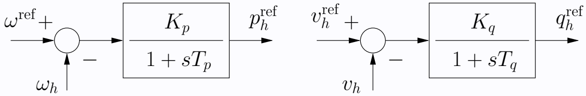


Example: Synchronous Machine and DER

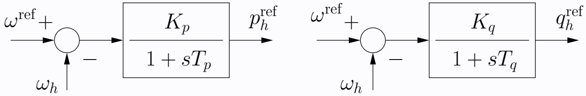


Example: Control of DERs – I

Control 1 (conventional)



Control 2 (based on complex frequency findings)



Example: Control of DERs – II

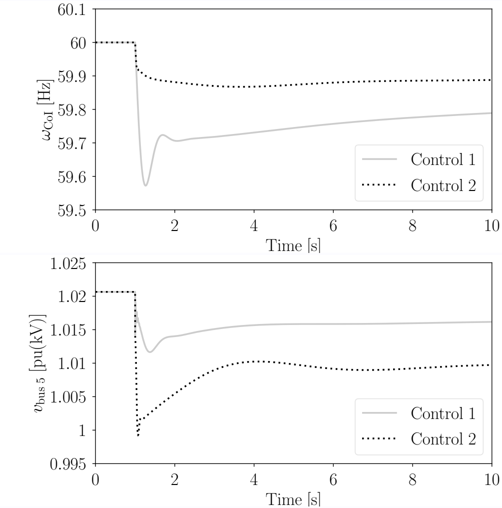


Table of Contents

Dual Grid-Forming Converter

We now consider a *dual* model for grid-forming (GFM) controlled converters.

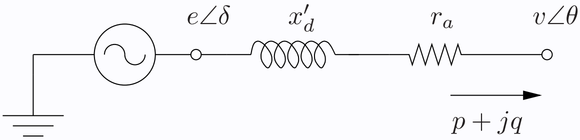
The model is inspired from the observation that the structures of the active and reactive power equations of lossy synchronous machine models are almost symmetrical in terms of armature resistance and transient reactance.

Conventional Synchronous Machine – I

Consider the power injections of the lossy electromechanical model of the synchronous machine shown in the figure below:

$$p = \frac{[ev\cos(\delta-\theta) - v^2]r_a + [ev\sin(\delta-\theta)]x'_d}{r_a^2 + {x'_d}^2}\,,$$

$$q = \frac{[ev\cos(\delta-\theta) - v^2]x'_d - [ev\sin(\delta-\theta)]r_a}{r_a^2 + {x'_d}^2}\,.$$



Conventional Synchronous Machine – II

In synchronous machines, the armature resistance r_a is small with respect to x'_d and is often neglected, thus leading to the well-known equations:

$$
\begin{aligned}
p &= \frac{ev \sin(\delta - \theta)}{x'_d} \, , \\
q &= \frac{ev \cos(\delta - \theta) - v^2}{x'_d} \, .
\end{aligned}
$$

Dual Model – I

Consider now the *dual* parts of the machine equations, that is, the terms that depend on the armature resistance and suppose that the reactance x'_d is zero or negligible:

$$\tilde{p} = \frac{ev\cos(\delta - \theta) - v^2}{r_a},$$

$$\tilde{q} = -\frac{ev\sin(\delta - \theta)}{r_a}.$$

Dual Model – II

For simplicity, assume that the virtual parameter that represents the armature resistance is negative, say $K = -1/r_a$, thus leading to:

$$\begin{aligned} \tilde{p} &= Kv^2 - Kev\cos(\delta - \theta)\,, \\ \tilde{q} &= Kev\sin(\delta - \theta)\,. \end{aligned}$$

For this hyptotheical device, the active power strongly depends on the magnitude of the internal emf e, while the reactive power strongly depends on the phase angle δ.

Swing Equations

Recall that the conventional swing equation is defined in terms of the machine rotor angle:

$$\begin{aligned} \dot{\delta} &= \omega - \omega_o \,, \\ M\dot{\omega} &= p_m - p(e, v, \delta, \theta) - D(\omega - \omega_o) \,, \end{aligned} \tag{1}$$

Dual Swing Equations – I

To obtain the dual swing equation, consider the complex quantity:

$$\bar{e} = e \exp(j\,\delta)\,.$$

Define $u = \ln(e)$, $e \neq 0$, then the previous equation becomes:

$$\bar{e} = \exp(u + j\,\delta)\,,$$

and its time derivative is:

$$\dot{\bar{e}} = (\dot{u} + j\,\dot{\delta}) \exp(u + j\,\delta) = (\varrho + j\,\omega)\,\bar{e}\,,$$

where ω is defined as in (1) and ϱ is:

$$\varrho = \dot{u} = \dot{e}/e\,.$$

Dual Swing Equations – II

Finally, the swing equation dual to (1) is defined as:

$$\begin{aligned} \dot{u} &= \rho\,, \\ \tilde{M}\dot{\rho} &= p^{\mathrm{ref}} - \tilde{p}(u, v, \delta, \theta) - \tilde{D}\rho\,, \end{aligned}$$

or, equivalently

$$\begin{aligned} \dot{e} &= \varrho\, e\,, \\ \tilde{M}\dot{\varrho} &= p^{\mathrm{ref}} - \tilde{p}(e, v, \delta, \theta) - \tilde{D}\varrho\,, \end{aligned}$$

Primary Controllers

The simplest first order model for the turbine governor can be written as:

$$T_m \dot{p}_m = \frac{1}{R}(\omega^{\mathrm{ref}} - \omega) + p_{m,o} - p_m \,,$$

For a 3-rd order machine model, a basic automatic voltage control has the form:

$$T'_{d0} \dot{e} = v_f - (x_d - x'_d) i_d - e \,,$$
$$T_r \dot{v}_f = K_r (v^{\mathrm{ref}} - v) - v_f \,,$$

Dual Primary Controllers

In the same vein, the primary active power control for the dual-GFM control is required to track ϱ, for example:

$$\tilde{T}_m \dot{p}^{\mathrm{ref}} = \frac{1}{\tilde{R}}(\varrho^{\mathrm{ref}} - \varrho) + p_o^{\mathrm{ref}} - p^{\mathrm{ref}} \,,$$

The dual to the AVR can be written as:

$$\begin{aligned} T_q \dot{\delta} &= K_q(q^{\mathrm{ref}} - \tilde{q}) - \delta \,, \\ \tilde{T}_r \dot{q}^{\mathrm{ref}} &= \tilde{K}_r(\omega^{\mathrm{ref}} - \omega) - q^{\mathrm{ref}} \,, \end{aligned}$$

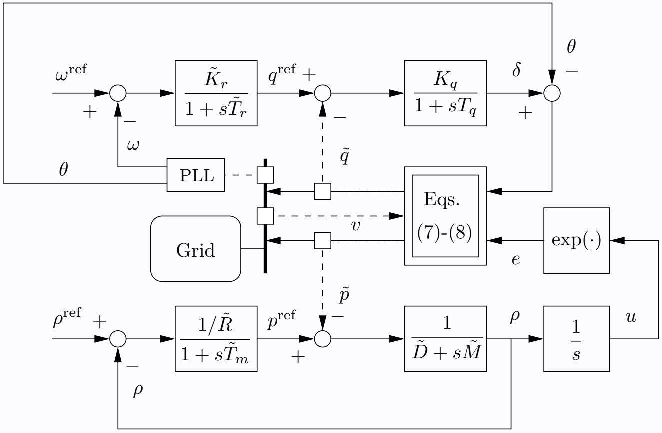
or alternatively, defining:

$$\delta_r = K_q \, q^{\mathrm{ref}} \,.$$

we obtain:

$$\begin{aligned} T_q \dot{\delta} &= \delta_r - K_q \, \tilde{q} - \delta \,, \\ \tilde{T}_r \dot{\delta}_r &= \tilde{K}_r'(\omega^{\mathrm{ref}} - \omega) - \delta_r \,, \end{aligned}$$

Complete Model of the Dual-GFM



Synchronism of Dual-GFMs

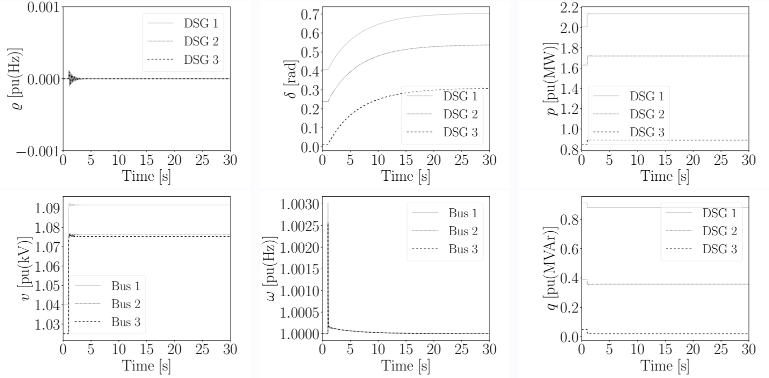
The virtual angular speed does not appear in the formulation of the dual-GFM control except for the time derivative of the internal signal δ in the reactive power control.

It is still necessary, of course, to fix the frequency in the system. The dual-GFM converter imposes the frequency at the bus through the regulation of the reactive power.

δ is relative to the phase angle θ of the voltage at the point of connection of the converter to the grid. Hence, to reach steady state, the rest of the grid must be synchronous at the rated frequency ω^{ref}.

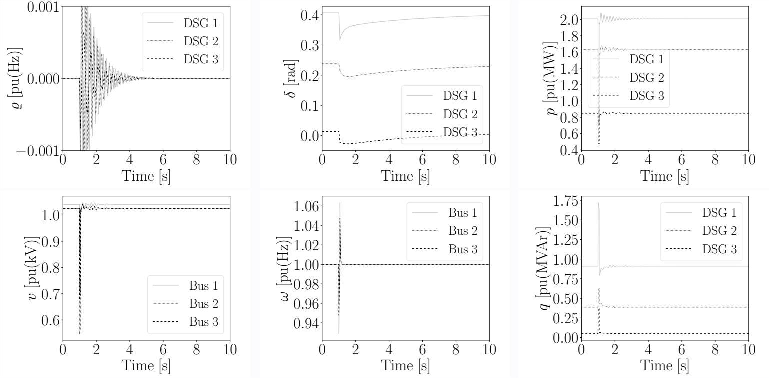
Example 1: WSCC 9-bus System – I

All generators are dual-GFMs. 20% of loss of load.



Example 1: WSCC 9-bus System – II

All generators are dual-GFMs. Short-circuit at but 7, cleared after 60 ms disconneting a line.



Example 2: Modified All-Island Irish System – I

We illustrate the dynamic performance of the proposed dual-GFM converter for a dynamic model of the all-island Irish transmission system.

The original system includes 1479 buses, 1851 transmission lines and transformers, 22 synchronous generators, along with their appropriate control systems, 169 wind power plants and 245 loads.

All wind power plants are assumed to be GFLs and not to provide any inertial response nor fast-frequency regulation.

We have substituted **all** synchronous machiens with dual-GFMs with same capacity.

Example 2: Modified All-Island Irish System – II

Outage of the largest infeed (connection with UK).

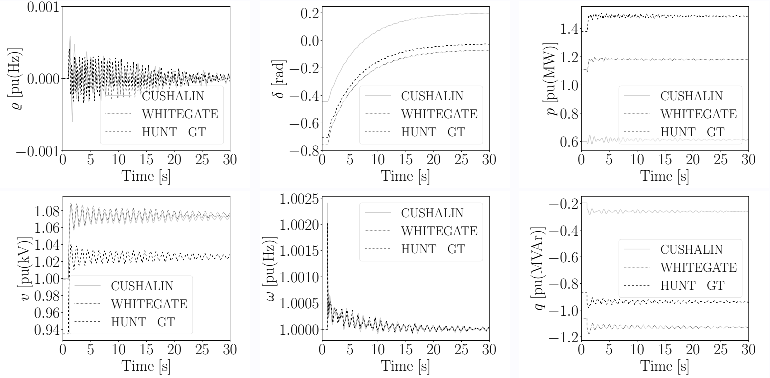


Table of Contents

Final Remarks

Robustness

The dual-GFM is particularly robust and stable following large contingencies.

Compatibility

The dual-GFM does not seem to work well combined with synchronous machines, but more tests are needed.

Universality?

The dual-GFM seems to be able to work both in AC and DC. Future work will explore this feature and the implementation of a prototype!

Table of Contents

References

- F. Milano, *Complex Frequency*, IEEE Transactions on Power Systems, vol. 37, no. 2, pp. 1230-1240, March 2022.
- F. Milano, *Dual Grid-Forming Converter*, IEEE Transactions on Power Systems, vol. 40, no. 2, pp. 1993-1996, March 2025. arXiv: 2408.13185

Further Reading – I

Works on applications of complex frequency.

- W. Zhong, G. Tzounas, F. Milano, *Improving the Power System Dynamic Response through a Combined Voltage-Frequency Control of Distributed Energy Resources*, IEEE Transactions on Power Systems, vol. 37, no. 6, pp. 4375-4384, Nov. 2022.
- D. Moutevelis, J. Roldán-Pérez, M. Prodanovic and F. Milano, *Taxonomy of Power Converter Control Schemes based on the Complex Frequency Concept*, in IEEE Transactions on Power Systems, preprint available. arXiv: 2209.11107
- R. Bernal, F. Milano, *A Complex Frequency-Based Control for Inverter-Based Resources*, Journal of Modern Power Systems and Clean Energy (MPCE), SGEPRI, accepted for publication in March 2025. arXiv: 2501.00448
- I. Ponce, F. Milano, *Local Synchronization of Power System Devices*. IEEE Transactions on Power Systems, vol. 40, no. 5, pp. 4194-4204, September 2025. arXiv: 2407.02661

Further Reading – II

Works on the generalization of complex frequency.

- F. Milano, *A Geometrical Interpretation of Frequency*, IEEE Transactions on Power Systems, vol. 37, no. 1, pp. 816-819, January 2022.
- F. Milano, *Equivalence between Geometric Frequency and Lagrange Derivative*, IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 72, no. 9, pp. 4800-4809, September 2025. arXiv: 2410.02340
- J. Gutiérrez Florensa, Á. Ortega, L. Sigrist, F. Milano, *Quasi Steady-State Frequency*, IEEE Transactions on Circuits and Systems I: Regular Papers, accepted for publication in September 2025. arXiv: 2505.21461

Thank you!