Online Reinforcement Learning in Large and Structured Environments

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Reinforcement learning

Reinforcement learning is concerned with learning to take actions to maximize rewards, by trial and error, in environments that can evolve in response to actions

- Traditional Search Goal: Find a policy with high total reward using as few interactions with the environment as possible

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- Traditional Search Goal: Find a policy with high total reward using as few interactions with the environment as possible
- Optimization Goal: Maximize total reward / Minimize regret (shortfall) in total reward compared to an optimal policy
  - **Applications:** Recommendation systems, Sequential investment, Dynamic resource allocation ....
  - No separate budget to purely exploring the environment
  - Exploration and Exploitation must be carefully balanced

Outline

Reinforcement learning algorithms for Regret Minimization in large and structured (unknown) environments

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- **Part 1:** Online learning in large scale **Multi-armed Bandits**
- **Part 2:** Online learning in large **Markov Decision Processes**

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Main Challenge: Generalizing learned knowledge across unseen states and actions

Part 1: Online Learning in large-scale Multi-armed Bandits[1]

[1] S. R. Chowdhury and A. Gopalan, "*On kernelized multi-armed bandits*", **ICML**, 2017.

Background: Multi-armed Bandit

1 **2** **3** ••• **N**

N arms with **unknown** parameters $\mu_1, \ldots, \mu_N$

(Think Bernoulli)

Background: Multi-armed Bandit

Time 1

1 $\boxed{\mathbf{2}}$ **3** ••• **N**

$y_1 \sim \mu_2$

Background: Multi-armed Bandit

Time 2

$\boxed{\mathbf{1}}$ $\mathbf{2}$ $\mathbf{3}$ ••• $\mathbf{N}$

$y_2 \sim \mu_1$

Background: Multi-armed Bandit

Time 3

$\mathbf{1} \quad \mathbf{2} \quad \boxed{\mathbf{3}} \quad \cdots \quad \mathbf{N}$

$$y_3 \sim \mu_3$$

Background: Multi-armed Bandit

Time 4

$$\mathbf{1} \qquad \boxed{\mathbf{2}} \qquad \mathbf{3} \qquad \bullet\bullet\bullet \qquad \mathbf{N}$$

$$y_4 \sim \mu_2$$

Background: Multi-armed Bandit

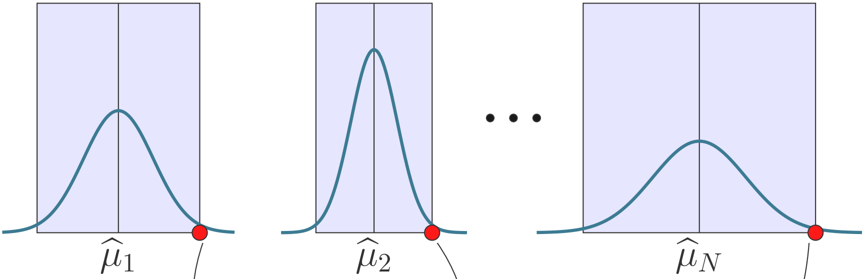
Time T

$\mathbf{1}$ $\mathbf{2}$ $\mathbf{3}$ ••• $\boxed{\mathbf{N}}$

$y_T \sim \mu_N$

Upper Confidence Bound algorithm [Auer et al.'02]

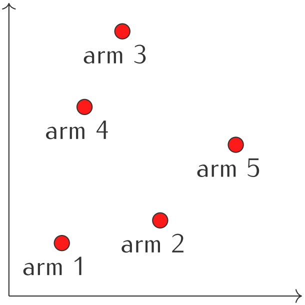
Idea: Be optimistic under uncertainty !



Play arm $i_t = \underset{1 \leqslant i \leqslant N}{\operatorname{argmax}} \left\{ \widehat{\mu}_i + \sqrt{\frac{2 \log t}{k_i}} \right\}$

Variation: Linear Bandits [Dani et al.'08, ...]

- Assumption in MAB: Arms' rewards are independent
- Often, more structure / coupling is present



Problem setting

- Each arm i is a vector $x_i \in \mathbb{R}^d$
- Playing arm i_t gives reward

$$y_t = \theta^\top x_{i_t} + \varepsilon_t$$

- $\theta \in \mathbb{R}^d$ is unknown and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ is noise

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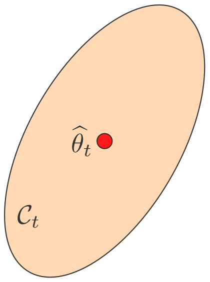
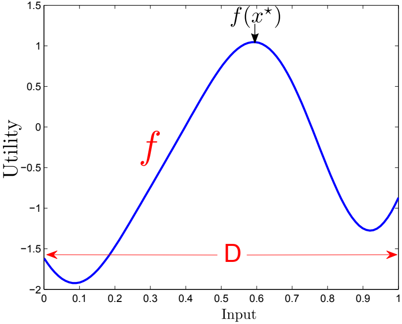


LinUCB algorithm

- Build a point estimate (least squares estimate) $\widehat{\theta}_t$ and a confidence region (ellipsoid) $\mathcal{C}_t$
- Play the most optimistic action w.r.t. this ellipsoid

$$i_t = \underset{1 \leqslant i \leqslant N}{\operatorname{argmax}} \max_{\theta \in \mathcal{C}_t} \left(\theta^\top x_i \right)$$

Generalization: Black-box optimization



Problem setting:

- Maximize an unknown utility function $f : D \to \mathbb{R}$
- Sequentially query f with points $x_1, x_2, \ldots, x_T$
- Noisy point evaluations: $y_t = f(x_t) + \varepsilon_t$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ is the noise

Goal: Maximize cumulative (expected) utility: $\sum_{t=1}^{T} \mathbb{E}[y_t]$

or, equivalently,

Minimize cumulative regret: $\sum_{t=1}^{T} \left(f(x^\star) - f(x_t) \right)$

Application: Hyperparameter tuning

Hyperparameters in DeepNN training

- Learning rate
- Regularizer
- Number of hidden layers
- Number of units in each layer
- Optimizer (SGD, Adagrad, Adam, ...)
- Nonlinearity (Relu, Softmax, ...)

DeepNN training as Black-box optimization

- D : all possible hyperparameter configurations
- $f(x)$: training error for configuration x
- $x^\star$: the best set of hyperparameters

Huge (possibly infinite) set of hyperparameters to choose from !!!

Possible approaches

Grid search, Random search

- Do not use information from previous searches
- Not good when point evaluations are expensive

Need to make an educated decision about where to search next

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Bayesian optimization

1. Learn a (probabilistic) model for f
2. Use model predictions and uncertainty to select query x_t
3. Update model with data (x_t, y_t) and repeat.

Regularity assumption

- f is an element of a reproducing kernel Hilbert space (RKHS) $\mathcal{H}_k$ associated with a kernel $k : D \times D \to \mathbb{R}$

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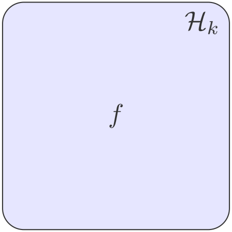

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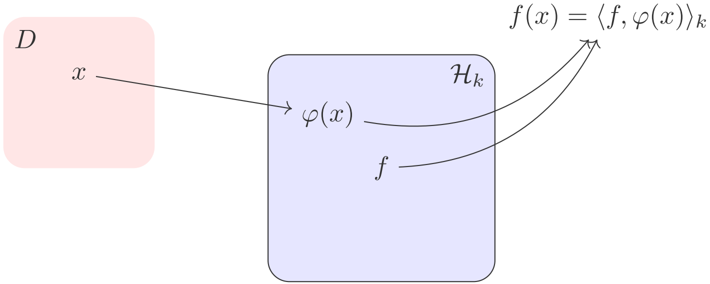

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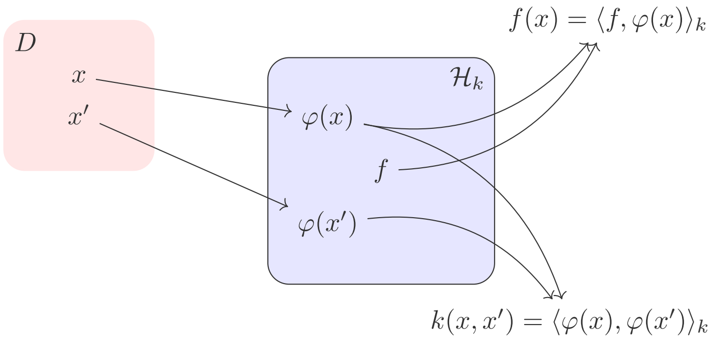

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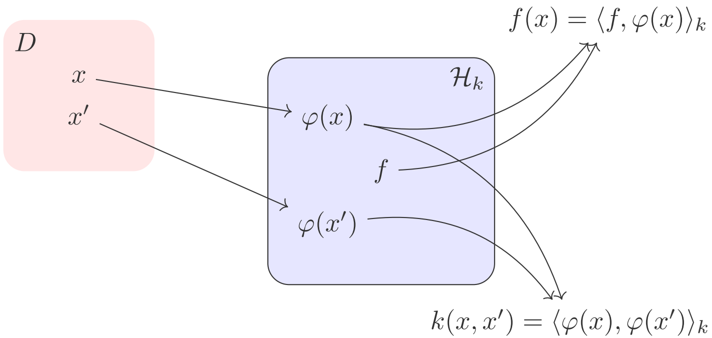
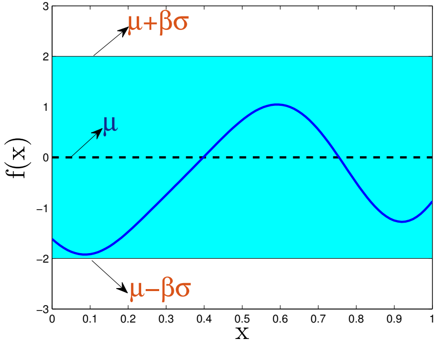


- Induces smoothness: $|f(x) - f(x')| \leqslant \|f\|_k \, \|\varphi(x) - \varphi(x')\|_k$

Algorithm Design

Idea: Gaussian Process (GP) regression



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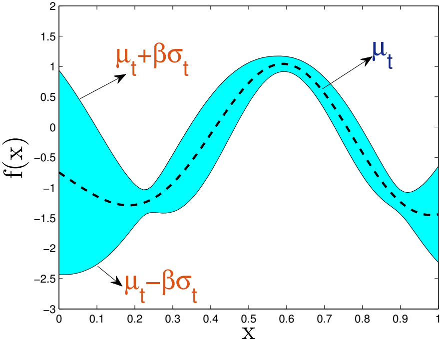


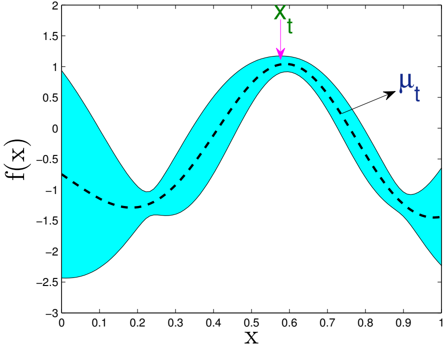
- Unknown utility function f modeled by Gaussian processes, $f \sim \mathcal{GP}\left(0, k(x, x')\right)$
- Posterior of f given t noisy observations $\mathcal{H}_t = (x_\tau, y_\tau)_{\tau=1}^t$ is a GP

$$f|\mathcal{H}_t \sim \mathcal{GP}(\mu_t(x), k_t(x, x'))$$

Posterior mean and covariance:

$$\begin{aligned} \mu_t(x) &= k_t(x)^\top (K_t + \sigma^2 I)^{-1} y_{1:t} \\ k_t(x, x') &= k(x, x') - k_t(x)^\top (K_t + \sigma^2 I)^{-1} k_t(x') \end{aligned}$$

Key Idea: Choose the point with highest Upper Confidence Bound



At each round t, choose the query point x_t using current GP posterior and a suitable parameter β_t:

$$x_t \in \operatorname*{argmax}_{x \in D} \mu_t(x) + \beta_t \sigma_t(x)$$

Algorithm 1: Improved GP-UCB

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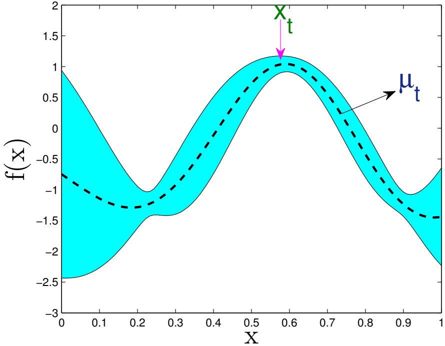

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First appeared as **GP-UCB** [Srinivas et al.'10] $\longrightarrow$ We provide a tighter regret bound ($O(\log T)$ improvement!)

IGP-UCB achieves sublinear regret [CG'17]

Regret bound: $O\left(\gamma_T\sqrt{T\log\left(\frac{1}{\delta}\right)}\right)$ with probability at least $1-\delta$

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"Information Complexity": Captures reduction in uncertainty after observing noisy rewards (depends on the kernel function)

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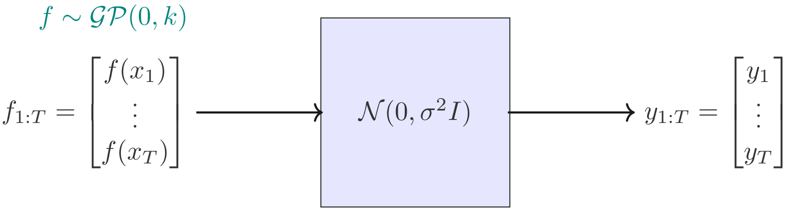

$$\gamma_T = \max_{\{x_1, \ldots x_T\}} \mathtt{I}(y_{1:T}, f_{1:T})$$

Algorithm 2: Gaussian Process Thompson Sampling

What would a "Bayesian" do?

- Sample a random function and choose its maximizer
- Prehistoric [Thompson'33]

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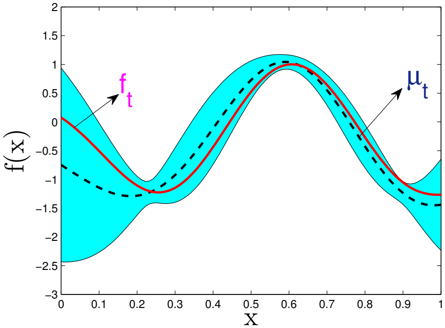

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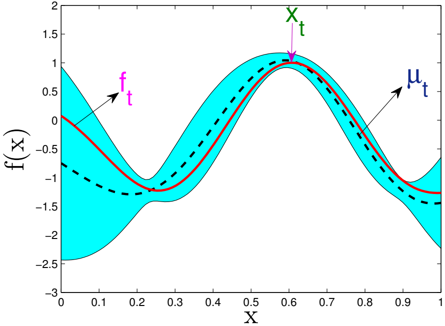

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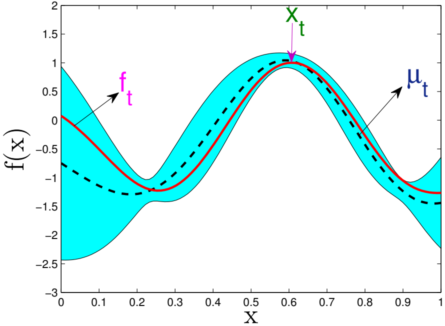

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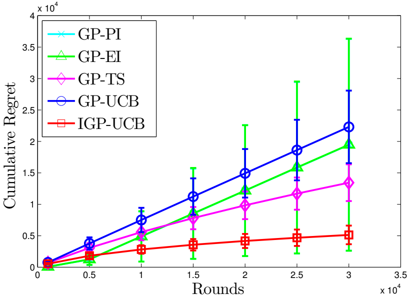
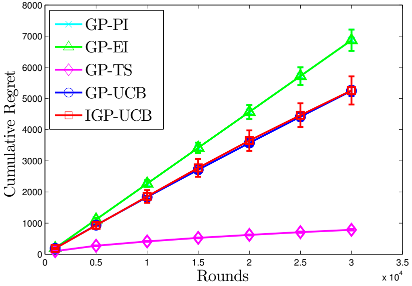
Numerics

f sampled from RKHS (RBF kernel)



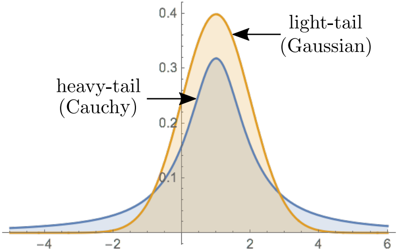
- IGP-UCB improves over GP-UCB ☺☺
- GP-TS fares well ☺

Temperature Sensor Data (Intel Berkeley Research lab)



- IGP-UCB performs similar to GP-UCB ✓
- GP-TS performs the best ☺

Variation: Heavy-tailed kernelized bandits[2]



eg. Student's-t, Pareto, Cauchy etc.

Motivation:

- Distribution of delays in communication networks
- Bursty traffic flow distributions
- Price fluctuations in financial and insurance data

- Heavy-tailed payoffs: $\mathbb{E}\left[|y_t|^2\right] < +\infty$
- **Results:** Regret upper and lower bounds of order $\approx \gamma_T \sqrt{T}$

Bounded second-moment sufficient for $O(\sqrt{T})$ regret!

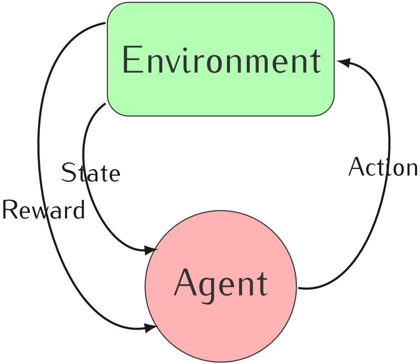
[2] S. R. Chowdhury and A. Gopalan, "*Bayesian Optimization under Heavy-tailed Payoffs*", **NeurIPS**, 2019.

Part 2: Online Learning in Large-scale Markov Decision Processes[3]

[3] S. R. Chowdhury, A. Gopalan and O.-A. Maillard, "*Reinforcement Learning in Parametric MDPs with Exponential Families*", **AISTATS**, 2021.

Markov Decision Process

We consider learning in an episodic MDP $\{\mathcal{S}, \mathcal{A}, R, P, H\}$



- State space $\mathcal{S} \subset \mathbb{R}^m$
- Action space $\mathcal{A} \subset \mathbb{R}^n$
- Transition probabilities $P : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$
- Reward function $R : \mathcal{S} \times \mathcal{A} \to [0, 1]$
- Finite episode length $H \in \mathbb{N}$

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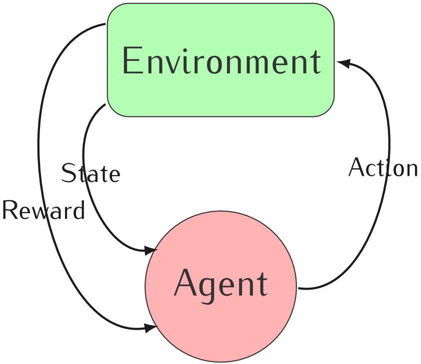

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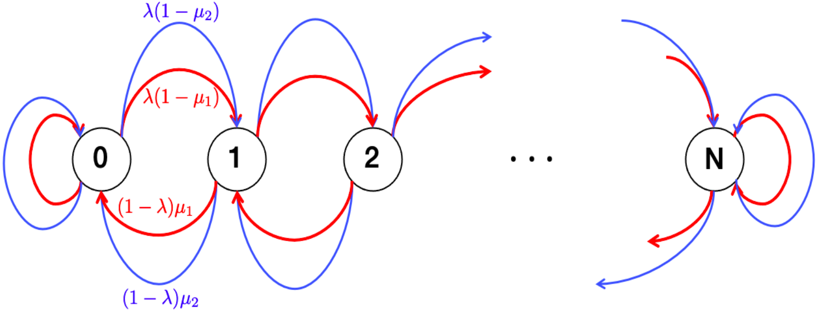
Parameterized MDP – Queuing System

Transitions are parameterized by θ – unknown to the agent a priori



- Single queue with N states, discrete-time
- `Bernoulli`(λ) arrivals at every state
- 2 actions: (`Bernoulli`) service rates $\{\mu_1, \mu_2\}$
- Assume service rates known, uncertainty in λ only

Online Reinforcement Learning

- MDP parameter θ is unknown to the decision maker a priori
- Must LEARN optimal policy – what action to take in each state to maximize the cumulative reward

$$\mathbb{E}\left[\sum_{t=1}^{T}\sum_{h=1}^{H} R(s_{t,h}, a_{t,h})\right]$$

or, equivalently, minimize the cumulative regret

$$\mathbb{E}\left[\sum_{t=1}^{T}\sum_{h=1}^{H} R(s_{t,h}, a^{\star}_{t,h})\right] - \mathbb{E}\left[\sum_{t=1}^{T}\sum_{h=1}^{H} R(s_{t,h}, a_{t,h})\right] \; ,$$

- **Trade-off:** Explore the state space or Exploit existing knowledge to design good current policy?

Online reinforcement learning

- **Upper confidence-based approaches:** Build confidence intervals per state-action pair, be optimistic!
  - Rmax [Brafman-Tennenholtz'01]
  - UCRL2 [Jaksch et al.'07]
- **Key idea:** Maintain estimates + high-confidence sets for transition probabilities for every state-action pair
- "Wasteful" if transitions have **structure/relations**

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- "Wasteful" if transitions have **structure/relations**

Parameterized MDP: Transitions are described by a finite dimensional parameter – unknown to the agent a priori

Example: Linear quadratic regulator

<u>Berstekas'04:</u>

State transitions: $s' = As + Ba + \eta$

- $A \in \mathbb{R}^{m\times m}$ and $B \in \mathbb{R}^{m\times n}$ are unknown matrices
- Noise $\eta \sim \mathcal{N}(0, \sigma^2\mathbf{I})$ with known variance

Rewards: $R(s,a) = s^\top Ps + a^\top Qa$

- $P \in \mathbb{R}^{m\times m}$ and $Q \in \mathbb{R}^{n\times n}$ are known matrices

Generic models: Linear MDPs

Yang and Wang'19: $P(s'|s,a) = \psi(s')^\top M \varphi(s,a)$

Jin et al.'19: $P(s'|s,a) = \theta^\top \nu(s', s, a)$

- Known feature functions: $\psi(s')$, $\varphi(s,a)$ and $\nu(s',s,a)$
- Unknown parameters: M (a matrix) and θ (a vector)
- **Special case:** Tabular finite-state, finite-action MDP

DO NOT cover several popular models:

- Linearly controlled systems [Bertsekas' 04]
- Factored MDPs [Kearns and Koller' 99]

Our Model: Linear Exponential Family [CGM'21]

log-transition probabilities are linear:

$$P_\theta(s'|s,a) = \exp\left(\theta^\top F(s',s,a) - Z_{s,a}(\theta)\right)$$

- $\theta \in \mathbb{R}^d$ is the unknown parameter of the model
- $F(s',s,a)$ are known features (sufficient statistic)
- $Z_{s,a}(\theta)$ is the log-partition function

(Captures a **wide range of distributions**, e.g., Gaussian, Bernoulli, Gamma, Chi-square, ...)

Algorithm: Exponential family UCRL (Exp-UCRL)

Key idea: Maintain estimates + high-confidence sets for θ

At each episode t:

1. Compute the penalized maximum-likelihood estimate (MLE) $\widehat{\theta}_t$ of θ from data $\{s_{\tau,h}, a_{\tau,h}, s_{\tau,h+1}\}_{\tau<t,h\leq H}$:

$$\widehat{\theta}_t \in \operatorname*{argmin}_{\theta\in\mathbb{R}^d} \sum_{\tau<t,h\leq H} -\log P_\theta(s_{\tau,h+1}|s_{\tau,h}, a_{\tau,h}) + \frac{1}{2}\|\theta\|^2$$

2. Build a confidence set around the MLE:

$$\mathcal{C}_t = \left\{\theta \mid \sum_{\tau<t,h\leq H} \mathtt{KL}\Big(P_{\widehat{\theta}_t}(\cdot|s_{\tau,h}, a_{\tau,h}), P_\theta(\cdot|s_{\tau,h}, a_{\tau,h})\Big) + \frac{1}{2}\left\|\widehat{\theta}_t - \theta\right\|^2 \leq \beta_t\right\}$$

3. Compute the optimal policy w.r.t. $\mathcal{C}_t$ (by value iteration, simulation,...) and **play actions prescribed by that policy**

Exp-UCRL attains sublinear regret [CGM'21]

Concentration inequality: $\mathbb{P}\big[\forall t \in \mathbb{N},\ \theta \in \mathcal{C}_t\big] \geq 1 - \delta$

- A **general** result to design high-confidence sets for adaptive regression in conditional exponential families
- Generalize known results for linear bandits [Abbasi-Yadkori et al.'11] and GLM bandits [Filippi et al.'10]

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- A **general** result to design high-confidence sets for adaptive regression in conditional exponential families
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Regret Bound: $O\left(\frac{\beta}{\sqrt{\alpha}} H^2 d \sqrt{T \log(1/\delta)}\right)$ with probability $\geq 1-\delta$

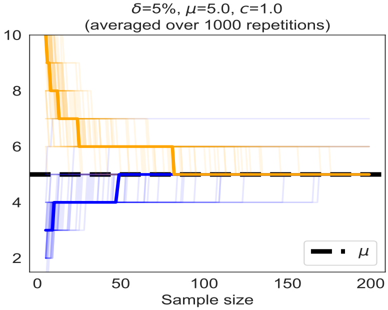
$$\alpha = \inf_{\theta,s,a} \lambda_{\min}\left(\mathcal{C}_\theta[\psi(s')|s,a]\right)$$

$$\beta = \sup_{\theta,s,a} \lambda_{\max}\left(\mathcal{C}_\theta[\psi(s')|s,a]\right)$$

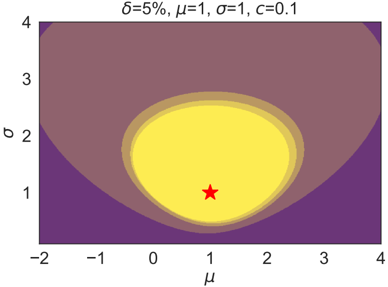
Model dependent constants — encode degree of **non-linearity**

Confidence Sets in Exponential Families [CSGM'22]

A general toolkit:[4] High probability confidence sets in exponential families — applications well beyond bandits and RL



Confidence sets for Chi-square distribution



Joint confidence sets for (μ, σ) for Gaussian distribution

[4] S. R. Chowdhury, P. Saux, A. Gopalan, O.-A. Maillard "*Bregman Deviations of Generic Exponential Families*", **arxiv**, 2022.

Variation: Privacy Concerns in RL

Reinforcement learning (RL) widely applied to personalized service:

- personalized healthcare
- virtual assistants
- social robots
- online recommendations
- ...

However, both states and rewards contain user's sensitive information

- healthcare: age, gender, treatment history
- virtual assistants: words, voice, sentences.
- social robots: facial expressions and scores on puzzles
- online recommendations: shopping habits

How to protect all these information in a rigorous way?

Differential privacy

Central Model:

- The learning agent has access to users' personal data
- Privacy protection: adversary cannot infer any particular user's data by observing the outputs the agent

Local Model:

- Each user protects her data at the local side
- The learning agent only has private data from users

Can we have a unified framework for both models in RL?

Privacy in Tabular MDPs

Private information:

- number of visits to a state-action pair $n(s,a)$ – bits $(0/1)$
- number of transitions to a given state $n(s'|s,a)$ – bits $(0/1)$
- rewards $r(s,a)$ – scalars $[0,1]$

The goal: release private counts with minimal amount of noise

Protection in local model:

- each user k, add independent noise, leads to sum of K noise

Protection in central model:

- Binary counting mechanism, only $\log K$ noise [Chan et al.'11]

Private Algorithms and Regret

Private Policy Optimization:[5]

- Central Model: $\widetilde{O}\left(\sqrt{S^2AH^3T} + S^2AH^3/\varepsilon\right)$
- Local Model: $\widetilde{O}\left(\sqrt{S^2AH^3T} + S^2A\sqrt{H^5T}/\varepsilon\right)$

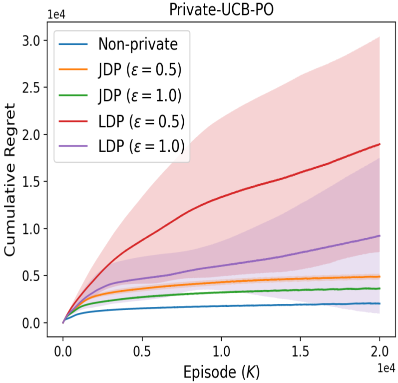
Private Value Iteration:

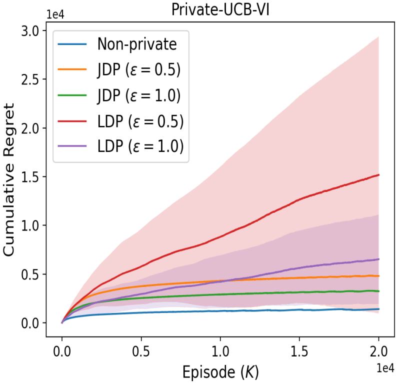
- Central Model: $\widetilde{O}\left(\sqrt{SAH^3T} + S^2AH^3/\varepsilon\right)$
- Local Model: $\widetilde{O}\left(\sqrt{SAH^3T} + S^2A\sqrt{H^5T}/\varepsilon\right)$

Regret increases if level of privacy increases (i.e., ε decreases)

[5] S. R. Chowdhury and X. Zhou, "*Differentially Private Regret Minimization in Episodic Markov Decision Processes*", **AAAI**, 2022.

Numerics [CZ'22]





Current/Future Work

Model selection in bandits and MDPs:

- Function class (in bandits) / class of transition distributions (in MDPs) unknown?
- How to identify the correct model class during learning?

Ethics/Privacy questions:

- How to protect users' sensitive information (e.g. recommendation systems, personalized treatment)?
- Fairness and transparency in policy selection?

Multi-agent systems:

- Decentralized peer-to-peer learning/ Federated learning?
- Impact of **network properties**, **delay in communication**, etc.?

Thank You