Improving the Nulling Beamformer with Subspace Suppression

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Introduction

Source localization in Magnetoencephalography (MEG) is a difficult ill-posed problem where the appropriate method depends on the nature of the analysis. Studies often involve analysis of a sparse set of Regions of Interest (ROIs) over the cortex, such as in (Vaina 2010). A simple solution involves computing the minimum norm estimate (MNE)(Hämäläinen 1994) and averaging over all vertex sources in each ROI. Due to the large point spread function, the regions should be far apart to avoid cross talk. The nulling beamformer overcomes the cross talk problem by "nulling" the gain produced by interfering source locations (Hui 2010). Each ROI's lead field matrix needs to be reduced in rank in order to prevent overconstraining the problem. Hui et al. perform a truncated Singular Value Decomposition (TSVD), but this will not necessarily produce matrices with minimal subspace angles, especially if two ROIs are close neighbors. We propose a new method, Subspace Suppression (SS), where the number of singular values along with a tuning parameter allow for controlling crosstalk through suppressing singular vectors that project into the lead field matrices of interfering ROIs.

The Crosstalk Problem and choosing Regions of Interest

It is often the case where there is interest in investigating cortical areas that are in close proximity. However, the point spread function caused by the source localization solution may cause leakage of signal from one ROI to another. Thus, it is essential to try to eliminate these effects.
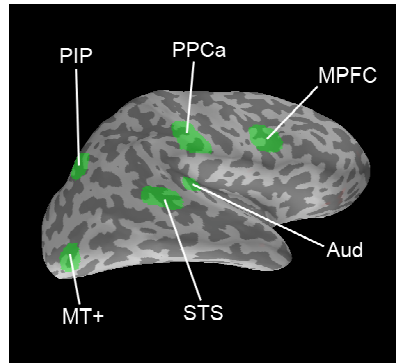

Figure 1

The Minimum Norm Estimate (MNE)

The minimum norm estimate (MNE) is a method of solving the source localization inverse problem. It provides a solution across all vertices over the decimated cortical surface. We derive the estimate by computing the inverse operator:

$$M = RG^T \left(GRG^T + I \right)^{-1}$$

where R is the data covariance matrix over the sensor space, G is the spatially whitened gain matrix, I is an identity matrix, and M is the inverse operator. Then, we apply the inverse operator to the whitened data through the following operation:

$$j(t) = Mx(t)$$

where x(t) is the spatially whitened data at time t and j(t) is the current sources at time t.

Nulling Beamformer

If we know the locations of activity apriori, we should be able to find a more accurate representation of our signal. The nulling beamformer is a method based on the Linearly Constrained Minimum Variance (LCMV) beamformer[5,6]. In the LCMV beamformer assumes that the source signal is generated from a small number N of sources on the cortex. In this case, the forward problem can be modeled as follows

$$m = \sum_{i=1}^{N} g(q_i)s(q_i)$$

Nulling Beamformer (continued)

The goal is to find a suitable inverse operator that maps m, the sensor measures, to s, the source estimates, that produces minimum variance in the source space. The nulling beamformer adds additional constraints that attempts to nullify the effects of all other confounding sources from the source location of interest. However, in order to avoid overconstraining the problem, the degrees of freedom from each ROI must be reduced. The nulling beamformer achieves this through the Truncated Singular Value Decomposition (TSVD).

$$G_p = U_L^p S_L^p V_L^{p^T}$$

where S_L is a diagonal matrix of L singular values and U_L and V_L include the L input and output singular vectors. With N ROIs, we construct the following matrices and weight vector:

$$H = [U_L^1 .. U_L^N]$$
$$B = [r_d^1 V_L^1 S_L^{1^{-1}} ... r_d^N V_L^N S_L^{N^{-1}}] \text{ where } r_d^p = 1_p \otimes I$$
$$w = R^{-1} H (H^T R^{-1} H)^{-1} B^T$$

where 1_p is an indicator vector of length N that is 1 only at element p and I is the identity matrix of size 3x3. Now we apply this weight vector to obtain the following source estimates for each ROI:

$$s(q_i) = w^T(q_i) m$$

TSVD and Subspace Suppression

The nulling beamformer runs into an interesting problem as ROIs become much closer, as is between STS and aud in Figure 1. If the truncation amount for the TSVD leaves very few singular values and ROIs are very close, it is likely that these most significant singular vectors overlap greatly. The TSVD will remove useful information that could be used to separate the signals between the ROIs. We fix this by finding the projections from a set of ROIs into the space of another ROI so that we can minimize them. We construct projections from one ROI (let's call this ROI 2) to another (let's call this ROI 1) as follows:

$$s_{2\rightarrow 1,1} = |u_{1,1}^T G_2 v_{1,1}|$$
$$\vdots$$
$$s_{2\rightarrow 1,n} = |u_{1,n}^T G_2 v_{1,n}|$$

We assemble this into a diagonal matrix:

$$S_{2\rightarrow 1} = \begin{pmatrix} s_{2\rightarrow 1,1} & 0 & \cdots & 0 \\ 0 & s_{2\rightarrow 1,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & s_{2\rightarrow 1,n} \end{pmatrix}$$

Next we perform a minimization:

$$S_{ss} = \max(S_{1,i} - \alpha \sum_{j=2}^{K} S_{j\rightarrow 1,i}, 0)$$

This quantity is our new diagonal singular value matrix that can be processed through the nulling beamformer method. Note that the TSVD will now extract the singular vectors that both well represent the signal in the ROI and, with tuning parameter alpha, have minimal projections in neighboring ROIs.

Results

We perform a simulation using the mne_simu function of the MNE toolbox. A 200 ms square pulse starting at 0 ms is generated in the aud label (see Figure 1) in the right hemisphere followed by another 200 ms square pulse generated in the STS label starting at 300 ms. This stimulus is repeated 300 times over 700 ms periods. In Figure 2, we show the results found for STS with the minimum norm estimate (MNE), the Nulling Beamformer, and the Nulling Beamformer with Subspace Suppression (SS). We find that, if a little extra energy from the neighboring area is tolerated, we can recover the signal that is completely suppressed with the nulling beamformer.

Figure 2
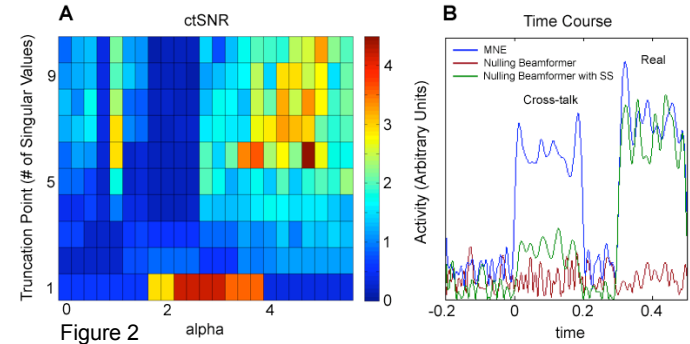
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