

Chapter 7. Quantum Description of Matter on the Smallest Scales

For reasons that will become clear in Chapter 11, in order to describe the universe on large scales, we need to understand it on the smallest scales as well. The quest to figure out the underlying structure of matter began in ancient times. In about 430 B.C., the Greek philosopher Democritus developed the concept of atoms, which he proposed to be tiny indivisible particles out of which all matter is composed. (The word “atom” derives from the Greek word “atomos,” which means “uncuttable.”) The idea was not universally accepted, however, until the 20th century.

During the Renaissance and the Enlightenment, scientists such as Robert Hooke, Robert Boyle, and Isaac Newton developed renewed interest in the possible existence of atoms. Boyle conducted an experiment in which he doubled the pressure of a sample of gas trapped in a sealed chamber and observed that this corresponded to cutting the volume in half. Hooke found that he could make sense of this if the force causing the pressure were the result of motions of individual atoms of gas contained in the chamber. The atomic theory was an example of the reductionistic approach that was becoming successful as a methodology for describing natural phenomena.

Atoms as a Model for Chemistry

From the late 18th through the 19th centuries, chemists sharpened the distinction between chemical reactions and simple mixing of substances. They accurately measured the weights of the substances before the reactions and of the products of the reactions. This allowed them to distinguish between elements — substances of uniform properties — and compounds, mixtures of materials that could be broken up. Chemical reactions could be explained in terms of atomic theory, but for a long time the theory was thought of as a useful abstract model that did not correspond to reality. There was no direct evidence for the existence of atoms, since they could not be resolved by the best microscopes of that period.

The measurements of the chemists led to determinations of the relative weights of the atoms of the different elements. One of these chemists, John Dalton, studied the reactions between two types of atoms, such as hydrogen and oxygen to make water, and termed the products “compound atoms,” later dubbed molecules. These are the smallest units of substances that are not pure elements. Dalton considered (correctly) that molecules are composed of two or more atoms, bound together. However, many chemical reactions were complex and difficult to interpret, which we now know to be the consequence of many molecules being composed of considerably more than two atoms.

In 1869, Dmitri Mendeleev made sense out of what had been a confusing collection of observed similarities in the chemical properties of different elements. For example, the element chlorine reacts readily with hydrogen, sodium, and potassium, as do the elements fluorine and bromine. Mendeleev organized the elements into a periodic table (presented in Appendix B of this book). Elements along any column of the table have chemical properties that are very similar to each other. For example, the elements in the rightmost column — which was missing from Mendeleev’s periodic table — do not react at all. These are the “noble gases” such as helium, neon, and argon, not discovered until the 1890’s. There were also holes in Mendeleev’s diagram when he drew it up, but these were filled as more elements were discovered. The periodic table was one of the crowning achievements of chemistry, although the explanation for it took more than half a century.

Atoms as a Useful Model in Physics

The physicists of the 19th century considered atomic theory from a different perspective, mostly related to the connection among motion, heat, and pressure. In 1842, James Joule found that the heat produced by a rotating paddle wheel (a miniature version of the type used in steamboats) depends directly on the energy expended in making the wheel turn. In 1847, Rudolf Clausius realized that, at least for gases, this made sense if the temperature is a measure of the kinetic energy (see Ch. 4) of the individual atoms or molecules. In other words, temperature is simply a measure of the thermal energy, which is the average microscopic kinetic energy of the sub-microscopic constituents of a substance (cf. Figure 7-1).

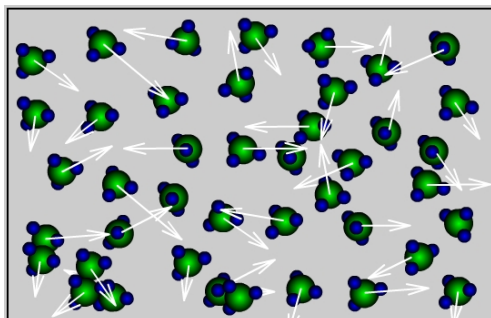


Figure 7-1. Molecules in random, "Brownian" motion, each with different kinetic energy, the average of which is given by eq. 7-1. The arrows represent the velocities of the molecules. The thermal energy, which is directly proportional to the temperature, is the average kinetic energy of the molecules, individual atoms, and individual free (*i.e.*, not bound inside an atom) particles present in the substance.

For a gas in which the constituents collide frequently, a formula expresses the average thermal energy of a particle (an atom, ion, electron, or molecule):

$$E_{\text{thermal}} = \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{3}{2} kT \quad (7-1)$$

E_{thermal} = thermal energy (in J), the angled brackets mean the average of the expression inside, m = mass (in kg) of the individual molecules, atoms, or particles in the substance, v = velocity of the individual molecules, *etc.*, T = temperature (in K), and k = Boltzmann's constant = 1.38×10^{-23} J/K.

The concept of conservation of energy (see Ch. 4) became clearer after the association was made between heat and energy. Energy that is apparently lost from a system, as measured by its total potential and kinetic energy, is converted to thermal energy. So, if you drop a piece of clay onto the ground, you observe that it does not bounce despite considerable initial potential energy and, just before it hit, considerable kinetic energy. This is explained by the fact that it became hotter after striking the ground. In fact, if a measurement of the temperature of everything involved were made initially and immediately after impact, the increase in thermal energy (of the clay and its surroundings) would be found to equal the potential energy just before the clay was dropped.

Because of the success of the atomic theory, by the end of the 19th century most chemists and physicists were convinced that atoms exist. Any doubt was removed when Albert Einstein developed a detailed molecular theory of microscopic (Brownian) motions in 1905. By that time, it was becoming clear that atoms were not indivisible particles, but had structure. We now know that atoms contain a tiny nucleus with protons and neutrons surrounded by "clouds" of electrons. Chapter 8 explores the structure of the nucleus. The remainder of this chapter is more concerned with the electrons and their relationship to light.

Light Emitted and Absorbed by Atoms: Spectral Lines

As mentioned in Chapter 5, opaque objects emit a continuous spectrum of light. In contrast, the light from hot, transparent gas appears in very narrow bands of wavelength, so thin that they are called spectral lines. In 1802, astronomer William Wollaston examined sunlight in detail with a high-quality prism and found that, rather than the completely continuous spectrum the eye perceives in a rainbow, the Sun's spectrum

(Fig. 7-2) actually contains many very thin, *dark* lines superimposed on the continuous spectrum. Ten years later, the wavelengths of more than 600 of these lines were measured by Joseph Fraunhofer.

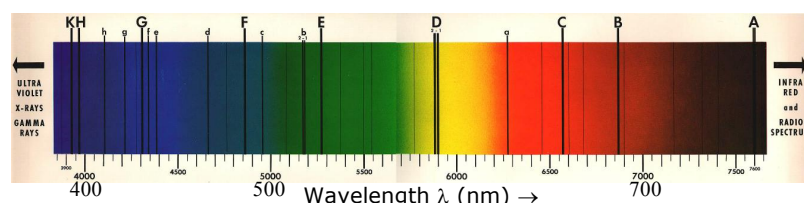


Figure 7-2. The visible spectrum of the Sun. It is filled with dark ("Fraunhofer") lines superposed on the continuous blackbody spectrum.

[Source: hesperia.gsfc.nasa.gov]

Later in that century, physicists found that a low-density, hot gas (in a glass tube) emits light with a spectrum of *bright* lines, with no continuous spectrum at all. They were able to study single isolated elements in this way, determining their emission-line spectra. **Each chemical element has a distinctive pattern of colored emission lines** (see Fig. 7-3), a sort of "fingerprint" of the element. In the 1880's, Johann Balmer found that the spectrum of hydrogen, known by then to be the lightest and probably simplest element, follows a pattern in which the lines are more closely spaced at shorter wavelengths (i.e., from red to violet; see Fig. 7-3). The atomic physicists of the early 20th century realized that any successful model of the atom must explain this pattern of wavelengths.

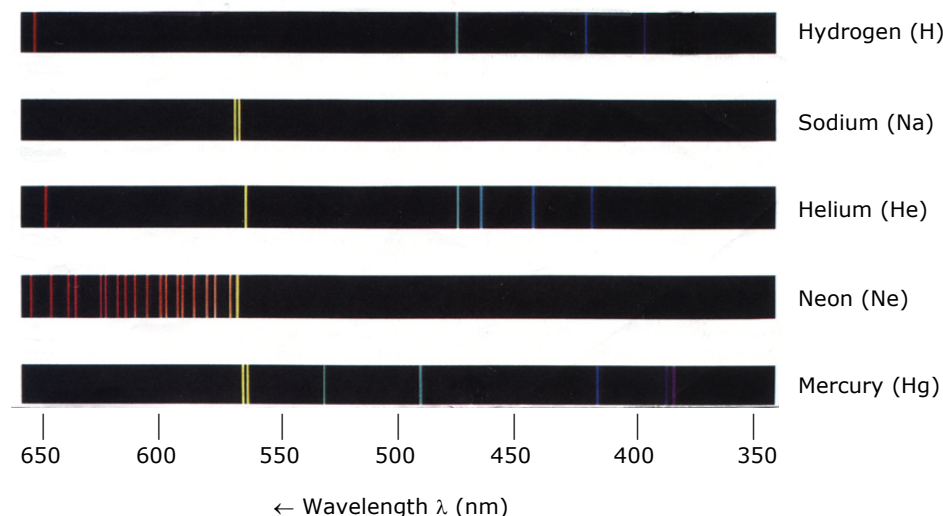


Figure 7-3. Emission-line spectra of five elements. [Note: the eye sees wavelengths between 380 and 430 nm to be violet, not the dark blue of this reproduction.] Note that wavelength decreases to the right, opposite to Fig. 7-2; there is no standard convention.

The Bohr Model of the Hydrogen Atom

In 1912, Niels Bohr, a young Danish theorist, endeavored to figure out how atoms generate emission lines. The best guess at the time was that negatively charged electrons orbit the positively charged nucleus. The energies involved in the orbits would naturally depend on the strength of the attractive electric force between each electron and the nucleus, which in turn would depend on the number of protons and hence the atomic number of the element. There were two other major clues:

1. Electrons orbiting the nucleus should lose energy by making light. This would cause them to spiral into the nucleus in only a millionth of a second. Something must prevent them from doing so.
2. The pattern of wavelengths of emission lines is always the same for each element. The visible-light spectrum of the hydrogen atom (see Fig. 7-3) is particularly simple, with 3 (or 4 for people whose eyes are sensitive to deep-violet light) lines that are closer together at shorter wavelengths.

Bohr's solution was to combine the orbit model with the finding that **light is grouped in packets (photons) of energy $hf = hc/\lambda$** , where h is Planck's constant, f is the frequency, and λ is the wavelength (see Ch. 5, eq. 5-3). That is, the energy of light is quantized. Bohr applied the concept of quantized energies to the orbits of electrons in an atom. The key idea was that an electron bound to an atom could only have certain discrete energies. The energies are negative because the electrons are bound to the atoms by the electric attraction to the nucleus. The lowest (most below zero) energy is called the ground state, while the higher (closer to zero) bound states are called excited states. The electron escapes from the atom if it absorbs enough energy to make its new total energy positive. The atom then becomes an ion (it becomes ionized). This additional energy can be supplied in one of two ways:

1. Absorption of a photon with sufficient energy — high enough frequency/short enough wavelength.
2. Transference to the electron of some of the kinetic energy of a particle — usually a free electron — that collides with the atom.

In Bohr's model, each excited state has a definite energy that is greater (closer to zero) than that of the ground state. If an electron absorbs a photon of just the right energy — exactly equal to the energy difference between two states — then the electron jumps up to the higher energy level. (The photon disappears since all of its energy has been used.) On the other hand, an electron that is already in an excited energy state will spontaneously “jump” to a lower (more deeply negative) energy level. It does this in a small fraction of a second. This downward jump causes light to be emitted in the form of a photon with energy exactly equal to the difference between the energies of the two states.

There are three ways for the electron to achieve an excited state: (1) it previously absorbed a photon, (2) it was free and then became “captured” by the atom, or (3) the atom gained energy from a collision, usually with a free electron.

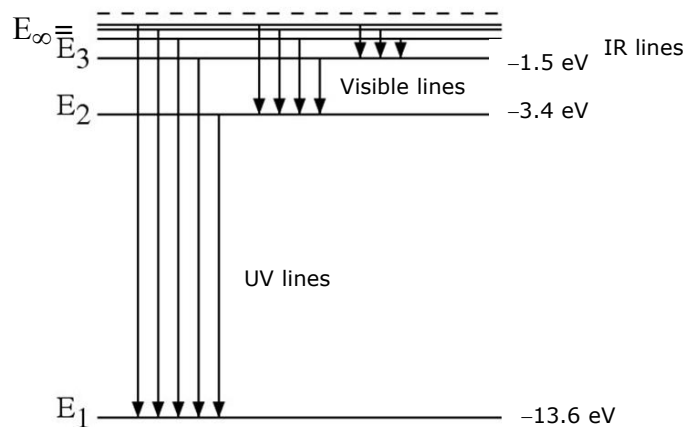


Figure 7-4. Energy levels in a hydrogen atom. The energies are marked for levels 1, 2, 3, and ∞ , where the last is the level beyond which the electron becomes free of the atom. The longer arrows have higher frequencies and shorter wavelengths. The four visible lines are, from shorter to longer arrows, red, blue-green, violet, and deeper violet (which most people cannot see); refer to Fig. 7-2.

Bohr's model thereby explained why the emission of light from atoms in a gas occurs in the form of lines: The atoms of a given element can only emit photons that have very specific energies and therefore very specific wavelengths. Each gaseous element has its own characteristic pattern of emission lines.

But could Bohr's model explain the values of the wavelengths of the emission lines? It is (by far) easiest to consider the hydrogen atom, since it contains only one electron. Bohr had suspected that Planck's constant h must play an important role in the energy levels of the electrons. He formulated the wavelength of light emitted during a transition from energy level n_2 (with energy E_2 , which is negative) to level n_1 (with energy E_1 , which is lower than zero by a greater amount), where n_1 and n_2 are integers (1, 2, 3, etc.), as

$$f = (E_2 - E_1)/h \quad (7-2a)$$

or, in terms of wavelength,

$$\lambda = \frac{c}{f} = \frac{hc}{E_2 - E_1} \quad (7-2b)$$

f = frequency (in Hz), E_1 = (negative) energy of lower level (in J), E_2 = (negative) energy of higher level (in J), h = Planck's constant = 6.63×10^{-34} J s, λ = wavelength (in m), c = speed of light (in m/s).

Earlier, Balmer had noticed that the frequencies of visible emission lines of the hydrogen atom follow the equation

$$f = 3.29 \times 10^{15} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ Hz} \quad (7-3a)$$

and the wavelengths are

$$\lambda = \frac{91.15 \text{ nm}}{\left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]} = 91.15 \left[\frac{n_1^2 n_2^2}{n_2^2 - n_1^2} \right] \text{ nm} \quad (7-3b)$$

f = frequency (in Hz), n_1 = number of lower energy level (integer, no units), n_2 = number of higher energy level (integer, no units), λ = wavelength (in nm).

The wavelength 91.15 nm (frequency of 3.29×10^{15} Hz) is the maximum wavelength that an absorbed photon needs to have in order to ionize a hydrogen atom whose electron was initially in the ground state ($n_1=1$).

Bohr recognized that these two formulas would be the same if the energy levels of the hydrogen atom obeyed the equation

$$E_n = -h(3.29 \times 10^{15} \text{ Hz})/n^2 = (2.18 \times 10^{-18} \text{ J})/n^2 = (13.6 \text{ eV})/n^2.$$

As indicated above, since 2.18×10^{-18} J is not a very convenient number, it is often converted to 13.6 electron volts (eV, a unit of energy), where $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ is the energy gained by an electron after passing through a 1-volt change in electric potential.

The energy levels of Bohr's model of the hydrogen atom is illustrated in Figure 7-4. Although the model was based on the wrong picture of the atom — the electron is not like a tiny charged ball orbiting the nucleus — the energy levels are correct for the hydrogen atom. Bohr realized that the model could not be a close representation of reality, since it could not explain *why* an electron must occupy one of the discrete energy levels rather than spiral into the nucleus. His model worked well only if it were simply assumed that electrons must occupy energy levels with integer values of n and that they emit no light while they stay at the same level. In addition, the values of the energy levels only worked for atoms or ions with a single electron. Bohr was, however, able to express the energy of level 1 in terms of the mass and electrical charge of the electron and Planck's constant h .¹ Quantum Mechanics, which we describe later in this chapter, provides the modern model of the atom as well as explanations for other phenomena on extremely small scales. Bohr's model was an important step toward the development of this more complete theory.

Light Emitted and Absorbed by Atoms: The Rules (Kirchoff's Laws of Spectra)

¹ For convenience, many scientists prefer to use the quantity $\hbar = h/(2\pi) = 1.06 \times 10^{-34}$ J s; when spoken, this constant is called "h bar".

In 1859, Gustav Kirchhoff² formulated three laws that describe the type of spectrum that is observed when the light from an object is viewed through a prism or diffraction grating (a flat piece of glass or transparent film etched with fine lines). This causes light to separate according to its wavelengths, thereby forming a spectrum. Here we state these rules, along with the explanation according to Bohr's model of the atom. Figure 7-5 illustrates the three situations governed by Kirchhoff's Laws. All of these play important roles in analyses of astronomical observations of stars, galaxies, and cosmic clouds of gas and dust.

1. The light from an opaque object has a continuous spectrum, that of a blackbody. This was discussed in Chapter 5. If the temperature is hot enough (more than about 1000 K), the object will glow in visible light; at lower temperatures almost all of the radiation is in the infrared or, for very low temperatures, the radio portion of the electromagnetic spectrum. If the temperature is hotter than about 10,000 K, most of the radiation will be in the ultraviolet part of the spectrum, although it will still shine at visible wavelengths as well. The visible color of an opaque object depends only on its temperature (unless it is viewed through some medium that changes the color).

Box 7-1. How a continuous spectrum forms in an opaque object

How does a continuous spectrum form from emission by atoms? A high temperature causes the atoms of the object to collide frequently with free electrons. The energy of a collision is often absorbed such that a bound electron jumps to an excited energy level. This electron then emits a photon as it returns to a lower, less excited energy state. If the object is opaque, the vast majority of such photons are absorbed by other atoms. The wavelengths of the photons are changed by the Doppler effect (see Ch. 5 and later in this chapter), since the atoms and electrons are moving at high speeds in the hot object. The light that emerges is the result of emission and absorption of photons at many wavelengths, with the wavelengths shifted by varying amounts through the Doppler effect caused by the random motions of the atoms. The net result of this complex set of interactions is that the emission lines all blend together, and the brightness of the light that escapes from the object changes smoothly with wavelength.

2. The light from a hot, transparent gas has an emission line (or "bright line") spectrum. (This assumes that there is no hotter solid object behind it — see item 3 below.) As in the case of the opaque object discussed above, the high temperature of the gas causes many high-speed collisions, which result in the electrons in many of the atoms jumping to excited energy states. When they spontaneously jump back down to lower energy levels (usually within a small fraction of a second), light is emitted. Since the gas is transparent, the light escapes, maintaining its original wavelength (perhaps altered by the Doppler effect; see below). The light therefore appears as colored lines in the spectrum, with a pattern of wavelengths that corresponds to the element that the atom represents (see Figs. 7-3 & 7-6). If more than one element is present in the gas, then the lines from all of these elements appear in the spectrum. The brightness of each line depends on the number of atoms of the corresponding element that are present in the gas, as well as on the temperature (which affects the level of excitation of the electrons in the atoms of each element).

² Kirchhoff was also famous for his laws describing the behavior of electric circuits.

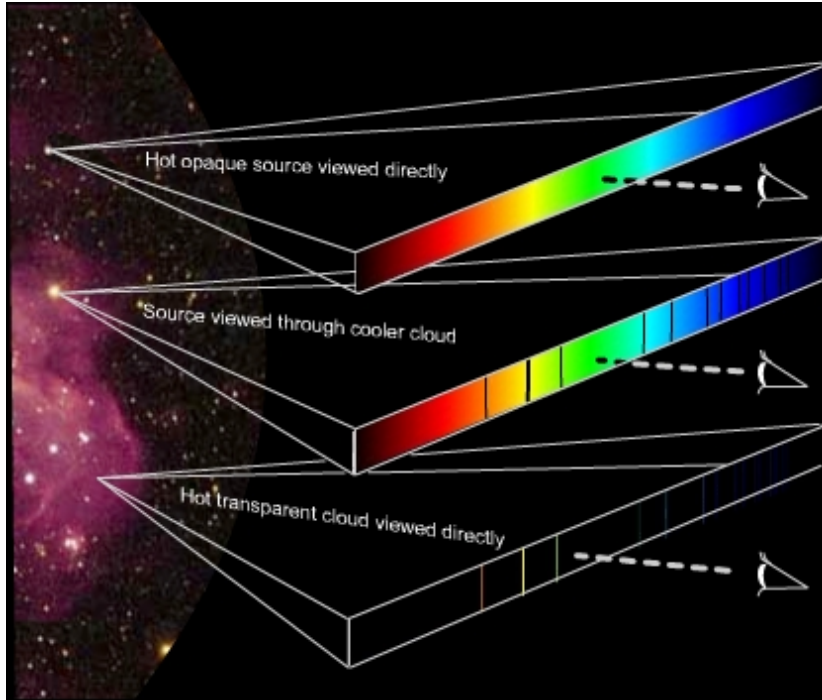


Figure 7-5. The three types of spectra. *Top:* Continuous spectrum from a hot, opaque object. *Middle:* Absorption-line ("dark line") spectrum from a hot, opaque object whose light passes through a cooler, transparent gas (e.g., from a cloud or an atmosphere). The spectra of most stars are absorption-line spectra; see Fig. 7-2. *Bottom:* Emission-line ("bright line") spectrum from a hot, transparent gas.

3. A hot, opaque object whose light passes through a foreground transparent gas that is *cooler than the opaque object* will have a (dark) absorption line spectrum. In such a situation (see, e.g., Fig. 7-2), the continuous spectrum that the opaque object emitted has dark lines in it. These lines have the same wavelengths as do emission lines from the same atoms in a hot gas (*i.e.*, case 2 above). The spectra of most stars are of this absorption-line type. This is because the light from the hot, opaque surface must pass through a transparent, cooler atmosphere before escaping into space. What happens is that the electrons in the atoms of the foreground gas absorb those photons from the continuous spectrum that have exactly the right energies for electrons in the atoms to jump up to higher energy levels.

The frequency and wavelength of the emission or absorption line resulting from a transition between levels n_2 and n_1 (or vice-versa) are given for a hydrogen atom by eq. (7-3).

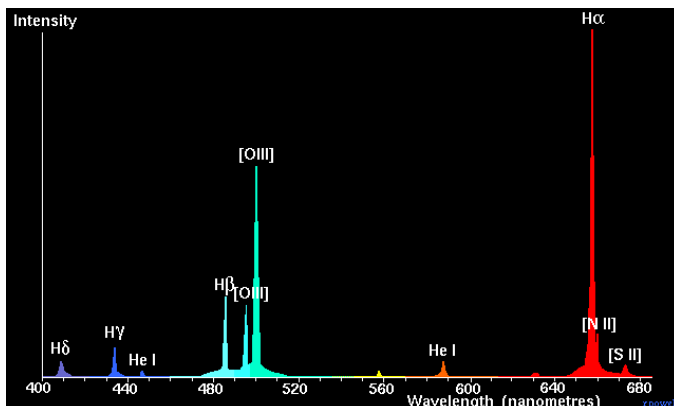
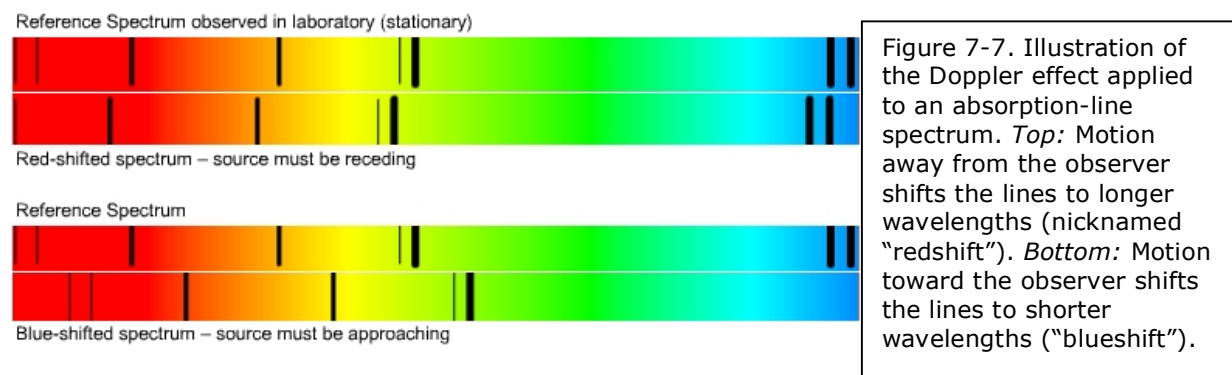


Figure 7-6. An emission-line spectrum of a hot, transparent cloud of gas in our Galaxy, in the form of a graph of brightness vs. wavelength. Lines from several elements – hydrogen (H), helium (He), oxygen (O), nitrogen (N), and sulfur (S) are prominent. These can be identified by the wavelengths of their emission lines. The same is true for an absorption-line spectrum, e.g., that of the Sun (Fig. 7-2). [Source: www.atlasoftheuniverse.com/nebulae/ngc3372.html]

The Doppler Effect (Revisited)

Chapter 5 introduced the Doppler effect: the wavelength of light is shortened (frequency is increased) if the source of the light is moving toward the observer (or the observer is moving toward the source of

light), and lengthened if it is moving away. In the case of a continuous spectrum, this is extremely difficult, if not impossible, to measure, because there is no distinctive feature in the spectrum that one can observe to shift. However, a line spectrum is ideal for observing the Doppler effect (see Fig. 7-7), and thereby for determining the velocity of the object relative to the observer.



The Doppler effect shifts the pattern of all lines from an element by the same multiplicative factor, so that the ratios of wavelengths (which correspond to the pattern of the spectrum) remain the same.

It is therefore usually easy to identify the element that corresponds to a given set of emission or absorption lines, even if there is a large Doppler shift. The rest wavelengths of spectral lines of all the different elements are determined by experiments in Earth-based laboratories. If the characteristic pattern of lines of an element is observed in a cosmic object but the wavelengths are different from those in the laboratory, then astronomers can determine quite accurately the velocity of the object relative to the Earth³ from the equation

$$v = \left(\frac{\lambda_{\text{obs}}}{\lambda_0} - 1 \right) c \quad (7-4)$$

v = velocity (negative toward, positive away from observer, in m/s), λ_{obs} = observed wavelength,
 λ_0 = rest wavelength, c = speed of light = 3.0×10^8 m/s.

Note: The Doppler effect does not alter the colors of stars in our own Galaxy — the speeds are not fast enough. However, there are certain types of objects, *e.g.*, quasars, in which gas clouds can move so fast that the colors of the emission lines are altered (for example, from green to yellow).

The Doppler effect is used by police to measure the speed of vehicles on the highway. A beam of radio waves of known frequency is emitted by the police radar gun. The beam then reflects off the vehicle, with an increase in frequency according to the Doppler effect. A radio receiver then measures the frequency of the reflected waves and a computer determines the speed of the vehicle required to reflect the waves at that frequency. A similar technique is used by meteorologists to measure the velocities of clouds in order to determine the speed of advancing storm fronts.

³ The velocities of rotation and orbital motion of the Earth, and in some cases even the velocity of the Sun as it moves through the Galaxy, are usually calculated and subtracted since these change during the day or year.

De Broglie's Particle Waves

After developing his “cartoon” model of the hydrogen atom in 1913, Niels Bohr felt that he was riding the wave of a revolution in science. Where would this new way of thinking lead? There were two main tasks: to determine why the electron could only occupy certain specific energy levels, and to expand the theory to include atoms more complex than hydrogen. Unfortunately, the progress of pure science in Europe was essentially halted by the devastating world war that soon engulfed the continent. It was therefore not until the 1920's that progress was again possible. Despite the delay, the research of that decade would indeed revolutionize our concept of matter. In the process, the consensus among scientists that the universe is a predictable, deterministic system would come unraveled.

The first breakthrough came in 1924, from a young French nobleman, Louis de Broglie. He considered the dual wave and particle nature of light, and how it might apply to Bohr's model of the atom. In a brilliant leap of intuition, he wondered whether electrons, which up to this point had been considered as tiny, solid particles, might also behave as waves. Since the momentum of a photon is the energy divided by the speed (eq. 5-4), perhaps one can use the same formula to express a particle's momentum in terms of its wavelength as $p = mv = h/\lambda$. The wavelength of a “particle wave” is then

$$\lambda = h/p = h/(mv) \quad (7-5)$$

λ = wavelength (in m), $p = mv$ = momentum (in kg m/s), m = mass (in kg), v = velocity (in m/s),
 h = Planck's constant = 6.63×10^{-34} kg m²/s = 6.63×10^{-34} J s.

De Broglie's idea received experimental confirmation through the discovery of interference patterns of electron waves.

The wave description explains why an electron can only occupy certain energy levels inside an atom: when it is bound to an atom, an electron must be a standing wave. Recall from Chapter 5 that standing waves occur when, for example, a vibrating string is bound at both ends, as on a guitar. Waves reflect back and forth off the boundaries. The standing waves are those with wavelengths that are integer multiples of the “fundamental” wavelength. These preferred wavelengths, called harmonics, can persist because of constructive interference. That is, crests of a wave moving in one direction coincide with the crests of a reflected wave moving in the opposite direction. All the oscillations at other wavelengths die out because of destructive interference. This is because the crests of waves passing in opposite directions are offset by random distances, so the average amplitude approaches zero after many reflections.

The energy levels of Bohr's model could be explained as the energies of the standing waves that can exist in an atom. But the cost of this advance was to abandon the picture of particles as tiny solid balls in favor of waves. There is, however, one reassuring aspect to equation (7-5): the mass of any macroscopic object (you, for example) is so high — and therefore the wavelength is so tiny — that the wave nature is not apparent. So, objects in our everyday experience do not appear wavy.

Schrödinger's Wave Equation

In 1925, Erwin Schrödinger of Austria developed the primary formulation of Quantum Mechanics by expanding de Broglie's result to the three dimensions of an atom. The equation that Schrödinger derived is the equivalent of Newton's 2nd Law (see eq. 4-3), but for waves instead of solid objects. It is more complex in form because the description of waves requires the use of calculus (as does Newton's theory in the case of changing acceleration). The properties of a wave (of a particle such as an electron) are represented mathematically by the wave function, given the symbol ψ (Greek letter “psi”). Schrödinger's

equation⁴ shows how the wave function changes with position and time under the influence of a force. It is therefore equivalent to (but, alas, not as simple as) Newton's 2nd Law that gives the change in motion of macroscopic bodies in the presence of a force. The connection of Schrödinger's equation to observations is through the square of the wave function (actually, the square of its absolute value), which is the amplitude of the wave at any given position and time.

When Schrödinger applied his equation to the hydrogen atom, he obtained Bohr's result for the energies of the allowed energy levels. But his formulation is *much* more powerful, for it allows the calculation of the energy levels of atoms that contain more than one electron. Furthermore, Schrödinger's theory indicates that electrons should not radiate light as long as the energy level remains the same.

This then answered the perplexing question of why electrons in orbit around a positively charged nucleus do not lose energy and spiral into the nucleus: the planetary model is not the proper way to visualize atoms. The electrons do not "orbit," they instead exist in 3-D standing wave patterns surrounding the nucleus (see Fig. 7-8). A photon is emitted only when that pattern changes to one that requires less energy. Schrödinger's equation indicates that the electron does not instantaneously jump from one level to another — a feature of Bohr's model that seemed contrived. Instead, the wave pattern of one energy level fades out while that of the new level gradually appears. ("Gradually" here is a relative term: the process occurs over only about 10 billionths of a second!)

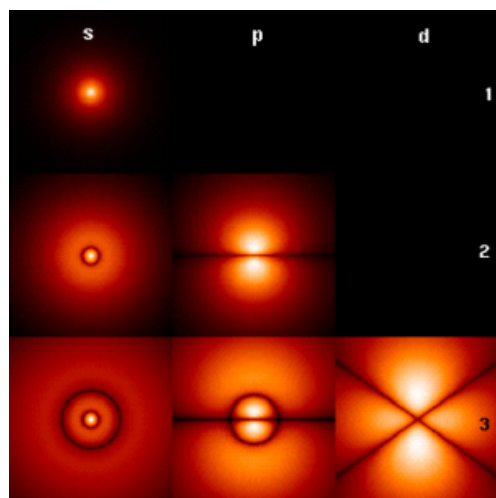


Figure 7-8. Examples of standing wave patterns of an electron in an atom. These are 2-D representations of 3-D patterns. Each corresponds to a distinct energy level. The probability of finding the electron at any particular location is proportional to the intensity of the wave at that location. There is zero probability of finding the electron in the dark regions. When there is no detector, the electron is spread across the entire wave pattern. [From wikipedia.org]

Wave-Particle Duality: Particles as Wave Packets

We are now ready to confront one of the great problems of modern science: how can a photon, electron, or any other type of tiny entity behave as both a particle and a wave? Each description explains some of the phenomena that occur on the smallest scales. For example, the common mass and electric charge shared by all electrons and the discrete energy of photons correspond to the particle picture, while diffraction and interference suggest waves. The solution of Schrödinger's equation in the "free" particle case — when no binding forces are present — provides the answer to the above question. A free particle is a wave packet, a wave whose amplitude decreases with distance from some central location (Fig. 7-9).

⁴ We will not reproduce the equation here, since it involves higher-level mathematics that is unfamiliar to most of the readers.

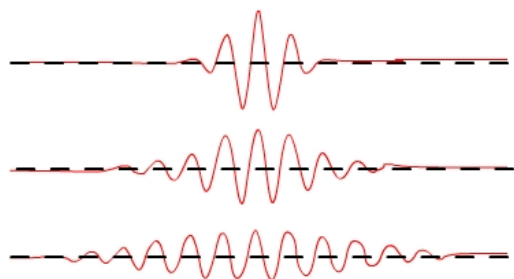


Figure 7-9. Sketch of a wave packet at three times. The square of the displacement of the wave is proportional to the probability that a detector will locate the wave at that position. *Top*: shortly after the particle is located by a detector, the wave is confined to a small region. *Middle and bottom*: the wave packet spreads out with time if there is no detector measuring its location. Note that a real wave packet is 3-dimensional.

We have discussed in Chapter 5 pressure waves in air (sound), waves of vibrating strings, and the more abstract electromagnetic waves that we call light, which can even propagate through a vacuum. What kind of wave is a particle like an electron? The answer to this question is not at all obvious and indeed gave Schrödinger and the other quantum theorists considerable conceptual problems. In 1926, Max Born postulated that the wave function ψ in Schrödinger's equation represents information about the location and other variable properties of the particle. The square of the displacement is the probability that the particle will be found — using some appropriate detector — at a particular position at a given time. [Note: the wave function can represent other physical states of the particle as well, such as its spin.] If we imagine that the wave packet represents some physical quantity, such as “particle juice,” then most of the “juice” would be found at the center of the wave packet. The physical state of the particle is a combination (a “hybrid”) of all the possible physical states. This view of the particle as being spread-out is valid when there is no attempt to detect the particle's position or physical state.

When a detector locates the position of the particle, however, the wave function needs to change. Its wave function “collapses” to a very small region, which corresponds to the place where the particle is detected. [Note that there is always some uncertainty in the measurement of the position, as with any other scientific measurement, so the wave function does not collapse down to a point.] After the detection, though, the solution to Schrödinger's equation indicates that the wave packet of the electron will subsequently spread out from this position as time advances. Figure 7-9 illustrates this expansion of an electron wave packet.

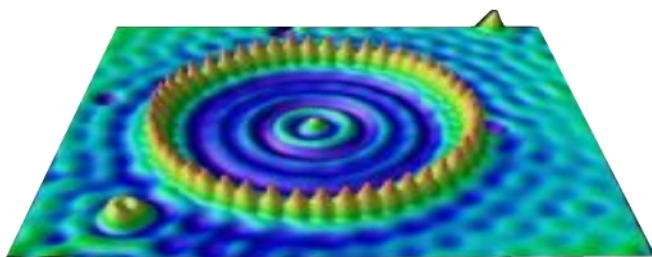


Figure 7-10. Image of the wave pattern of an electron trapped inside a “corral” of iron atoms (the yellow spikes forming a circle). [Courtesy of M. Crommie]

Since the concept that a particle is really a wave of probability is so abstract, it is very appropriate to question whether this idea actually corresponds to reality. In fact, the predictions of Schrödinger's equation and other principles of Quantum Mechanics have been verified by many experiments to a high degree of accuracy. A vivid representation of an electron wave is shown in Figure 7-10, which is the result of an experiment in which physicists “imprisoned” an electron inside a “corral” of iron atoms. The electric field of the electron was measured over a period of time to determine the places where the electron was located. In keeping with Schrödinger's wave mechanics, the electron was more likely to be present at some locations than at others, with the pattern following that of a standing wave.

Quantum Theory

The Uncertainty Principle

The successful predictive capability of Schrödinger's equation stirred the imagination of the great physicists of the time, who puzzled over its implications. The theoretical interpretation that they devised is called "quantum theory." One of these great minds was Werner Heisenberg, who in 1927 realized that there is a limit to the level of precision of measurements that can be made of basic physical quantities on tiny scales. He formulated his famous uncertainty principle: the uncertainty in the position of a particle (Δx) times that of its momentum (Δp) always exceeds Planck's constant h divided by 4π [or $\hbar/2$; recall that $\hbar = h/(2\pi)$]:

$$\Delta x \Delta p > h/(4\pi) = 5.3 \times 10^{-35} \text{ kg m}^2/\text{s} \quad (7-6)$$

" Δ " before a symbol means "the uncertainty in" that quantity, Δx = uncertainty in position (in m), Δp = uncertainty in momentum (in kg m/s), h = Planck's constant = $6.63 \times 10^{-34} \text{ J s}$ = $6.63 \times 10^{-34} \text{ kg m}^2/\text{s}$.

Since momentum is just mass times velocity, this means that the position and velocity of individual particles cannot both be measured exactly at the same time.

The uncertainty principle underscores the difference between the description of a particle as a wave packet and the earlier notion of particles as tiny solid spheres. There are two ways of seeing why the position and velocity cannot both be known exactly at the same time:

1. If both the position and velocity could be known precisely, then the future position and velocity could be predicted precisely from knowledge of the forces involved. But the wave function provides only probabilities of the location or state of a particle, so the future state cannot be specified exactly. This implies that there must be uncertainty in our knowledge of the current position and velocity of the particle.
2. The detection of a particle involves a physical interaction with it. For example, an electron can be detected by beaming some light toward it. If a photon of that light reflects off the electron, we can determine where the electron was by observing the photon. But the photon then transfers some momentum to the electron. As a consequence, the detector interacts with the electron and changes its physical state. The uncertainty principle dictates that we will not know exactly both the position and momentum of the electron after this interaction.

A second version of Heisenberg's uncertainty principle, the time-energy inequality, is similar to the first, and has even broader implications. The product of the uncertainty in the energy (ΔE) of a particle and the interval of time (Δt) over which it is measured always exceeds Planck's constant divided by 4π :

$$\Delta E \Delta t > h/(4\pi) = 5.3 \times 10^{-35} \text{ J s} \quad (7-7)$$

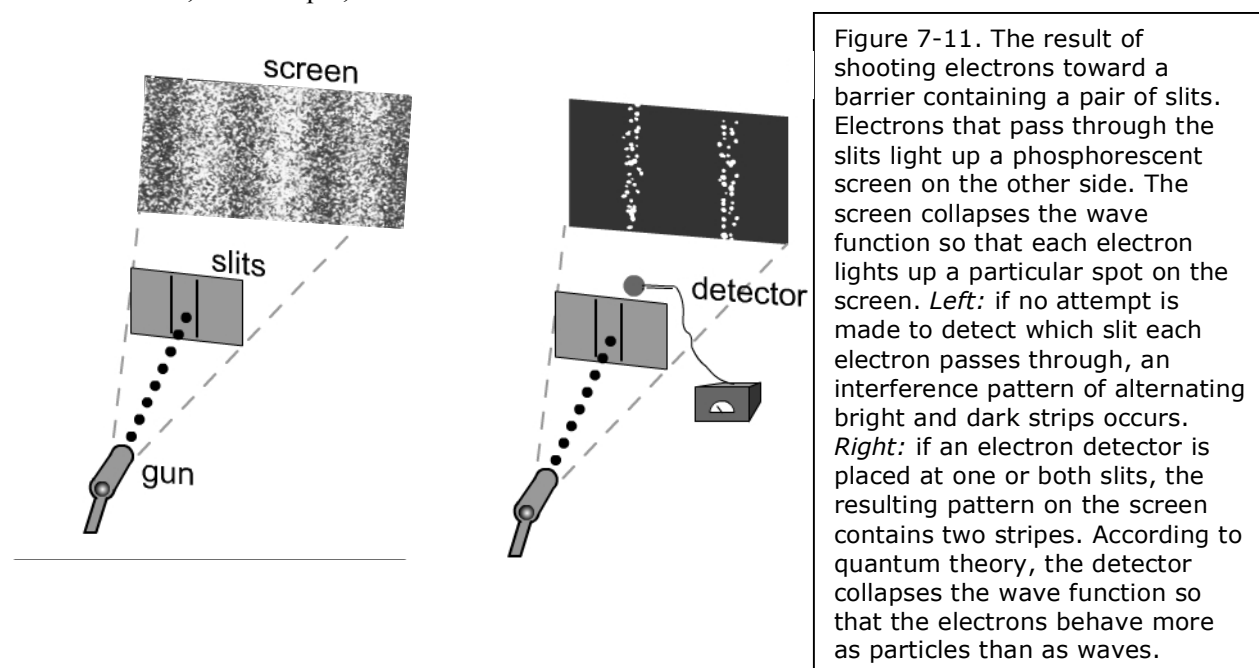
ΔE = uncertainty in energy (in J), Δt = time over which the measurement is made (in s), h = Planck's constant = $6.63 \times 10^{-34} \text{ J s}$.

The meaning of eq. (7-7) is that, when measured over a very short time interval, the energy of a particle has a value that is uncertain by at least $h/(4\pi\Delta t)$. This form of Heisenberg's uncertainty principle has an important implication. Since the product of the uncertainty in energy and the time interval over which the energy is measured always exceeds some value $h/(4\pi)$, then over a very, very short time interval the energy of a particle can be quite large (or very small) compared to its energy averaged over longer time

periods. It even allows for “virtual particles” to pop into and out of existence over extremely short intervals of time. This facet of quantum theory is very important for our understanding of particle physics (Ch. 8) as it applies to the very early universe (Ch.10).

The Measurement Problem

Since the measurement of the position of a particle “collapses” the wave function, it seems that an observation actually affects the wave function that provides information about the state of the particle. But does it affect the particle itself? The answer appears to be “yes!” In Chapter 5, we discussed what happens when a wave passes through a slit or hole in a barrier: waves with the same wavelength as the original wave, and centered on each slit, continue to propagate beyond the barrier (the process called **diffraction**). If there are two slits, then the waves pass through both slits. Beyond the barrier, these two waves interfere with each other. Since particles are actually wave packets, such an interference pattern should occur if, for example, a beam of electrons encounters a barrier with two slits.



Indeed, this is exactly what is observed in experiments, but only if no attempt is made to determine through which slit any of the individual electrons passes. This is illustrated in the left panel of Figure 7-11. However, as shown in the right panel, the result is quite different if a detector is placed at one (or both) of the slits. Instead of the interference pattern, the electrons behave almost as “bullets” rather than as interfering waves. That is, if no attempt is made to detect the electrons, they behave as waves that in fact pass through both slits. However, the presence of the detector collapses the wave functions by determining the positions of the electrons as they pass by the barrier.

In other words, the act of observation changes the nature of the electrons! This occurs because the observation requires an interaction between the detector and the particle. For an electron, this requires the application of an electric or magnetic field. For such tiny particles, there is no such thing as a completely passive observer. This intimate connection between the act of observation and the outcome of the event being observed, called the “measurement problem,” is very disturbing from a philosophical viewpoint. It seems to require that an observation is a physical event that does not obey Schrödinger’s equation, since the equation describes a wave function that behaves in a smooth manner. There is also the unanswered question of who or what constitutes a valid “observer.”

The Correspondence Principle

The development of Quantum Mechanics was guided in part by Bohr's correspondence principle: all the formulas should be consistent with the behavior of the macroscopic world when macroscopic phenomena are observed. Indeed, de Broglie's wavelength of matter (formula 7-5) is very, very short for a massive particle. Therefore, the wave nature of macroscopic objects has utterly negligible consequences. The uncertainty in position or velocity (expression 7-6) of such an object is so small compared with its size or velocity that no device can measure positions and speeds with sufficient accuracy to notice the effect. Finally, Schrödinger's wave equation is essentially the same as Newton's 2nd Law (see Chapter 4) when applied to macroscopic phenomena.

Quantum Mechanics and the Periodic Table

Among the great successes of Quantum Mechanics is that Schrödinger's equation explains the entire periodic table of the elements. Application of the equation reproduces the energy levels of the hydrogen atom proposed by Bohr and those of every other type of atom. The interference of the 3-D wave functions of the different electrons in the atoms of elements beyond hydrogen leads to a fundamental principle of atomic physics and chemistry: the Pauli exclusion principle. As proposed by Wolfgang Pauli in 1925, the principle states that no more than two electrons can occupy the same energy level; these two must possess opposite spin values, one "up", the other "down". (Here "spin" is actually a quantum property that has some of the properties of spin, but it is not really a clockwise or counterclockwise rotation of the particle.) This provides structure to the occupation of the different energy levels by electrons. The natural tendency toward the lowest total energy makes certain combinations of atoms to form molecules more favorable than others. This is the reason why elements in columns I and II of the periodic table (reproduced in Appendix B) readily react with elements in columns VI and VII. In fact, all of chemistry is explained by Quantum Mechanics.

Box 7-2. Quantum theory and the nature of reality

Humans have an intuitive sense of what is meant by "reality" in our everyday, macroscopic world. A solid wall is real because we cannot walk through it, and if we try, we can experience pain and injury. But what is the basis of this reality? Quantum theory suggests that the tiny building blocks of matter – the particles – behave in a way that is alien to our everyday world. How can we reconcile what we experience with the behavior of the nanoworld?

The primary answer comes from the correspondence principle and statistics. The wave nature of macroscopic objects is far too small to be measurable. And the number of events that occur in the nanoworld every second is so large that the uncertainty of probabilities on tiny scales becomes statistical certainty on larger scales. If you flip a coin, you know that there is a 50% probability of either head-up or tail-up, and if you predict which will occur, you will be right only half the time. But if you flip a coin a million times, it will land head-up very, very close to 50% of the time and tail-up the other 50%. The statistics of the large number of events allows very definite predictions of macroscopic phenomena.

Quantum theory wreaks havoc with our concepts of reality only when we consider direct interactions with particles through an act of observation. Such an act causes the particle or event to take on a definite value, the probability of which is predicted by Schrödinger's equation. Before the observation, the particle was in a hybrid state: a superposition of all the possible outcomes of the measurement. Albert Einstein opposed this interpretation, declaring that "God does not play dice [with the universe]!"

Einstein, along with colleagues Boris Podolsky and Nathan Rosen, composed one of his famous *gedanken* (thought) experiments to try to demonstrate the absurdity of the "Copenhagen" interpretation of

quantum theory championed by Bohr and his colleagues. Imagine that two electrons with equal velocities in opposite directions collide head-on. We know from conservation of momentum (see Ch. 4) that the initial total momentum was zero (all electrons have identical masses), and so the final momentum must be zero as well. So, after the collision, the electrons must have equal and opposite momenta. But, according to the Copenhagen interpretation, until a measurement of momentum is made, each particle has a range of possible momenta. Einstein, Podolsky, and Rosen considered what would happen if the measurement of one electron's momentum were made after it had become very distant from the other. For the other electron then to "know" that it must instantaneously have momentum with the same magnitude would require information to travel faster than the speed of light from the electron whose momentum is measured. This demonstrated to Einstein that the Copenhagen interpretation must be wrong. Instead, he thought, the momentum of each electron must have been determined when the particles collided, even if the observer did not yet have any knowledge of what that momentum is. In 1962, John Bell showed that Einstein's view corresponds to different probabilities of the outcomes of measurements than does the Copenhagen interpretation. The results of experiments from the 1970's to the present have agreed with the predictions of the Copenhagen interpretation rather than those of more deterministic interpretations. Nevertheless, the Copenhagen interpretation requires that an act of observation is a special event, but it is not clear how that can be true.

Schrödinger devised a thought experiment, referred to as the "Schrödinger's Cat" conundrum, that he thought demonstrated the absurdity of the Copenhagen interpretation. His idea was to have a nearly direct interaction between a single event in the nanoworld and the macroscopic world. Schrödinger imagined the construction of a box that, when closed, caused the contents to be completely unobservable by the outside world. Inside the box are a cat, air, a radioactive atom, a Geiger counter, and a vial of poisonous gas. The gas will be released and kill the cat if the Geiger counter detects the products of the decay of the radioactive atom. We can imagine that the atom has a 50% probability of decaying within 1 hour. Such a decay will emit a particle that triggers the Geiger counter. The decay is a physical state governed by the probability laws of Quantum Mechanics. The cat therefore has a 50% probability of being alive after that hour. According to the Copenhagen interpretation, while the box is closed, the radioactive atom is in a hybrid state of half decayed, half undecayed, at least as far as the outside observer is concerned. What then is the state of the cat – half alive and half dead? Schrödinger considered this to be silly and therefore rejected the Copenhagen interpretation. But the concept of hybrid states has since received experimental support. Does this mean that there are two different realities, one experienced by the cat and the other by the outside observer?

A number of solutions to this dilemma have been proposed. Three that are currently still in contention are:

1. The system of the atom, the cat, the Geiger counter, and the poisonous gas is too complicated to be described as a wave function. Instead, the quantum states are not "pure" (or "coherent") but rather so complex that the statistical nature of quantum theory applies because so many atoms, electrons, etc. are contained in the box. This solution, called decoherence, now has some experimental support: the purity of a quantum state has been shown to decay with distance from the apparatus that put the particle into that quantum state. We do not, however, understand the theory of decoherence when the state of a single atom (the radioactive one) can affect directly the outcome of a macroscopic event.

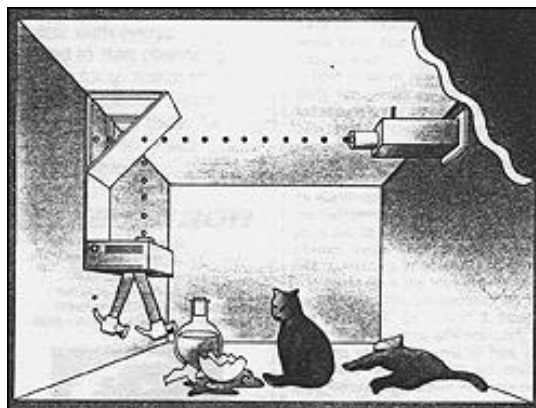


Figure 7-12. Sketch of the set up imagined by Schrödinger to demonstrate the absurdity of the Copenhagen interpretation of Quantum Theory. A cat is placed inside a sealed box, along with a Geiger counter, a vial of poisonous gas, and a radioactive atom having a probability of 50% of decaying within 1 hour. If the atom decays during the hour, it will release a particle that will trigger the Geiger counter, which will send a light beam into a machine that causes a hammer to swing and break open the vial of toxic gas. The box is opened after one hour, revealing the cat to be either dead or alive. But what was the state of the cat before this observation was made? [Source: <http://www.dhushara.com/book/quantcos/penrose/cat.jpg>]

2. Hugh Everett's many worlds hypothesis. The act of measurement does not actually collapse the wave function. For example, if the radioactive atom can either decay or not, it does both! At that point, the universe splits into two parallel universes: in one, the cat is dead after one hour, while in the other, the cat remains alive. Neither logic, mathematics, nor the spirit of quantum theory is violated. Schrödinger's equation is obeyed at all times, since there is no collapse of the wave function. However, many people think that this proposal comes at the expense of adding complexity to the world, since it requires that another parallel universe be created each time that an event happens that has more than one possible outcome, as is the case many, many times every second. Nevertheless, the mathematical simplicity of the many worlds hypothesis and its possible application to the question of where the universe came from (discussed in Chapter 11) have made it popular with a number of theorists.

3. One of God's roles in the universe is to act as the observer whose measurements collapse all the wave functions when an event occurs on tiny scales. This is familiar, since Newton proposed that God interacts continuously with the universe to prevent gravitational collapse. Most scientists reject this notion, preferring instead to rely on natural explanations for natural phenomena.

Summary

Although it took a long time to establish, the theory that matter is composed of atoms explains many of the properties of substances. The periodic table devised by Mendeleev organizes the elements according to their chemical properties. The theoretical basis for the periodic table was understood with the development of Quantum Mechanics in the 1920s. The temperature of a substance is a measurement of the average kinetic energy of the microscopic motions of the atoms, molecules (two or more atoms bound together), ions, and electrons inside it.

Niels Bohr developed a simplistic picture of the hydrogen atom in which the electron can only occupy certain “quantized” energy levels. This model provided very accurate predictions of the wavelengths of emission lines from hydrogen atoms. But it was unsatisfactory because there was no theoretical basis to explain why only these energy levels are allowed. The solution required a revision in our concept of the nature of a particle such as an electron.

When an observer views a hot, transparent gas (e.g., from a clear fluorescent light tube or a hot cloud in space), the spectrum consists of bright emission lines with a pattern of wavelengths — which at visible wavelengths correspond to specific colors — that is specific to the element(s) in the gas. This is called an emission-line spectrum. When the same elements are present in a cold gas cloud (or atmosphere) that lies in front of a hot, opaque source of light (e.g., the surface of a star), the entire blackbody spectrum (see Ch. 5) will be observed, except for dark lines. This is called an absorption-line spectrum. The dark lines have exactly the same pattern of wavelengths as the emission lines from those elements. The Bohr model explained the emission and absorption of light as the consequence of electrons changing energy levels, with the energy of the photon corresponding to the energy lost (for emission lines) or gained (absorption lines) by the electron. The spectrum of the gas contains a line for every possible transition between two energy levels in the atom, although most of these lines have wavelengths outside the visible range (e.g., at infrared or ultraviolet wavelengths). When there is relative motion between the observer and the gas, the entire pattern of the wavelengths of the lines is shifted toward longer wavelengths — redshift — if the motion is away from the observer and toward shorter wavelengths — blueshift — for motion toward the observer. The faster the motion is, the greater the shift of wavelengths is.

At the beginning of the 20th century, physicists conceived of particles as tiny solid spheres. However, after Bohr and others developed a model that could reproduce the observed spectra of atoms, it became clear that many phenomena on tiny (“nano”) scales could be explained better if particles were instead

waves. De Broglie wrote down a simple formula (7-5) that relates the wavelength to the momentum of the particle. Schrödinger formulated an equation that specifies the behavior of the wave function that describes the state of either a free particle or an electron bound to an atom. The wave is, in essence, a wave of information about the physical state of a particle, *e.g.*, its position or spin. The mathematical form of the wave function of a free electron (or a photon of light) leads to a picture (Fig. 7-9) in which a particle is a wave packet, a spherical wave whose amplitude decreases from the central position. The behavior of particles and the outcomes of events on the smallest scales can only be determined probabilistically rather than calculated precisely. This means that the exact same measurement of two systems with the exact same initial conditions will not always produce an identical result. When there is no active measurement of the physical state of the particle, it possesses a combination of all the possible states. However, when the position, spin, or other variable physical property of a particle is observed, the act of measurement disturbs the state of the particle. This causes the particle to behave more as a tiny solid object (the old view of a particle) than as a wave. The inability to observe a particle without disturbing it is known as the “measurement problem.”

The modern model of an atom contains a tiny, positively charged nucleus at the center, surrounded by 3-D standing waves representing the possible positions of the electrons (see Fig. 7-8). Different geometries of the standing waves correspond to different energy states. This Quantum Mechanics model accounts not only for the wavelengths of photons that are emitted or absorbed when an electron changes its energy state, but also for the chemical properties of the elements.

Heisenberg’s uncertainty principle states that a particle’s position and momentum (and therefore velocity) cannot both be measured exactly at the same time. Similarly, the time-energy inequality dictates that the uncertainty in the measurement of energy of a particle is inversely proportional to the time interval over which it is measured. The uncertainty in energy and time allows a particle to have a very large energy over an extremely short time period. It can even cause particles to “pop” in and out of existence.

The strangeness of the tiny “nanoworld” governed by Quantum Mechanics can be reconciled with the macroscopic world that we experience. In accordance with Bohr’s correspondence principle, the extremely short wavelengths of massive objects means that their wave nature is so slight that it can be completely ignored. The large number of nanoscale events, governed by probability laws, that occur in macroscopic phenomena lead to a statistical certainty in the behavior on everyday size scales.

Glossary

Element: A name given to similar atoms. All atoms of a given element have the same number of protons in the nucleus. (See Ch. 8 for a discussion of the nuclei of atoms.)

Molecule: A combination of atoms that are bound together.

Particle: A tiny bit of matter, *e.g.*, an electron or a proton. (See Ch. 8 for more examples.)

Periodic Table: An organization of the chemical elements into rows and columns such that the elements in any given column have similar chemical properties.

Temperature (Symbol: T): A quantity used to measure the thermal energy of an object or medium (see Ch. 5). Most conveniently measured in kelvins (K). At a temperature of 0 K, the thermal energy equals zero.

Thermal Energy: The average microscopic kinetic energy of particles in a substance.

Absolute Zero ($0\text{ K} = -273\text{ }^{\circ}\text{C}$): Temperature at which all microscopic motion stops, corresponding to zero thermal energy.

Ion: An atom that has more or fewer electrons than protons, so that it is electrically charged.

Electron: A particle with negative electric charge that is a primary component of an atom.

Bohr Model of the Hydrogen Atom: An incomplete description of an atom in which electrons can only occupy certain energy levels that occur in discrete (“quantized”) steps.

Energy Level: A quantized state of an electron in an atom described by its negative binding energy under the attractive force of the positively charged nucleus.

Ground State: The lowest, most stable energy level in an atom.

Excited State: Any energy level other than the ground state.

Photon (often denoted by the symbol γ): Quantized unit of light, described as a wave packet. The energy and momentum of a photon depend on its frequency (wavelength); see eqs. (5-3) and (5-4).

Spectrum: A representation of the amplitude of a group of waves as a function of wavelength or frequency. For electromagnetic waves (light), a spectrum indicates how brightness changes with wavelength. A commonly seen example for visible light is a rainbow.

Continuous Spectrum: The case when the brightness varies smoothly with wavelength. Contrasts with a line spectrum (see below).

Blackbody: An opaque object. Any blackbody that has a temperature higher than absolute zero (0 K) emits a continuous spectrum of electromagnetic waves. See eqs. (5-5–5-7) for relations between the electromagnetic radiation (light) of a blackbody and its physical properties.

Spectral Lines: Sharp features (i.e., occurring over a very short range of wavelengths) in a spectrum.

Emission-line Spectrum: Bright lines that appear in the spectrum of a hot, transparent gas.

Absorption-line Spectrum: Dark lines that are superimposed on a continuous spectrum, when the light of a hot, opaque object shines through a cooler gas cloud.

Kirchoff’s Laws: Three guidelines that describe the type of spectrum (continuous, emission-line, or absorption-line) that will be observed from an opaque or transparent object.

Doppler Effect/Doppler Shift: The change in wavelength (or frequency) caused by relative motion between the source of waves and the observer. See eqs. (5-2) & (7-4) and Fig. 7-7.

Redshift: Nickname given to a Doppler shift toward longer wavelengths because of motion away from the observer.

Blueshift: Nickname given to a Doppler shift toward shorter wavelengths because of motion toward the observer.

Quantum Mechanics: The branch of physics that describes the behavior of atoms, molecules, and particles. The main entities on very tiny scales are particle waves. A primary feature is that physical properties like energy occur in discrete steps (*i.e.*, they are “quantized”).

Quantum theory: The interpretation of Quantum Mechanics.

Particle wave: Description of a particle in terms of a wave, with the wavelength given by eq. (7-5).

Schrödinger’s equation: One of the primary formulas of Quantum Mechanics. The equation describes how the physical states (such as position, energy, or spin) of a particle-wave depend on time and location. The equation uses calculus that is beyond the mathematical level of this book.

Wave function (symbol: ψ): The mathematical representation of a particle wave. The square of the absolute value of the wave function gives the probability that a given property of a particle wave (*e.g.*, position or spin) will have a particular value when measured.

Collapse of the wave function: This refers to what happens to the wave function when a physical measurement is made of the particle wave. The wave function momentarily becomes very compact, after which it spreads out.

Wave packet: A description of a particle as a wave whose amplitude decreases with distance from the point where its wave function has its maximum value. (See Fig. 7-9.)

Uncertainty principle: Statement by Heisenberg that the product of the uncertainty in position Δx times that of the momentum Δp of a particle always exceeds the value $h/4\pi$, which is $5.25 \times 10^{-35} \text{ kg m}^2/\text{s}$, where h is Planck’s constant. (See eq. 7-6.)

Time-energy inequality: Variation of Heisenberg’s uncertainty principle: The product of the uncertainty in energy ΔE times the time interval Δt over which the energy is measured always exceeds the value $h/4\pi$, which is $5.25 \times 10^{-35} \text{ J s}$, where h is Planck’s constant. (See eq. 7-7.)

Measurement problem: The logical puzzle over the inability to measure the physical property of a particle without disturbing it and causing that property to have a particular value. The physical property possesses a combination of all possible values, governed by probability rules, until a measurement is made.

Diffraction: Phenomenon that occurs when part of a wave passes through a small hole or slit in a barrier. A new wave, centered on the hole, emerges on the opposite side of the barrier. This new wave has the same wavelength as the incident wave, although its amplitude is lower.

Interference pattern: Observed arrangement of areas of high and low amplitude (bright and dark in the case of light) areas caused when two or more waves cross each other, as in the two-slit experiment (Fig. 7-11). This is a phenomenon only of waves; bullet-like objects do not interfere with each other.

Correspondence principle: Bohr’s rule that any equation that describes phenomena on the smallest scales must be consistent with observations of the macroscopic world when the appropriate values of properties like mass and velocity are used.

Nanoscales & Nanoworld: Realm of very tiny length scales, comparable to the sizes of individual atoms. The term is derived from a nanometer, a unit of length, where $1 \text{ nm} = 1 \times 10^{-9} \text{ m}$ (one-billionth of a meter).

Questions for Discussion

A. When you shine (invisible) ultraviolet light (“black light”) on certain materials, they glow with brilliant colors. How might this behavior be explained in terms of the Bohr model of the atom? What type of spectrum do you expect to be produced? [Hint: the colors come from the transparent outer layer of the material.]

B. In Bohr’s model of the atom, electrons are constrained to occupy certain allowed energy levels. Otherwise, they would radiate light continuously as they orbit the nucleus, thereby losing energy and spiraling into the nucleus in a fraction of a second. In this case, atoms as we know them could not exist and therefore we could not exist. A universe in which we exist must therefore have atoms with such properties. Is this anthropic (human-centered) reasoning a satisfactory explanation for the existence of quantized energy levels?

C. Some stars have a combination of an absorption-line spectrum with the wavelengths all shorter than expected by the same fractional amount (*e.g.*, all are 0.995 times the expected wavelengths) plus a bright emission-line spectrum with the lines at their expected wavelengths. What is a possible explanation for this complex spectrum?

D. Does the Bohr model of the atom explain the periodic table of the elements? What features might need to be added to do this?

E. Although it at first seems nonsensical that solid matter might be composed of waves, what is required for something to possess the characteristic of being “solid”? Is there any fundamental reason why a wave packet could not have this characteristic?

F. When Einstein learned of the idea that events on the smallest scales are probabilistic rather than strongly deterministic, he considered that this meant that there was something wrong with quantum theory. Does a probabilistic foundation for matter make sense to you?

G. Where is an electron 0.0000001 s after it is measured to be at a specific location? According to Quantum Mechanics, its position can only be determined probabilistically. Does this mean that it has spread out so that it is essentially everywhere it is allowed to be according to the probabilities, or is it in one specific location even if the observer can only determine probabilistically where that location is? Does it make any difference which interpretation is adopted?

H. Do you find the many worlds hypothesis attractive as an explanation for the strange behavior of the nanoworld? Why or why not?

Sample Problems on the Quantum Nature of Matter

1. A gas containing only ionized hydrogen (*i.e.*, with only nuclei and free electrons) has a temperature of 10,000 K. How many times faster do the electrons move, on average, than do the nuclei? The mass of a hydrogen nucleus is 1836 (roughly 43^2) times that of an electron. [Although it is not exact, use the approximation that $\langle \frac{1}{2}mv^2 \rangle \approx \frac{1}{2}m\langle v \rangle^2$, where the curved equal sign means “approximately equals.”]

Answer: Use formula (7-1) (ignore the first part) and solve algebraically for $\langle v^2 \rangle$:

$$\langle \frac{1}{2}mv^2 \rangle \approx \frac{1}{2}m\langle v \rangle^2 = \frac{3}{2}kT$$

$$\langle v \rangle^2 \approx \frac{3kT}{m}$$

Now form an algebraic ratio (see Appendix A):

$$\frac{\langle v_e \rangle^2}{\langle v_N \rangle^2} \approx \left(\frac{3kT}{m_e} \right) / \left(\frac{3kT}{m_N} \right) = \frac{m_N}{m_e},$$

where the subscripts “N” and “e” refer to the nucleus and electron, respectively. Now take the square-root of both sides to get the desired ratio:

$$\frac{\langle v_e \rangle}{\langle v_N \rangle} \approx \sqrt{\frac{m_N}{m_e}} \approx \sqrt{43^2} = \underline{43}.$$

So, the electron has a velocity that is 43 times that of the nucleus. Note that the value of the temperature was not needed, since it conveniently cancelled out when the algebraic ratio was formed.

2. Calculate the wavelength of an emission line that is formed when electrons in hydrogen atoms jump from the $n=4$ to the $n=2$ energy level. Consult the section “Light and the Electromagnetic Spectrum” of Chapter 5 to determine the color (including, if relevant, infrared or ultraviolet) of this emission line.

Answer: Use formula (7-3b) (ignoring the middle part):

$$\lambda = 91.15 \left[\frac{n_1^2 n_2^2}{n_2^2 - n_1^2} \right] \text{ nm} = 91.15 \left[\frac{2^2 4^2}{4^2 - 2^2} \right] \text{ nm} = 91.15 \left[\frac{4 \times 16}{16 - 4} \right] \text{ nm} = 91.15 \left[\frac{64}{12} \right] \text{ nm} = \underline{486.1 \text{ nm}},$$

where $n_1=2$ and $n_2=4$.

According to the relevant text in Chapter 5, this wavelength corresponds to a blue color (more precisely, it is a turquoise color).

3. A human of mass 100 kg is running at a speed of 5 m/s. What is the wavelength of the person?

Answer: Use formula (7-5):

$$\lambda = h/(mv) = (6.63 \times 10^{-34} \text{ kg m}^2/\text{s}) / [(100 \text{ kg})(5 \text{ m/s})] = 1.3 \times 10^{-36} \text{ m}.$$

This wavelength is incredibly small: a human does not have measurable wave properties.

4. The momentum of an electron (mass $m = 9.11 \times 10^{-31}$ kg) is measured by a detector with an uncertainty of 2.12×10^{-25} kg m/s. Calculate the minimum uncertainty in the position of the electron.

Answer: Use formula (7-6), solve algebraically for Δx , and then plug in the numbers:

$$\Delta x \Delta p > 5.3 \times 10^{-35} \text{ kg m}^2/\text{s}$$

$$\Delta x > (5.3 \times 10^{-35} \text{ kg m}^2/\text{s})/\Delta p = (5.3 \times 10^{-35} \text{ kg m}^2/\text{s})/(2.12 \times 10^{-25} \text{ kg m/s}) = \underline{2.5 \times 10^{-10} \text{ m}}.$$

Since the uncertainty cannot be less than this, 2.5×10^{-10} m is the minimum uncertainty in the position.

5. As mentioned in this chapter, a particle can pop into and out of existence as long as its energy is less than the limit given by expression (7-7). One such type of particle is called a “virtual photon.” Calculate the minimum uncertainty in energy of a virtual photon that lives for 5.3×10^{-22} s. (Use this as the time over which its energy is measured.) Compare this with the minimum (rest-mass) energy of an electron, $E_{\text{rest}} = mc^2 = 8.2 \times 10^{-14}$ J.

Answer: Use formula (7-7), solve algebraically for ΔE , and then plug in the numbers:

$$\Delta E \Delta t > 5.3 \times 10^{-35} \text{ J s}$$

$$\Delta E > (5.3 \times 10^{-35} \text{ kg m}^2/\text{s})/\Delta t = (5.3 \times 10^{-35} \text{ J s})/(5.3 \times 10^{-22} \text{ s}) = \underline{1.0 \times 10^{-13} \text{ J}}.$$

Since the uncertainty cannot be less than this, 1.0×10^{-13} J is the minimum uncertainty in the energy.

Dividing this by 8.2×10^{-14} J, we find that the uncertainty is 1.2 times the energy of an electron at rest (see the next chapter for a discussion of rest-mass energy).

Homework Problems

1. A gas is composed of only hydrogen (H) and helium (He) atoms at some temperature T . The mass of a He atom m_{He} is 4 times the mass of a H atom, m_{H} .

a. What is the ratio of the average velocities of the H to He atoms? [Use the same approximation as in sample problem 1 above.]

b. By how many times does the average velocity of a hydrogen atom increase if the temperature is $9T$ instead of T ?

2. A strange hypothetical atom has only 4 energy levels that its electrons can occupy. Determine how many spectral lines will appear in its spectrum. Show how you arrive at your answer by drawing a sketch similar to Figure 7-4 (but with no energy levels beyond $n=4$). [Hint: electrons can “jump” from any level onto any other level.]

3. A strange hypothetical atom has only 5 energy levels that its electrons can occupy. Determine how many spectral lines will appear in its spectrum. Show how you arrive at your answer by drawing a sketch similar to Figure 7-4 (but with no energy levels beyond $n=5$). [Hint: electrons can “jump” from any level onto any other level.]

4. Calculate the wavelength of an emission line that is formed when electrons in hydrogen atoms jump from the $n=5$ to the $n=3$ energy level. Consult the section “Light and the Electromagnetic Spectrum” of Chapter 5 to determine the color (including, if relevant, infrared or ultraviolet) of this emission line.
5. Calculate the wavelength of an emission line that is formed when electrons in hydrogen atoms jump from the $n=4$ to the $n=1$ energy level. Consult the section “Light and the Electromagnetic Spectrum” of Chapter 5 to determine the color (including, if relevant, infrared or ultraviolet) of this emission line.
6. A certain star with a surface temperature of 10,000 K is composed of only hydrogen and helium. As do most stars, it has an atmosphere that is cooler than the surface. Draw a sketch of the visible spectrum of the star, indicating the colors (or using color pencils or pens if you have them). Use Figures 7-2 & 7-3 as a guide; the format of your sketch should be as in one of the panels of Fig. 7-3, *i.e.*, a strip with wavelength decreasing to the right.
7. A strange star with a surface temperature of 5,000 K is composed of only hydrogen and sodium. As do most stars, it has an atmosphere that is cooler than the surface. Draw a sketch of the visible spectrum of the star, indicating the colors (or using color pencils or pens if you have them). Use Figures 7-2 & 7-3 as a guide; the format of your sketch should be as in one of the panels of Fig. 7-3, *i.e.*, a strip with wavelength decreasing to the right.
8. An electron of mass 9.11×10^{-31} kg moves at a speed of 1.5×10^8 m/s. What is the wavelength of the electron?
9. A proton of mass 1.67×10^{-27} kg moves at a speed of 2.0×10^7 m/s. What is the wavelength of the proton?
10. The position of an electron (mass $m = 9.11 \times 10^{-31}$ kg) is located by a detector with an uncertainty of 1.0×10^{-11} m.
 - a. Calculate the minimum uncertainty in the momentum of the electron.
 - b. Compare this with the momentum ($p = mv$) of a fast electron traveling at a velocity of 3.0×10^7 m/s, which is 0.1 times the speed of light. To compare, divide the uncertainty from part (a) by the momentum.
 - c. Comment on whether the uncertainty in momentum is important or if it instead can be neglected when measuring the position and momentum of an electron.
11. a. Calculate the minimum uncertainty in energy (in J) of a “virtual” photon (you will encounter these in Ch. 8) that lives for 2×10^{-22} s. [Use this as the time over which its energy is measured.]
 - b. Compare this uncertainty in energy with the rest-mass energy of an electron, 8.2×10^{-14} J. To compare, divide the uncertainty from part (a) by the rest-mass energy.
 - c. If the photon’s energy is at least twice that of an electron, it can turn into particles. Use the uncertainty in energy you calculated in part (a) as the actual energy of the photon. Can it turn into particles?