# Chapter 4. Motions According to Newton

The new view of the universe proposed by Kepler and Galileo was a giant leap forward in humanity's understanding of nature. Although the leaders of the Roman Catholic Church resisted this break from the traditional science of ancient Greece, many scholars embraced it. Similar departures from the old ways of thinking were occurring at the same time. Thus began a new intellectual era: the Age of Reason, also known as the Enlightenment.

Renè Descartes, the famous French philosopher of that era, suggested a logical method for solving scientific problems called "rational analysis," a form of reductionism: divide the problem into smaller components, solve each component separately, and then build a complex solution to the larger problem by assembling the individual pieces. This method has become one of the most fundamental – and successful – guiding principles followed by modern scientists. An example of application of this principle is the analysis of two-dimensional motion discussed below.

Isaac Newton was born in 1643 into this intellectual climate, in which the human mind was thought capable of understanding nature. Newton's equations describing motion and the attraction of massive objects (gravity) provided the major advances needed for a basic understanding of physical phenomena. His formulation is extremely accurate except when we consider motions almost as fast as the speed of light, phenomena in the vicinity of very massive, condensed objects, or the universe as a whole.

*Motion: Basic Definitions*

The description of motion includes equations, since mathematics links theory to reality. The formulas allow us to make definite predictions of the behavior of moving objects. As part of our description, we must define precisely the terms and symbols involved.

*Distance traveled*: The distance ***d*** measured from start to finish. The preferred units of distance are meters, abbreviated "m." The direction in which the object moves is often important to the description.

*Velocity*: The distance traveled per unit time, combined with the direction of the motion. The magnitude of the velocity is the speed ***v***. The preferred units are meters per second (m/s).

*Acceleration*: The change in velocity per unit time, a change in either speed or direction of motion (or both). The symbol used is ***a*** and the preferred units are meters per second per second ($m/s^2$).

*The Basic Equation Relating Distance to Velocity and Acceleration*

We can calculate the motion of an object under constant acceleration with algebra. We would need calculus to do so when the magnitude of the acceleration changes with time (Newton *invented* calculus to do this!), so we will avoid such cases here.

The basic equation that describes the distance moved *along a particular direction* is

$$d = v_o t + (1/2)at^2. \tag{4-1}$$

$d$ = distance moved (in m), $v_o$ = initial velocity (in m/s), $a$ = acceleration (in $m/s^2$), $t$ = time (in s).

Here the subscript "o" corresponds to the initial value. (The distance moved $d$ could be expressed as $d$-$d_o$ and the time interval as $t$-$t_o$, but we will avoid this complication by defining $d_o$ and $t_o$ such that the initial position is at $d_o$=0 at time $t_o$=0.)

*Two-dimensional Trajectories*

Some explanation of what is meant by "along a particular direction" is required. Some cases are one-dimensional, *i.e.*, the motion is along a straight line. In that situation, the distance $d$, velocity $v$, and acceleration $a$ are all along the direction of the line. However, other cases are two-dimensional, for example, curved motion on a flat surface or the trajectory of an object thrown into the air at some angle to the ground. Still others require the use of all three spatial dimensions, for example, the motion of a flying bird. In these more complicated cases, we choose a set of directions that are all perpendicular to each other. Then equation (4-1) applies independently to each direction.

For example, the trajectory of an object shot or thrown into the air is generally two-dimensional, since the curved path lies in a single plane. The two directions in this case are horizontal and vertical. For horizontal motion, we define the forward direction as positive and backward as negative. For vertical motion, up is usually positive and down negative.

We then split equation (3) into two equations: one for distance moved in the horizontal direction – which we will refer to as direction $x$ – and the other for distance moved vertically – direction $y$:

$$d_x = v_{ox}t + (1/2)a_xt^2 \tag{4-1a}$$
$$d_y = v_{oy}t + (1/2)a_yt^2 \tag{4-1b}$$

$d_x$ = distance moved in the horizontal direction (in m), $v_{ox}$ = initial horizontal component of the velocity (in m/s), $a_x$ = horizontal component of the acceleration (in m/s$^2$), $t$ = time (in s, same for all directions); $d_y$, $v_{oy}$, and $a_y$ have similar meanings but refer to the vertical components.

Three-dimensional (3-D) motion can be treated in the same way, by adding yet another dimension, usually given the symbol $z$. Since all the principles involved are present in the simpler 2-D motion, we will not consider the 3-D case here. Note that time (symbol $t$, with preferred units of seconds, abbreviated "s") is the same for all directions. Such a quantity is termed a "scalar," whereas those quantities such as velocity and acceleration that have a direction are referred to as "vectors."

The second equation of interest gives the velocity after some time interval in terms of the initial velocity $v_o$, the acceleration $a$, and the time interval $t$:

$$v = v_o + at. \tag{4-2}$$

$v$ = velocity measured at time $t$ (in m/s), $v_o$ = initial velocity (in m/s), $a$ = acceleration (in m/s$^2$), $t$ = time (in s).

As above, this is valid for each direction that is perpendicular to the others:

$$v_x = v_{ox} + a_xt \tag{4-2a}$$
$$v_y = v_{oy} + a_yt \tag{4-2b}$$

$v_x$ = horizontal component of the velocity at time $t$ (in m/s), $v_{ox}$ = initial horizontal component of the velocity (in m/s), $a_x$ = horizontal component of the acceleration (in m/s$^2$), $t$ = time (in s, same for all directions); $v_y$, $v_{oy}$, and $a_y$ have similar meanings but refer to the vertical components.

There is a simple case for which we can use our intuition in addition to the equations. If there is no acceleration ($a$ = 0), then the speed must be constant; indeed, in this case eq. (4-2) gives $v = v_o$. Then the distance traveled is simply equal to the speed multiplied by the time interval; mathematically, eq. (4-1) with $a$=0 does in fact reduce to $d = v_o t$. Another important case is when the object starts from rest, which mathematically means that $v_o = 0$. The velocity is then $v=at$ and the distance traveled is $d = (1/2)at^2$.

*Example of a Two-dimensional Trajectory*

A good example of 2-D motion that illustrates the power of rational analysis provided by equations (4-1a,b) and (4-2a,b) is that of a ball thrown in a horizontal direction at an initial velocity $v_{ox}$ from a height $h$ above the ground. (If the direction is not horizontal, the approach still works, but the solution is slightly more complicated mathematically.) There are only two dimensions in this problem, horizontal in the forward direction ($x$) and vertical ($y$). Gravity pulls downward on the ball such that $a_y = -g = -10$ m/s$^2$. Here, $g$ is the magnitude of the acceleration of gravity on or near the surface of the Earth and we will neglect the effects of air resistance. (A more accurate value of the acceleration of gravity near the surface of the Earth is $g$ = 9.8 m/s$^2$, but for our purposes an error of 2% is too small to be of concern. In addition, calculations that use $g$ = 10 m/s$^2$ often can be performed easily without the use of a calculator.)

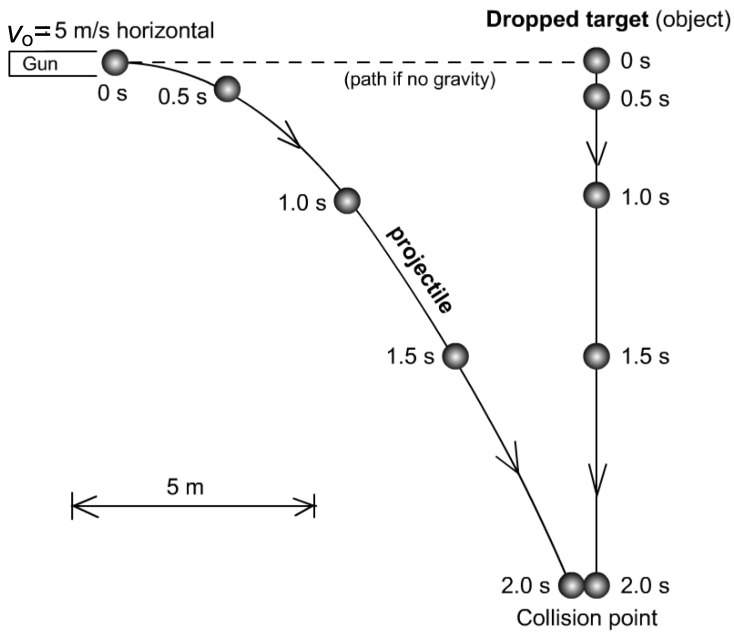


Figure 4-1. Example of the trajectory of an object thrown horizontally (*left side*) and a second object dropped from rest (*right side*). The time since each object was thrown or dropped is marked at each point shown. Note how the rate of descent is exactly the same for the two objects: only the horizontal motion is affected by an initial velocity in the horizontal direction.

The first question we need to answer is *how long will it take for the ball to hit the ground?* The second question is *what is the horizontal distance that the ball will travel before it lands?* We have two equations, (4-1a) and (4-1b). In (4-1a), we can set $a_x$=0 since there is no acceleration in the horizontal direction after the ball is released. However, in this equation we have two unknown quantities, $d_x$ and $t$, so we cannot yet solve it. In equation (4-1b), we can set the initial vertical speed $v_{oy}$=0 since the ball is thrown horizontally. We know both the acceleration $a_y = -10$ m/s$^2$ and the distance $d_y = -h$ that the ball will travel in the vertical direction before hitting the ground. So, the only unknown quantity is the time interval $t$, for which we can solve algebraically:

$$-h = 0 + \tfrac{1}{2}a_y t^2 \qquad \text{or} \qquad t^2 = -2h/a_y.$$

We can then take the square-root of both sides (recall that a mathematical action is legal in algebra if taken on both sides of an equation) to obtain the time interval as

$$t = \sqrt{\frac{-2h}{a_y}}$$

$t$ = time (in s), $h$ = height above the ground at the start (in m), $a_y$ = vertical component of the acceleration.

If we had specified the initial height to be 5 m, then the answer would be

$$t = \sqrt{\frac{-2(5\text{ m})}{-10\text{ m/s}^2}} = \sqrt{1\text{ s}^2} = = 1\text{s}.$$

Now that we have found $t$, it simply must be plugged into equation (4-1a) to obtain the horizontal distance traveled: $d_x = v_{ox}t$. If we had specified the initial horizontal speed as 15 m/s, then for $t$ =1 s the horizontal distance traveled would be 15 m. Note that the horizontal velocity does not change, since there is no acceleration in the horizontal direction.

Thus, splitting the problem into two halves, one that considers only horizontal quantities and the other only vertical, allows the solution of the entire problem. This implies that the time it takes for the ball to hit the ground does not depend on whether — or how fast — it is thrown horizontally (see Figure 4-1). In fact, non-intuitive as this result might seem, it is correct!

## Newton's Laws of Motion

Newton was particularly interested in determining mathematical expressions for the acceleration $a$ needed in the above equations. He recognized that acceleration is the result of a force applied to the object. He then formulated three principles that are now called *Newton's Laws of Motion*, which he developed around the year 1670.

1. *The law of inertia*: "Every body persists in its state of rest or of uniform motion unless it is compelled to change that state by forces impressed on it." This means that an object moving with constant velocity in one direction keeps doing so until acted upon by a force. If this velocity is zero, the body will stay at rest until a force acts on it. This tendency is called inertia.

2. The acceleration $a$ of an object of mass $m$ in response to an applied force of magnitude $F$ is given by the equation

$$a = F/m. \tag{4-3}$$

$a$ = acceleration (in m/s$^2$), $F$ = force (in Newtons, or N, where 1 N = 1 kg m/s$^2$), $m$ = mass (in kg).

(This is customarily expressed as $F$=$ma$, but here we are usually interested in determining the acceleration from the force and mass.) Newton's 2nd Law of Motion agrees with common sense: the stronger the force applied, the greater the acceleration that results. Also, the more massive an object is, the less it is accelerated by a given force. In fact, mass is a property that contributes to inertia, which can be defined as an object's resistance to changes in motion (where rest is a state of zero motion).

If more than one force is operating, then the force $F$ used in equation (4-3) is the net force. For example, a force of 2 N acting downward and another of 2 N acting upward would cancel each other, resulting in zero net force. As with equations (4-1a,b) and (4-2a,b), it is best to express the equation separately in the $x$ and $y$ (and, if three dimensions are appropriate, $z$) directions. Mass is a scalar quantity, so it is the same for all directions.

3. Every action has an equal and opposite reaction. This means that whenever a force is exerted by one object on another, the second object exerts an equal force on the first object in the opposite direction. Or, if part of an object is expelled in one direction, the rest of the object will move in the opposite direction. One example of this is the kick-back of a gun: as the bullet is ejected in the forward direction, the gun jerks backward.

When you weigh yourself on a scale that sits on the floor, the force of gravity pulls you down, while a spring inside the scale pushes back with an equal force, such that you do not move. The scale measures the force of the spring, which must equal your weight, which is the force of gravity on your body ($-mg$, where $m$ is your mass). In fact, when you bounce a ball off the ground, the ball bounces back and the Earth moves downward with an equal force. But, since the Earth is so much more massive (mass $M_E$ = $6.0 \times 10^{24}$ kg) than the ball, its change in motion $a=F/M$ is negligible (although non-zero).

Another example that illustrates Newton's 3rd Law of Motion is a rocket. Many people think that a rocket works by pushing against the air. If this were the case, a rocket would not work in space, yet the motions of many satellites and space probes have been changed by their rockets. Instead, the principle involved is one aspect of "equal and opposite reaction." The controlled explosion of the fuel ejects a stream of exhaust gases out the rear of the engine. This must be balanced by the rest of the rocket or spacecraft moving in the opposite, or forward direction. An untied blown-up balloon becomes a good example of a rocket when released.

*Momentum: A Conserved Quantity*

Quantitatively, the principle behind rockets and similar phenomena can be explained by the concept of conservation of momentum, which was introduced after Newton's time. Momentum (symbol $p$, preferred units of kg m/s) is a measurement of the inertia of a moving object and is simply the mass times the velocity:

$$p = mv. \tag{4-4}$$

$p$ = momentum (in kg m/s), $m$ = mass (in kg), $v$ = velocity (in m/s).

If there is no net external force acting on an object or set of interacting objects, **momentum is conserved**:

$$p = p_o$$

or

$$mv = m_o v_o \tag{4-5}$$

$p$ = momentum (in kg m/s), $p_o$ = initial momentum (in kg m/s), $m$ = mass (in kg), $v$ = velocity (in m/s), $v_o$ = initial velocity (in m/s).

The sum of the momenta of all objects at the start is equal to the sum of the momenta of all objects at some later time. (There may be more or fewer objects then.)

In the case of a rocket initially at rest in outer space, the initial momentum is zero. The net momentum must remain zero if no outside forces are in effect. So, the backward (negative) momentum of the exhaust once the fuel is ignited must equal the forward (positive) momentum of the rocket. Consider, for example, a rocket with a mass (without fuel) of $m$=1,000 kg. Add and ignite 10 kg of fuel such that it shoots backward out the nozzle at a speed of 10,000 m/s. The momentum of the fuel is then

$$p(\text{fuel}) = (10 \text{ kg})(-10{,}000 \text{ m/s}) = -100{,}000 \text{ kg m/s}.$$

The final momentum of the rocket must be equal and opposite, or +100,000 kg m/s. So, we have $p=mv$, or

$$v = p/m = (100{,}000 \text{ kg m/s})/(1000 \text{ kg}) = 100 \text{ m/s}.$$

The rocket will therefore be propelled forward at a velocity of 100 m/s.

## Newton's Law of Universal Gravitation

In 1665-66, at his family's farm, Newton contemplated the possible similarity between the motion of the Moon about the Earth and the falling of an apple from its tree. He eventually published his results in 1687. In this work, Newton presented his Laws of Motion as well as his Law of Universal Gravitation. In words, the latter states that there is an attraction between any two bodies, with a force proportional to the product of the two masses divided by the square of the distances between their centers. Here the term "center" means "center of mass," which is a sort of mass-averaged center, but for spheres such as stars and planets, the two terms are essentially the same. If we define the mass of the heavier and lighter object to be $M$ and $m$, respectively, and assign the symbol $r$ to the distance between their centers, then Newton's Law of Gravitation can be described by the equation

$$F = -\frac{GMm}{r^2} \tag{4-6}$$

$F$ = force (in N, equivalent to kg m/s$^2$), $G$ = gravitational constant = 6.67x10$^{-11}$ m$^3$/(kg s$^2$) or N m$^2$/kg$^2$, $M$ = mass of the heavier object (in kg), $m$ = mass of lighter object (in kg), $r$ = distance between the centers of the two objects (in m).

Because of the dependence of the force on one divided by the square of the distance, equation (4-6) is termed an "inverse square law." The small value of $G$ means that gravity is a weak force unless at least one of the masses is very large. On the other hand, it operates over very long distances since the gravitational forces of different masses add rather than cancel, as electric and magnetic forces can do.

The negative sign in equation (4-6) means that the force is attractive. This is an arbitrary convention that an attractive force is negative and a repulsive force is positive. It has the advantage that, when an acceleration is computed on the surface of a massive object, such as the Earth, the acceleration is negative, which we have already defined to be downward.

Newton inserted the word "universal" into his title of the law of gravitation because he considered the force to exist throughout the universe — on the surface of the Earth as well as in space. This was an extremely important concept at the time: motions and other phenomena in the heavens are subject to the same laws as are more mundane events on the Earth.

How does gravity operate? The mathematics implies that it might simply cause "action at a distance." That is, the mere presence of a massive object causes other objects to accelerate because of the gravitational force, even across empty space. Many scientists of Newton's time criticized this concept as "occult" (Gottfried Leibniz) or "absurd" (Christiaan Huygens). Even Newton was troubled by this and admitted that he had no suitable solution for how gravity is transmitted. A more philosophically pleasing explanation of gravity had to await Einstein's ideas in the early 20th century, which is covered in Chapter 9.

*Newton's Laws and Falling Bodies*

We can combine Newton's $2^{nd}$ law of motion and law of gravitation to explain Galileo's experimental result that two objects of different masses fall at the same rate when dropped to the ground. Since the $2^{nd}$ law describes the acceleration of an object when a force is applied and the law of gravitation gives that force in terms of physical quantities, we can combine equations (4-3) and (4-6) to derive the equation

$$a = \frac{F}{m} = -\frac{GMm}{mr^2} = -\frac{GM}{r^2} \tag{4-7}$$

> $a$ =acceleration (in m/s$^2$), $F$ = force (in N, equivalent to kg m/s$^2$), $G$ = gravitational constant = $6.67\times10^{-11}$ m$^3$/(kg s$^2$) or N m$^2$/kg$^2$, $M$ = mass of the heavier object (in kg), $m$ = mass of lighter object (in kg), $r$ = distance between the centers of the two objects (in m).

In the case of the Earth, $a = -9.8$ m/s$^2 \approx -10$ m/s$^2$.

Note that the mass of the object divides out, so that the only mass in the final equation is that of the more massive object — the Earth in Galileo's experiments. This is why Galileo found that the rate of descent of a falling body does not depend on its mass $m$. It does, though, depend on the mass $M$ and the radius $r$ of the planet onto whose surface it is falling.

*The Equivalence Principle*

It is important to notice one assumption in the derivation of eq. (4-7): in order to be able to divide out the mass $m$, we must equate the inertial mass of Newton's Laws of Motion with the gravitational mass of Newton's Law of Universal Gravitation. In other words, we have assigned the same symbol, $m$, and the same name, mass, to two different concepts, one relating to an object's resistance to changes in motion and the other controlling how strong a gravitational force it causes. The assumption that these two physical traits correspond to the same physical property termed "mass" is called the Equivalence Principle. Although experiments have thus far confirmed its validity to within the accuracy of the measurements, there is as yet no accepted, more general theory that explains why it should be true.

*Is Newton's Law of Universal Gravitation Beautiful?*

Recall that we should expect the best scientific descriptions of nature to possess beauty. In particular, they should be what scientists call "elegant": simple and symmetric. Does Newton's Law of Universal Gravitation pass this test? Well, it is in some sense simple: the equation is relatively straightforward and it applies to all masses in the universe. And it is indeed symmetric: the force is felt equally by both bodies in question. The fact that gravity is directed toward the center of an object causes very massive bodies to be spherical, which is the most symmetric geometrical shape in three dimensions. So, it wins the beauty contest!

## Application of Newton's Laws to Planetary Motions

Newton's Laws finally fulfilled the great goal of the Renaissance and early Enlightenment scientists: to explain the motions of the planets and moons in the solar system. Since Kepler had already shown that the orbits were ellipses, the law of gravitation requires that the forces – and hence accelerations – change as a planet's distance from the Sun varies. The mathematics of such changing accelerations requires the use of calculus, and indeed Newton employed the calculus he invented to obtain a mathematical description of elliptical orbits. Even without the use of calculus, though, it is still possible to describe the special case of circular orbits and show that Kepler's 3*rd* Law is explained by Newton's Laws. In order to do so, we must use the mathematics of circular motion.

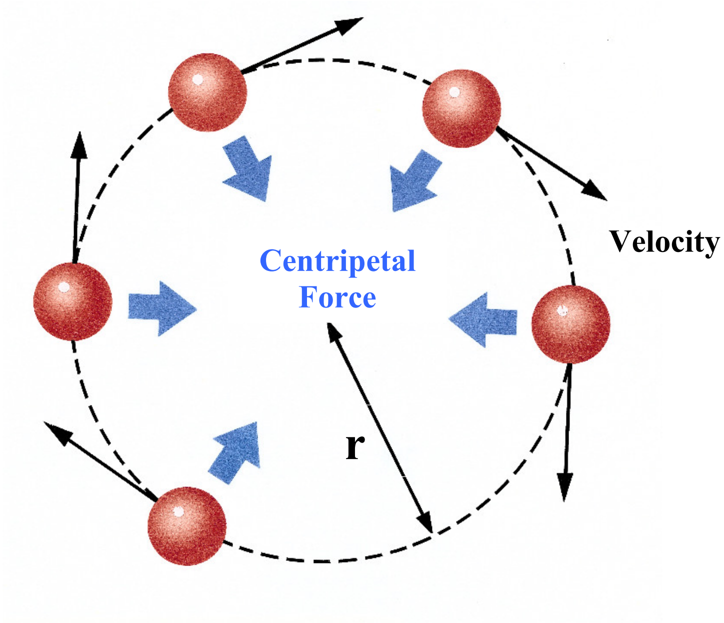


Figure 4-2. In order to keep an object in circular motion, a centripetal force (thick arrows) must continuously act toward the center of the circle. At any given instant, the velocity (thin arrows) is directed along the tangent to the circle.

As Figure 4-2 illustrates, a force is required to keep an object moving in a circle. This centripetal force must act toward the center of the circle. The equation that relates the force to the mass of the object, its velocity, and the radius of the circle, is

$$F_c = -\frac{mv^2}{r} \tag{4-8}$$

$F_c$ = centripetal force (force directed toward the center of the circle of motion, in N or kg m/s$^2$), $m$ = mass in kg, $v$ = velocity (in m/s), $r$ = radius of the circle of motion (in m); the negative sign means that the force is inward, toward the center.

For an orbit, the centripetal force is supplied by gravity, the equation for which is (4-6). If we set the right-hand sides of both equations equal to each other, we can then derive the period of the orbit in terms of other physical quantities. As a first step, we derive an algebraic expression for the orbital speed $v$:

$$-\frac{mv^2}{r} = -\frac{GMm}{r^2} \quad \rightarrow \quad -\frac{v^2}{r} = -\frac{GM}{r^2} \quad \rightarrow \quad v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}} \tag{4-9}$$

$v$ = velocity (in m/s), $G$ = gravitational constant = $6.67\times10^{-11}$ m$^3$/(kg s$^2$) or N m$^2$/kg$^2$, $M$ = mass of the heavier object (in kg), $r$ = radius of the circle of motion (in m).

Equation (4-9) gives the formula for the velocity of an object moving in a circular orbit.

The period ***P***, the time it takes for the planet to complete one orbit, is the circumference of the orbit ($2\pi r$) divided by the speed $v$:

$$P = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi\sqrt{r^3}}{\sqrt{GM}}$$

or, casting it into the form of Kepler's 3$^{rd}$ Law by squaring both sides,

$$P^2 = \frac{4\pi^2 r^3}{GM}.$$

We now not only have the proportionality expressed by Kepler's 3$^{rd}$ Law, we also have the constant of proportionality. If we had used calculus to derive the expression appropriate for elliptical orbits, the only change would be to replace the radius of the orbit $r$ by its semi-major axis $A$:

$$P^2 = \frac{4\pi^2}{GM} A^3. \tag{4-10}$$

$P$ = period (in s), $G$ = gravitational constant = $6.67 \times 10^{-11}$ m$^3$/(kg s$^2$) or N m$^2$/kg$^2$, $M$ = mass of the heavier object (the Sun in the case of a planet's orbit, in kg), $A$ = semi-major axis of the orbit (in m).

Equation (4-10) is a very powerful expression, since it contains (1) the period $P$, which is easily measured through observations, (2) the semi-major axis $A$, which is also measurable, although with greater difficulty since we must determine the distance from the object to the Sun, and (3) the mass $M$, which would be extremely difficult even to estimate roughly if it were not for this formula. The mass of the Sun can be determined by measuring the periods and semi-major axes of the orbits of the planets, while the masses of the planets can be determined by measuring the periods and semi-major axes of their moons.

For planets without moons and for the moons themselves, the mass is much more difficult to determine. In the past the masses of Venus and Mercury were estimated by their small gravitational influences on the Earth. During the past several decades, we have determined the masses of moons and other small bodies in the solar system by their gravitational influence on space probes sent from Earth. Even the masses of other stars besides the Sun can be determined through equation (4-10), since many of these are in binary (or multiple) star systems, so we can measure the periods and semi-major axes of the orbits of the two stars through observations spread over time. However, in the case of binary star systems, the mathematics is slightly more complex because both masses are comparable, as opposed to the solar system case of one object having much more mass than the others.

## Energy

A useful concept is that of energy, which is a common term but one that is difficult to define in words. Among many other virtues of this concept, expressing motions in terms of energy allows us to solve some problems with changing accelerations without the use of calculus. The units of energy are "joules," abbreviated "J," where 1 J = 1 kg m$^2$/s$^2$. This is a scalar quantity, so direction is not important, except – in the cases of some forms of energy – toward or away from the source of a force. Energy of motion, kinetic energy, has the formula

$$E_{\text{kin}} = \frac{1}{2} m v^2 \tag{4-11}$$

$E_{\text{kin}}$ = kinetic energy (in J, equivalent to kg m$^2$/s$^2$), $m$ is the mass (in kg), $v$ = speed (in km/s).

The 2nd type of energy is potential energy, which is the force $F$ times the distance $d$ (in the direction in which the force acts) from some reference point: $E_{pot} = (\pm)\ Fd$. (The sign used depends on the choice of reference point.) For example, when considering the potential energy of gravity for an object that might eventually fall to the ground, the ground is usually the best choice for the reference point, and $d$ then equals the height $h$ above the ground. Since the force of gravity on an object of mass $m$ at the surface of the Earth is $F = -mg$, the potential energy when the object is positioned at a height $h$ above the ground is

$$E_{pot} = -Fd = mgh. \qquad \text{(for object at height } h \text{ above the ground)} \qquad (4\text{-}12)$$

$E_{pot}$ = potential energy (in J, equivalent to kg $m^2/s^2$), $F$ = force (in N, equivalent to kg $m/s^2$), $d$ = distance (in m), $m$ = mass (in kg), $g$ = acceleration of gravity (in $m/s^2$, = 10 $m/s^2$ for objects near the surface of the Earth), $h$ = height above the ground or other reference point (in m).

The main reason why energy is such a useful concept is that, like momentum, energy is conserved. That is, the sum of the energy at some initial time is the same as the sum at a later time, plus/minus the energy added/ subtracted by some external agent between the two times. The energy added or subtracted is called work. Work is the force applied by an external agent – for example, air resistance – times the distance that the object is moved along the direction of that force. For example, imagine that the object is a book weighing 0.5 kg and that is initially on the ground. If you lift it and place it on a table that is 1 m from the ground, the initial potential energy is 0 and the final potential energy is

$E_{pot,f} = mgh = (0.5 \text{ kg})(10 \text{ m/s}^2)(1 \text{ m}) = 5 \text{ kg m}^2/\text{ s}^2 = 5 \text{ J}.$

Since the book is at rest at the beginning and end, its initial and final kinetic energy are both zero. And the initial potential energy is zero since the book was on the ground. You therefore must have expended 5 J of work into lifting the book in order to change its potential energy to +5 J.

*Escape Velocity*

An important quantity that can be derived from Newton's Laws using this energy formulation is the escape velocity $v_{esc}$. This is the speed, in a direction away from a massive body, that an object must have in order (eventually) to escape the gravitational influence of that body. Since gravity is a very long-range force, this implies, mathematically, that the object should be able to move to an infinite distance from the massive body. In practical terms, the escape velocity is then the initial speed at an initial distance $r$ that results in a final speed 0 at a final distance that is much, much greater than $r$.

The potential energy at a distance $r$ is

$$E_{pot} = -\text{GMm}/r \qquad \text{(for object at distance } r \text{ from center of massive body)} \qquad (4\text{-}13)$$

$E_{pot}$ = potential energy of object with mass $m$ at position $r$ [using as a reference point $r = \infty$ where the potential energy = 0] (in J, equivalent to kg $m^2/s^2$), $G$ = gravitational constant = $6.67 \times 10^{-11}$ $m^3/(kg\ s^2)$ or N $m^2/kg^2$, $M$ = mass of the heavier body (in kg), $m$ = mass of object whose potential energy is being calculated, $r$ = distance between the centers of the object and the more massive body.

This is the initial potential energy of the object. In order to escape the influence of the massive body, $r$ must increase to a very large value, which is essentially infinity. So, to calculate the final potential energy, we set $r = \infty$ in this equation, with the result $E_{pot,f} = 0$. The initial kinetic energy is $E_{kin,o} = \frac{1}{2}mv^2_{esc}$, while the final kinetic energy is zero, since the escape velocity is the lowest initial speed that will allow the body to escape. This means that it will eventually coast to a stop at a great distance from its starting point. We assume that no external forces are acting, so no work is done between the beginning and end. Conservation of energy then gives us:

$$E_{\text{kin},0} + E_{\text{pot},0} = E_{\text{kin,f}} + E_{\text{pot,f}}$$

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{r} = 0 \quad \rightarrow \quad \frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{r} \quad \rightarrow \quad v_{\text{esc}}^2 = \frac{2GM}{r}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}} \tag{4-14}$$

$v_{\text{esc}}$ = escape velocity (in m/s), $G$ = gravitational constant = $6.67 \times 10^{-11}$ $m^3/(kg\ s^2)$ or N $m^2/kg^2$, $M$ = mass of the heavier body (in kg), $r$ = distance between the centers of the object and heavier body.

We can calculate the escape velocity from the surface of the Earth, for which $M = 6.0 \times 10^{24}$ kg and $r$ is the radius of the Earth, $6.378 \times 10^6$ m: $v_{\text{esc}}$ (Earth) = 11,200 m/s = 11.2 km/s (upward direction), or about 7 miles per second. This is a very fast velocity by everyday standards.

As one would expect intuitively, the escape velocity increases as the mass of the massive body increases and as the distance from its center decreases. These effects result in a stronger initial gravitational force or, equivalently, a more deeply negative potential energy.

Negative potential energy corresponding to an attractive force is often referred to as binding energy, since it is a property of objects that remain bound by such a force. This requires that their speeds remain lower than the escape velocity.

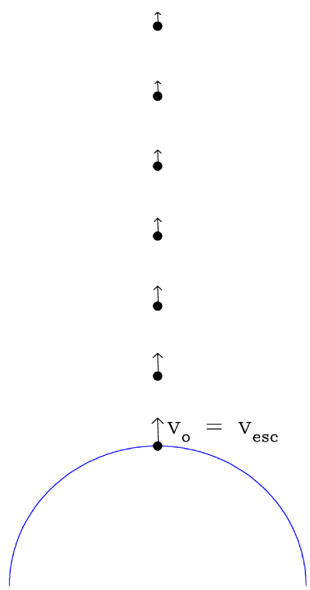


Fig. 4-3. Change in speed (indicated by the lengths of the arrows) of an object given an initial upward velocity equal to the escape velocity from the surface of a massive body. The gravity of the massive body slows the motion, but not to zero until a very great distance is reached, so the object eventually escapes the gravitational pull of the body. If the initial velocity were lower, the object would slow down, stop, and then fall back to the surface.

## General Characteristics of Orbits

As Kepler stated and Newton derived mathematically, all orbits of objects around a single high-mass body are ellipses, with the high-mass body at one of the two foci. This, of course, refers only to *bound* orbits, *i.e.*, those in which the velocity is always less than the escape velocity (eq. 4-14). If the speed ever exceeds the escape velocity, the object will slow down, but not enough to allow it to fall back toward the massive body.

We can think about orbits in the following way (refer to Fig. 4-4):
1. The velocity given by equation 4-9 is the “just right” speed to maintain a circular orbit, in which the force of gravity and the inertia of the object are always in balance. A velocity higher or lower than this

circular velocity causes the gravitational force and inertia of the object to be out of balance, and the orbit will be a non-circular ellipse.

2. If the speed is less than the circular velocity at some position, then the object will move toward the massive body, accelerating to a higher velocity as it does. It is, in a sense, “falling” toward the massive body. But the velocity is in a direction that is *not* straight toward the massive body. This causes the object to “miss” the massive body and orbit around it instead of hitting it. Once it reaches the position of closest approach, the velocity is so high that inertia carries the object away from the massive body. Beyond this point the velocity decreases, just as a ball thrown upward from the surface of the Earth slows down as it rises. Eventually, the object’s inertia will be too low to counteract the gravity of the massive body. The object will then head back inward along an elliptical path.

3. If the velocity at the original point is faster than the circular velocity, then the object will move away from the massive body, decelerating as it does. It will eventually slow down so much that its inertia will be too low to counteract the gravity of the massive body. The object will then head back inward along an elliptical path, after which the description is similar to that of item (2).

A circular orbit is therefore a perfect equilibrium between gravity and inertia. In an elliptical orbit, the two are out of equilibrium, with gravity being stronger at the farthest distance in the orbit and inertia being stronger at the closest position to the massive body. Figure 4-4 shows the magnitudes and directions of the velocity (representing inertia) and the gravitational force at various points of an elliptical and a circular orbit.

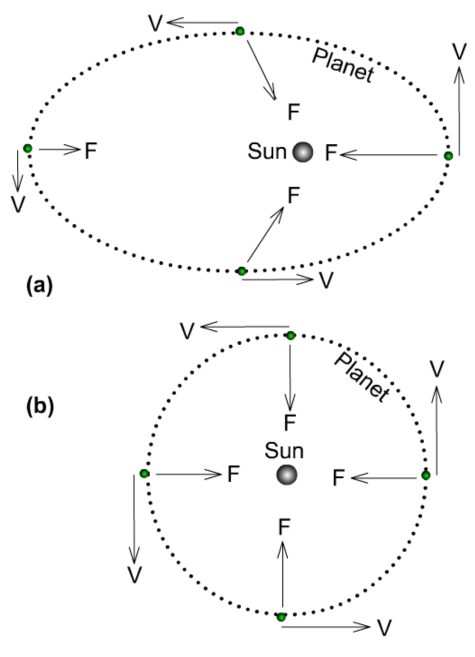


Figure 4-4. An elliptical (*top*) and circular (*bottom*) orbit. The arrows point in the directions of the forces (*F*) and velocities (*v*), and their lengths indicate their strengths or speeds.

## Philosophical Implications of Newton’s Laws: Determinism

Newton’s formulation of the laws of motion and gravity have had a profound and lasting effect on intellectual thought. The predictions of the equations were found to agree quite accurately with experiments and observations, and the mathematics is elegant. For this reason, during the Enlightenment a feeling of confidence developed in the ability of humans to formulate the underlying principles of nature.

This impression that the world is mechanistic led to the philosophy of determinism: all events, all actions, and perhaps even all thoughts in the universe are the inevitable consequences of the initial state of the universe combined with the natural laws that govern the universe. The astronomer-mathematician Pierre Simon-Laplace expressed this concept in clear terms. Here we paraphrase: **Imagine that we could measure the mass, position, velocity, and any other property that affects motion, of every piece of matter in the universe at one time. Then the past and future state of the universe and everything in it could – in principle – be calculated from the mathematical formulation of all the forces and other natural laws of the universe.**

The attraction of such a philosophy was compelling for many people, especially the Enlightenment humanists who believed that everything occurs naturally, meaning that there is no need to invoke divine intervention. Indeed, physical motions of objects seem to follow this principle since such motions apparently can be explained solely through natural laws. However, the implications of extending this to everything in the universe, including human behavior, are difficult for many people to accept. Strict determinism requires that no deity interfere with the universe, and implies that sentient creatures such as humans are simply mechanical beings controlled by the universe. All choices are made according to the laws of nature, so that free choice is merely an illusion of a complex mind. Determinism is one of the most provocative concepts in humanity's attempts to understand the underlying nature of the universe and of humans themselves. Even today, there is no generally accepted resolution to the issues that it raises.

## Chaos

We are actually saved from the concept of a predictable deterministic world by a mathematical concept called chaos. If a system of objects and their interactions is complex enough – for example, more than three massive bodies in a system of stars and planets – small changes in the positions or velocities can result in great departures in the future behavior. This is often expressed as the "Butterfly Effect." Since the weather is an example of a system subject to chaos, a butterfly flapping its wings in California could affect the atmosphere enough to cause a tornado at a particular time and location in Kansas. Mathematically, this is called sensitivity to initial conditions. A set of initial values of variables that is slightly different from another set can result in completely different outcomes in a chaotic system.

The implication is that unless we can measure the conditions of a complex system exactly at one time, we cannot predict its detailed behavior at a much later time even if the system is governed exactly by a known set of physical principles. This applies to the orbits of asteroids and moons in our solar system, and it should apply as well to the behavior of people even if they have no free will. While the behavior of a system may be predictable over a certain time period whose duration depends on the complexity of the system, all but the simplest systems are eventually subject to chaos.

But can we, at least in principle, measure all the positions, velocities, and forces exactly? The answer is "no." On very tiny size scales, Heisenberg's uncertainty principle applies: the position and velocity of a particle – a tiny piece of matter – cannot both be known precisely (see Ch. 7). Therefore, while it is possible that we are machines without true free will, we at least are not completely predictable machines!

## Summary

The Enlightenment period saw the development of a new theory of motion by Kepler, Galileo, and Newton. Newton's three laws of motion and Law of Universal Gravitation explained the laws of orbits discovered by Kepler. They also provided the formulation by which future positions and velocities of objects could be predicted given their masses as well as their positions and velocities at a specific time. This is valid for motions in space as well as on the Earth. The same equations are still in wide use, although we now know that they are inaccurate under extreme conditions, such as near extremely compact, very massive objects or when the speed approaches that of light. In cases in which the motion is complex, such as two-dimensional trajectories, rational analysis – in which the problem is separated into sub-problems that are each solved and then combined into an overall solution – is often very useful. This approach is a very important tool of the modern scientist.

An important concept introduced in this chapter is conservation, which means that the value of certain key quantities does not change unless some external force acts on the system. Two conserved quantities are momentum – the product of mass and velocity – and energy. Energy has various aspects; two of these are kinetic energy corresponding to motions – which includes the heat energy of microscopic motions – and stored potential energy. The energy added or subtracted by an external force is called work. Energy can be converted from one form to another. An application of conservation of momentum is the acceleration of a rocket forward while the exhaust is propelled backward. Examples of conservation of energy include the increase in speed of a falling object and the back-and-forth motion of a pendulum.

The formulation of the principles that govern motions led to the promotion of the philosophy of determinism, in which the universe is considered to be completely mechanistic. This concept seems to leave no room for the free will that most humans sense they possess. However, even small variations in the initial conditions of a complex system can produce wildly different behavior at a later time because of chaos. The initial conditions cannot be known well enough to predict all future behavior. So, if humans are machines, we at least are not completely predictable machines.

## Glossary

Rational analysis (or reductionism): The technique of solving a problem by dividing it into simpler parts, solving each part, and then synthesizing the solution to the original problem.

Velocity (symbol: $v$): The speed and direction of motion, *i.e.*, the change in distance per unit time. Measured in units of meters per second (m/s).

Acceleration (symbol: $a$): The change in velocity per unit time (including any change in direction). Measured in units of meters per second per second ($m/s^2$).

Force (symbol: $F$): Agent that causes the acceleration of objects on which it acts. Direction in which the force acts is important. Measured in units of Newtons (N).

Inertia: The resistance of a body to changes in its state of motion (or of rest). Inertia is a concept, not a physical quantity. It is related to mass and momentum.

Momentum (symbol: $p$): The mass times the velocity (including direction), $p=mv$. Measured in units of kg m/s. Momentum is a conserved quantity in a system.

Gravity: Force of attraction between two objects. Proportional to the product of the two masses divided by the square of the distance between the centers of the objects.

Mass (symbol: $m$ or $M$): Fundamental property of an object that determines its level of inertia as well as its force of gravity. Measured in units of kg.

Equivalence Principle: Assumption that the gravitational mass equals the inertial mass. This needs to be valid for the acceleration from gravity to be independent of the mass of the falling body.

Vector: A directional quantity. Examples: velocity, acceleration, force.

Scalar: A quantity that does not depend on direction. Examples: mass, time, energy.

Energy (symbol: $E$): A scalar property of an object that relates to its state of motion, potential motion, or strength of bonding by an attractive force. Measured in units of J (Joules, equivalent to kg $m^2/s^2$).

Kinetic energy (symbol: $E_{kin}$): Energy of motion, $E_{kin} = (½)mv^2$.

Potential energy (symbol: $E_{pot}$): A stored form of energy. If positive, it is a measure of the amount of other forms of energy that a body can attain if the stored energy is released. If negative, it is a binding energy.

Binding energy: Negative potential energy corresponding to the (equal and opposite) energy that must be added to release a body from an attractive force.

Work: Change in energy of a body when it is moved while under the influence of a force. Equal to force times distance moved.

Centripetal force: Force of attraction that keeps a body in orbital or curved motion, equal to $mv^2/r$. For circular motion, $r$ is the radius of the circle.

Determinism: The philosophical concept that actions in the universe are specified by the initial positions and velocities of all the bodies in the universe and the forces that act upon them.

Chaos: A characteristic of complex systems of objects and their interactions: small differences in initial conditions lead to major differences in behavior after a sufficiently long time.

## Questions for Discussion

A. Give some examples of things and phenomena that can be understood completely through a reductionistic approach. What cannot be understood at all through reductionism? Give some examples that can be understood partly through reductionism but also require an integrated approach to understand fully.

B. Compare the word “inertia” as used in physics with the more casual meaning used in everyday language. Do the same for the words “momentum” and “energy” as defined in this chapter.

C. Why is it not obvious that the mass that determines the force of gravity should be the same as the mass that determines a body’s resistance to changes in motion (inertia)?

D. If you are in a sailboat on a lake and the wind completely dies, can you propel your boat forward by blowing on the sail? What principle is involved?

E. Is the concept of "action at a distance" an unsatisfactory explanation for the transmission of a force such as gravity? What characteristics would a "satisfactory" explanation have?

F. Does unpredictability mean that a system is non-deterministic? Consider an experiment, *e.g.*, the motion of a feather dropped into the air, in which you can repeat the initial condition (the exact position of the feather and the direction its flat part faces) only within some uncertainty (*e.g.*, 0.1 mm and 1°, respectively). Since the motion of the feather is chaotic, the position of the feather at a later time cannot be predicted. Is this system non-deterministic? Would the motion of the feather have been predictable had you known the exact initial conditions of it and the air? Would the motion of the feather be the same if you could repeat its falling with the exact same initial conditions, including the state of the air?

## Examples of How to Solve Problems in the Physics of Motion

1. A car starts at rest and speeds up with an acceleration of 5 $m/s^2$.
a. How long does it take to reach a forward velocity of 50 m/s?

Answer: Use equation (4-2):

$$v = v_o + at$$

and solve algebraically for $t$. Here, $v_o$=0 since the car starts at rest.

$$at = v - v_o$$
$$t = (v - v_o)/a$$
$$= (50\ \mathrm{m/s} - 0\ \mathrm{m/s})/(5\ \mathrm{m/s^2})$$
$$\underline{t = 10\ \mathrm{s}}$$

b. How far does the car travel before reaching a forward velocity of 50 m/s?

Answer: Use equation (4-1):

$$d = v_o t + (½)at^2$$

and solve for $d$ using the value of $t$ found in part (a) above (10 s):

$$d = (0\ \mathrm{m/s})(10\ \mathrm{s}) + ½\ (5\ \mathrm{m/s^2})(10\ \mathrm{s})^2$$
$$= 0\ \mathrm{m} + ½\ (5\ \mathrm{m/s^2})(100\ \mathrm{s^2})$$
$$d = \underline{250\ \mathrm{m}}$$

2. A diver drops from the top of a sheer cliff into the ocean 20 m below.
a. How long does it take for the diver to hit the water? Neglect air resistance.

Answer: To determine the time, we deal only with the vertical direction. Use equation (4-1b):

$$d_y = v_{oy}t + ½a_y t^2$$

and solve for $t$. We know that $v_o$= 0 m/s and $a_y = -g = -10\ \mathrm{m/s^2}$. The equation becomes

$$d_y = (0)t + ½a_y t^2 = ½a_y t^2$$
$$t^2 = 2d_y/a_y$$
$$t = \sqrt{\frac{2d_y}{a_y}} = \sqrt{\frac{2(-20\ \mathrm{m})}{-10\ \mathrm{m/s^2}}} = \sqrt{4\ \mathrm{s}^2} = 2\ s.$$

Notice that both $d_y$ and $a_y$ are negative, since they are directed downward.

b. If the diver takes a running (horizontal) leap at 5 m/s, how far will he be from the bottom of the cliff (measured in the horizontal direction) when he hits the water? Neglect air resistance.

Answer: Use equation (4-1a):

$$d_x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

and solve for $d_x$ using the value $t = 2$ s derived in part (a), $v_{ox} = 5$ m/s (given), and $a_x = 0$ m/s$^2$ (no horizontal acceleration):

$$d_x = (5 \text{ m/s})(2 \text{ s}) + \tfrac{1}{2}(0 \text{ m/s}^2)(2 \text{ s})^2 = \underline{10 \text{ m}}$$

3. You drop a ball from a height of 5 m. What is its speed the instant before it hits the ground? Ignore air resistance.

Answer: This can be answered by first calculating the time as in part (a) of sample problem (2) and then, using the calculated value of $t$, by the method of part (a) of sample problem (1). However, a more direct method is to use conservation of energy:

$$E_{\text{kin,0}} + E_{\text{pot,0}} = E_{\text{kin,f}} + E_{\text{pot,f}}$$

$$\frac{1}{2}mv_{\text{o}}^2 + mgh = \frac{1}{2}mv_{\text{f}}^2 + mg(0) = \frac{1}{2}mv_{\text{f}}^2$$

Since $m$ appears in every term, we can divide by $m$ on both sides to obtain

$$\frac{1}{2}v_{\text{o}}^2 + gh = \frac{1}{2}v_{\text{f}}^2$$

The initial velocity $v_o = 0$ since the ball is dropped from rest. The equation then becomes

$$gh = \frac{1}{2}v_{\text{f}}^2$$

This is solved algebraically for $v_f$ to obtain

$$v_{\text{f}}^2 = 2gh \quad \text{or} \quad v_{\text{f}} = \sqrt{2gh} = \sqrt{2(10 \text{ m/s}^2)(5 \text{ m})} = \sqrt{100 \text{ m}^2/\text{s}^2} = 10\ m/s..$$

4. An ice skater of mass 50 kg stands still while holding a 10 kg weight. She then throws the weight horizontally in the forward direction at a speed of 10.0 m/s. How fast and in what direction does she move after throwing the weight, assuming that there is no friction between her skate blades and the ice and negligible air resistance?

Answer: This can be answered by applying conservation of momentum, which means that the sum of $mv$ of all things at the end equals the sum at the beginning:

$p = p_o$

$m$(skater) $v$(skater) + $m$(weight) $v$(weight) = $m$(skater+weight) $v_o$(skater+weight)

But $v_o$=0, so the entire right-hand side equals 0. We then have

$m$(skater) $v$(skater) = – $m$(weight) $v$(weight)

$v$(skater) = – $m$(weight) $v$(weight)/$m$(skater)

$v$(skater) = – (10 kg)(10.0 m/s)/(50 kg) = – <u>2.0 m/s</u>.

The negative sign means that she moves backward.

5. The Moon has a mass of $7.35\times10^{22}$ kg and a radius of $1.74\times10^{6}$ m.
a. What is the acceleration of gravity near the surface of the Moon?

Answer: We use eq. (4-7) and plug in the numbers:

$$a = -\frac{GM}{r^2} = -\frac{[6.67\times10^{-11}\ \mathrm{m^3/(kg\ s^2)}](7.35\times10^{22}\ \mathrm{kg})}{(1.74\times10^6)^2} = \underline{-1.62\ \mathrm{m/s^2}}.$$

This is about 1/6 times the acceleration of gravity on the Earth.

b. What is the escape velocity from the surface of the Moon in km/s?

Answer: We use eq. (4-14) and plug in the numbers:

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2[6.67\times10^{-11}\ \mathrm{m^3/(kg\ s^2)}](7.35\times10^{22}\ \mathrm{kg})}{1.74\times10^6}} = \underline{2370\ \mathrm{m/s}} = \underline{2.37\ \mathrm{km/s}}.$$

6. Calculate the period of the circular orbit of a satellite that has an altitude of 200.0 km. The radius of the Earth is 6378 km and its mass is $6.0\times10^{24}$ kg.

Answer: We use eq. (4-10) but first we need to add the Earth's radius and altitude to get the semi-major axis (= radius here) of the orbit, then convert km to m: $A$ = 200.0 km + 6378 km = 6578 km (1000 m/km) = $6.578\times10^{6}$ m.

$$P^2 = \frac{4\pi^2}{GM}A^3 = \frac{4\pi^2}{[6.67\times10^{-11}\ \mathrm{m^3/(kg\ s^2)}](6.0\times10^{24}\ \mathrm{kg})}(6.578\times10^6\ \mathrm{m})^3 = 2.8\times10^7\ \mathrm{s^2}$$

$$P = \sqrt{P^2} = \sqrt{2.8\times10^7\ \mathrm{s^2}} = \underline{5300\ \mathrm{s}} = \underline{88\ \mathrm{min}} = \underline{1.47\ \mathrm{hr}}.$$

(Unless asked, you do not need to convert seconds to minutes or hours. But here the conversion shows that the period of a "low-earth" orbit – meaning at an altitude much less than the Earth's radius – is close to 1.5 hours. This is true for most satellites that have been placed into orbit.)

## Homework Problems

**Note:** You should use the value $g = 10\ \text{m/s}^2$ for the magnitude of the gravitational acceleration near the surface of the Earth. You should solve each problem algebraically (with the symbol of the item you wish to obtain on the left-hand side of the equation) before plugging in the numbers. **You must show your step-by-step work to get full credit.** Show each step for all mathematical calculations and explain your reasoning when using logic and/or inspection of the equations to determine your answer. **If you use a calculator, round off the answer that appears on your calculator to 2 or 3 digits unless the statement of the problem uses more**.

1. John wishes to visit his parents in a city that is a distance of 600 km from his home. He travels in a train that moves at an average velocity of 150 km/hr. How long will the trip take him?

2. Mary decides to try out auto racing. Her car, which has a mass of 600 kg (including Mary's mass), accelerates uniformly from a speed of zero to 200 m/s in 10 s. What force in Newtons (N) must be exerted on the car (by the reaction of the road to the turning of the wheels) in order to accomplish this?

3. Professor Jones is traveling down the highway at a speed of 100 m/s when a moose runs onto the road 210 m ahead of the car. Loving animals, not to mention her car, she applies the brakes immediately, stopping with a uniform acceleration (deceleration) of $-25\ \text{m/s}^2$.

  a. How long does it take for the car to stop?
  b. How far does the car travel in this time?
  c. Does it stop before hitting the moose?

4. Professor Smith decides to try out skydiving from a horizontally moving airplane that is at an altitude of 2500 m. He knows that he must pull on the cord that releases his parachute when he reaches an altitude of 500 m (after this his speed will be too great when he hits the ground and before this he will drift into the ocean). So, he uses a stopwatch to determine how much time he has been falling and rolls himself into a ball so that air resistance can be neglected.

  a. How far will he fall before he needs to pull the cord?
  b. How many seconds must elapse (as indicated by his stopwatch) before he should pull on the cord? (Assume that he drops from the plane with an initial vertical velocity of zero.)
  c. What will be his vertical velocity the instant before he pulls the cord?

5. An astronaut floating at rest in space outside a spaceship shoves, directly away from the spaceship, a satellite with a mass 5 times her own mass at a speed of 1.0 m/s. What is the velocity (speed and direction) of the astronaut immediately after this?

6. A typical egg will break if it hits the floor with a downward velocity faster than $-0.5$ m/s. What is the maximum height from which it can be dropped from rest without breaking it? Neglect air resistance. (Warning: Do not try this at home unless you want to make scrambled eggs: Not all eggs are typical!)

7. The dwarf planet Eris has a mass of $1.67\times10^{22}$ kg and a radius of $1.16\times10^{6}$ m.

  a. Calculate the acceleration of gravity near the surface of Eris.
  b. Calculate the escape velocity from the surface of Eris.

8. The dwarf planet Ceres has a mass of $9.39 \times 10^{20}$ kg and a radius of $4.73 \times 10^{5}$ m.

a. Calculate the acceleration of gravity near the surface of Ceres.
b. Calculate the escape velocity from the surface of Ceres.

9. A daredevil rides his/her bicycle off a cliff. She wants to land on a plateau on the other side of a canyon that is a horizontal distance of 180 m from the cliff and a vertical distance of 45 m (see the figure below). What is the minimum horizontal speed with which he/she must take off the cliff in order to land on the plateau?

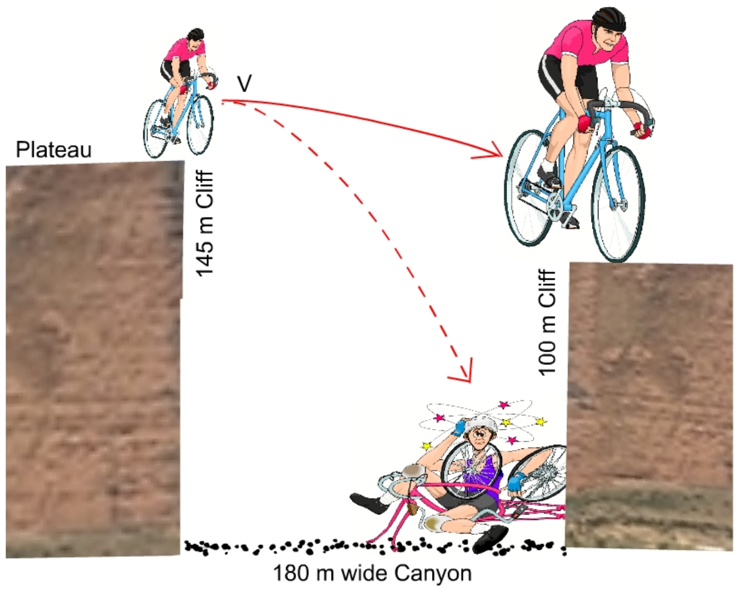


Figure for Problem 8.

10. The Moon has a mass of $7.35 \times 10^{22}$ kg and a radius of $1.74 \times 10^{6}$ m. Calculate the orbital period of a spacecraft that executes a circular orbit around the Moon at an altitude of 1000 km (= $1.00 \times 10^{6}$ m).

11. An asteroid and a comet, originally in circular orbits in opposite directions around the Sun with radius equal to 2.5 AU, collide head-on, which stops their motion. They stick together to form a single body, which then proceeds to fall directly toward the Sun.

a. What is the speed of the asteroid/comet body when it hits the Sun? The Sun's mass is $2.0 \times 10^{30}$ kg and its radius is $7.0 \times 10^{8}$ m, 1 AU = $1.5 \times 10^{11}$ m, and G = $6.67 \times 10^{-11}$ $m^3/(kg\ s^2)$. [*Hint:* Use conservation of energy, with eq. 4-13 to calculate the potential energy.]

b. Use Kepler's 3$^{rd}$ Law (from the previous chapter) to determine approximately how long (in years) it takes for the meteor/comet body to hit the Sun. [Hint: a line is also an extreme ellipse (minor axis = 0), with the length of the line equal to 2 times the semi-major axis of the ellipse. As an approximation for this part, treat the Sun as a point, which is fairly accurate since its radius is much smaller than the initial distance of the asteroid and comet from the Sun.]