Chapter 9: The Fabric of the Universe 2: Relativity and the Nature of Space-Time

At about the same time that Rutherford was exploring the atom, the German physicist Albert Einstein was staging another scientific revolution that would change dramatically our understanding of the nature of the universe on cosmic as well as tiny scales. Einstein started by comparing measurements made by different observers studying the same event from different reference frames, perspectives like that of an observer in a spaceship with windows. These reference frames might be in motion relative to each other or relative to the event being observed. The dependence of measurements on the relative motion of the observer and the event being observed is called relativity.

One example of relative motion in everyday life is the observation that the Sun, Moon, and stars all appear to rise in the east and set in the west on a daily basis. This results from the rotation of the Earth relative to the stars. Normally, we are attached to the surface of the Earth, moving only a small fraction of its circumference during a typical day. The Earth, however, spins such that a point at the latitude of Boston (42°), for example, moves in a circle at 1244 km/hr. Furthermore, the Earth orbits the Sun at a speed of about 30 km/s (note the time unit!). Our sense that the point at which we are located is stationary is therefore misleading. It is good to keep this in mind when the results of Einstein's Theory of Relativity seem to violate our intuition, just as the concept of a moving Earth did to most ancient Greek scientists.

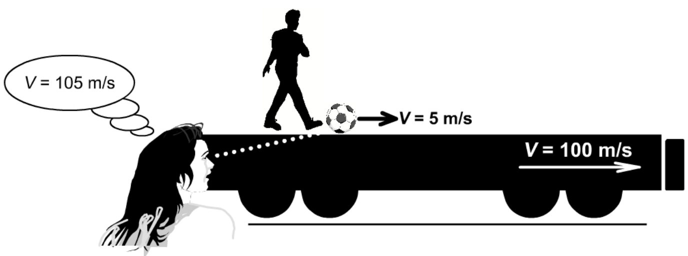

Fig. 9-1. A passenger kicks a ball forward on a moving flatbed train car. The passenger sees the ball move at the velocity at which he kicked it, v=5 m/s in the example shown. An observer standing still on the ground measures the speed of the ball to be the speed of the train, V=100 m/s in this example, plus the speed v at which the passenger kicked the ball. A different formula would be needed if one or more of the speeds involved were close to the speed of light.

Humans do have some sense of relativity in ordinary life, as when riding a train. A ball kicked forward by a passenger is measured by an observer standing outside on the ground to have a velocity equal to that of the train plus the velocity at which the passenger kicked the ball. Before Einstein proposed his theory of Special Relativity in 1905, this sort of relativity was well understood and considered to be the full story. However, since motions of reference frames in everyday life are much, much less than the speed of light c, we cannot assume that our understanding of relativity at low velocities will be valid when the relative motions are close to c.

The Constant Speed of Light

What if the velocities involved are not much less than the speed of light? Then we must take into account the fact that the train moves during the time that the ball's change in position (or any other property) is measured. In order to figure out what the observer will measure, we need to know two things: the speed of light and whether this speed depends on the motion of the source of light.

Is the speed of light constant? Actually, it is lower if the light is propagating through a medium rather than through a vacuum. But a more interesting question is whether the speed of light depends on the motion of the source of the light. Light has a wave nature, and waves might not be expected to behave in the same manner as solid objects like balls.

In a number of experiments between 1880 and 1920, U.S. physicist Albert Michelson measured the speed of light passing through a vacuum to be $c = 3.00\text{x}10^8$ m/s, the value that we adopt in this book. In 1887, he and Edward Morley used a clever experiment to show that there is no dependence of the speed of light on direction despite the Earth's orbital velocity of 30 km/s around the Sun. (Recall from Chapter 5 that the speed of sound also does not depend on the source of the sound waves, only on the properties of the medium through which the sound travels.) In 1905, Einstein used algebra and clear thinking to determine the consequences of this: the nature of space and time is quite different from our everyday notions when motions occur near the speed of light.

Special Relativity: How Motion Affects Space and Time

In 1905, Einstein proposed a new way of thinking about physical phenomena that takes into account that light takes time to reach its destination. He interpreted the Michelson-Morley experiment to imply that the speed of light is the same for all observers, independent of their motion or the motion of the source of light. He then proceeded to develop the theory of Special Relativity that applies to physical measurements in systems — reference frames — moving at constant velocities. Einstein reasoned that the results of an experiment performed by an observer in such a non-accelerating reference frame should not depend on the motion of that frame relative to any other non-accelerating frame. In other words, there is no "preferred" or absolute reference frame. Rather, all non-rotating, non-accelerating frames are equivalent, and phenomena within each such frame all appear "normal." Imagine, for example, that one person is in a spaceship traveling at a uniform velocity of $0.99c$ relative to another person's planet. Within the spaceship, measurements of length, time, and motion will be normal. Measurements are also normal within the reference frame of the person on the planet.

But what if a person in one reference frame measures a phenomenon that takes place in another frame whose motion is near the speed of light? Einstein found that measurements of such fundamental quantities as length and time are affected! In order to understand how this occurs, consider that we measure phenomena by observing the light emitted or reflected by the objects that are involved. If the motions are near the speed of light, the distance between an object and the observer changes significantly during the time interval within which the measurement takes place. This causes the interval between the times when the light — and therefore information — leaves the object and when it reaches the observer to keep changing while the measurement is in progress. Because of this, a time interval measured in the reference frame of the moving object is not the same as the time interval measured by the observer. Since time and distance (or length) are related through the velocity (see Ch. 4), the difference in the measured time interval implies a difference in the measurement of distance (or length). These effects are non-intuitive because we do not encounter objects moving near the speed of light in everyday life. We therefore need to rely on mathematical equations that we can derive using basic algebra and trigonometry.

Perhaps the most astonishing result of Special Relativity is that <u>the rate at which time passes is relative</u>. Imagine that, standing on a stationary planet, you observe an event that occurs in a passing spaceship that moves near the speed of light. To be specific, a friend on the spaceship throws a ball against a wall and catches it. Let's say that this event takes 2 s in the friend's reference frame. How long does it take in your frame? The answer is longer than 2 s! Time (as measured by an outside observer) passes more slowly the faster the platform on which an event occurs moves relative to the observer. The word "event" has a very liberal meaning here: it can be the return of a bounced ball or the beating of a heart. In the example above, your friend's heart would actually beat more slowly than normal as measured by you.

Box 9-1 derives the precise relationship between (1) the time t of events as measured in the observer's frame and (2) the time t_{rest} of the same events as measured in another frame moving relative to the observer's frame:

$$t = \Gamma\, t_{rest} \tag{9-1}$$

t = time measured by the observer, t_{rest} = time measured in the moving frame, Γ = Lorentz factor. The quantity Γ, the Lorentz factor, equals one for zero velocity (v=0) and is always greater than one for moving objects (see Fig. 9-3). Equation (9-1) therefore indicates that the observer measures events to take a longer time. This shatters our notion of time being an absolute quantity that passes in the same, steady way throughout the universe. We discuss this matter further below.

Box 9-1. The passage of time in a moving reference frame

The effect of relative motion on time is actually straightforward to calculate. One way of measuring a time interval is to determine how long it takes for something moving at a known speed to travel a known distance. Light is good to use since its speed does not depend on the relative motion between the source and the observer. Imagine that, as in Figure 9-2, we have a spaceship with a floor-to-ceiling height H and that a flash of light is directed vertically from the floor to the ceiling.

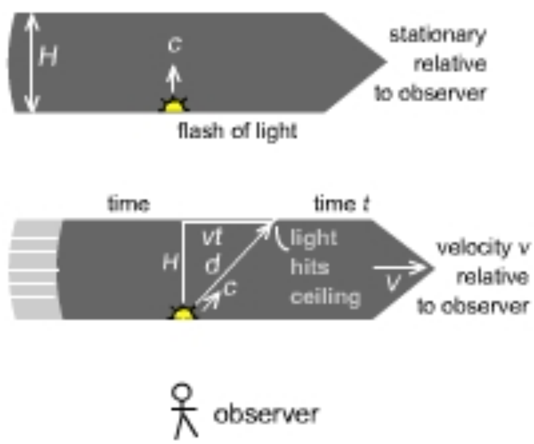

Fig. 9-2. Time on a spaceship as measured by an outside observer. In the top frame, the spaceship is stationary and the time it takes for a pulse of light to travel from the floor to the ceiling at height H is $t_{rest} = H/c$. In the bottom frame, the spaceship moves relative to the observer at a horizontal speed v. The time it takes for the pulse of light to travel from floor to ceiling is now longer, $t = d/c$.

1. As measured by a person inside the spaceship (or, more generally, at rest relative to the spaceship), the light will reach the ceiling in a time $t_{rest} = H/c$.

2. Now consider the case when the spaceship is moving at a horizontal speed v relative to the observer. The light still travels a distance H in the vertical direction, but also a distance vt in the horizontal direction, with t measured in the observer's frame. Since the speed of light has not changed, the time it takes is equal to the distance that the light has traveled from the floor to the ceiling divided by the speed of light, $t = d/c$. Using trigonometry, we know that this distance $d = \sqrt{H^2 + (vt)^2}$. The time interval is $t = d/c$, so we can replace d on the left-hand side by ct. Since we want to obtain an expression for t, we must square both sides to get $c^2t^2 = H^2 + v^2t^2$, or $c^2t^2 - v^2t^2 = H^2$. We can factor out a c^2t^2 on the left-hand side to get $c^2[1-(v^2/c^2)]\, t^2 = H^2$. Solving this for t and then taking the square-root of both sides, we get $t = \frac{H}{\sqrt{1-(v^2/c^2)}}$. Since $H = ct_{rest}$, $t = \frac{t_{rest}}{\sqrt{1-(v^2/c^2)}} = \Gamma t_{rest}$, which is the same as equation (1), where we have defined the Lorentz factor $\Gamma = \frac{1}{\sqrt{1-(v^2/c^2)}}$.

Equation 9-1 suggests that the Lorentz factor Γ represents the importance of the effects of Special Relativity. When the velocity v is less than about 20% of the speed of light, $v < 0.2c$, the Lorentz factor is very close to one. For example, at $v = 0.2c$, $\Gamma = 1.02$. However, as the velocity approaches the speed of light, Γ increases: At $v = 0.9c$, $\Gamma = 2.3$, at $v = 0.98c$, $\Gamma = 5.0$, and at $v = 0.995c$, $\Gamma = 10.0$. Figure 9-3 presents a graph of Γ as a function of v. The fact that the Lorentz factor is so close to one at low velocities explains the reason why we have no everyday sense of Special Relativity: motions of objects in our common experience are so slow relative to light that our world appears to obey the "classical" equations of Newton and others. If we were to live instead on spaceships that regularly zip through the Galaxy at a speed exceeding 98% of the speed of light, we would be accustomed to the effects that relative motion has on time and other measurable quantities.

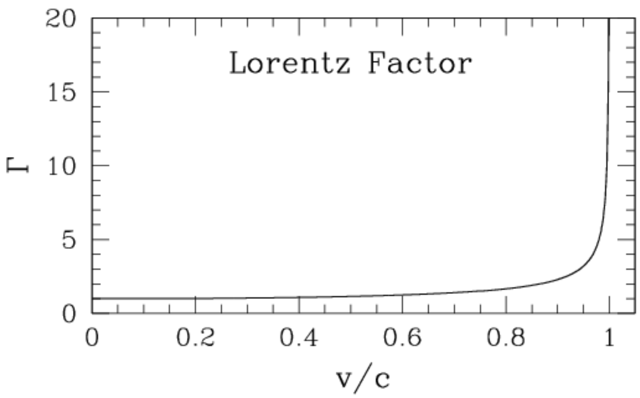

Fig. 9-3. The value of the Lorentz factor Γ as a function of velocity v divided by the speed of light c. It becomes infinitely large as v approaches c.

The equation for the Lorentz factor (see the bottom of Box 9-1) contains a singularity, which is a mathematical point at which something special happens: Γ becomes infinite if the velocity equals the speed of light ($v = c$). In addition, the term in the square-root becomes negative if the velocity exceeds c. Einstein realized that this implies that <u>no velocity in the universe can exceed the speed of light.</u> In fact, <u>no information</u> can propagate faster than the speed of light. While some scientists have proposed that particles called "tachyons" might exist that always travel at speeds greater than c, this concept remains completely hypothetical, with no solid evidence to support it.

Since the measurement of time is affected by relative motion, we should expect the same for measurements of length. Indeed, as shown in Box 9-2, the length L of an object in a moving frame is measured to be shorter than the length L_{rest} when it is not moving. Distances are similarly contracted if measured from a frame that is moving. The equation that expresses this is

$$L = L_{rest}/\Gamma. \tag{9-2}$$

L = length (or distance) in the direction of motion; no subscript: as measured by the observer;
L_{rest}: as measured in a reference frame in which the objects is at rest; Γ = Lorentz factor.
The effect applies only to the dimension in the same or opposite direction of the motion, however. Lengths and distances in directions perpendicular to that of the motion are the same in both frames.

Consider the example of a space traveler whose vessel moves at a speed of $0.98c$; the Lorentz factor is then $\Gamma = 5.0$. The distances to stars in the forward and reverse directions that are measured by the space traveler will be 5.0 times shorter than would be the case if the spaceship were stationary relative to the stars. In the direction perpendicular to the motion, on the other hand, the distances would be the same as they would be if the spaceship were stationary relative to the stars.

Recall that in everyday relativity, at speeds much less than c, the observed velocity of a ball thrown forward with velocity v in a frame moving at velocity V is simply $v+V$. This cannot be true when both v

and V are close to c, since then the sum would be greater than c. Einstein derived a formula for the addition of velocities that is valid even for speeds close to that of light. In the case in which both velocities are in the same direction, the formula for the total velocity measured by the observer, relative to whom the frame moves with velocity V, is

$$v_{\text{obs}} = \frac{v+V}{1+(Vv/c^2)}. \tag{9-3}$$

The value of v_{obs} never exceeds the speed of light.

Box 9-2. The effect of relative motion on measurements of length and distance

Imagine that you have a spaceship that has length L_{rest} when it is not moving. You can determine L_{rest} by standing at the rear and sending a pulse of light that reflects off a mirror at the front, then measuring the time t_{rest} it takes for the reflected pulse to return to the rear (see Figure 9-4). This must equal $t_{\text{rest}}=2L_{\text{rest}}/c$.

The situation is more complicated for an observer relative to whom the spaceship is in motion. Imagine that part of the pulse of light, as well as part of the reflection, is directed to the outside observer so that it can be detected. By the time the light pulse arrives at the position where the front of the spaceship was located earlier — when the light from the initial pulse at the rear was emitted — the spaceship has moved and the front is no longer there. The time t_1 that it takes for the light pulse to arrive at the front in the outside observer's frame is given by the equation $t_1 = L/c + vt_1/c$. Here L is the length of the spaceship as measured by the outside observer and vt_1 is the distance that the front of the spaceship advances during the time interval t_1 that it takes for the light pulse to travel from the rear to the front. If we solve the above equation for t_1 (note that it appears on both the right and left sides of the equation), we get $t_1 = L/(c–v)$.

After reflection off the mirror in the front, it takes a time interval t_2 for the light pulse to reach the rear, which by that time will have advanced by a distance vt_2. The time t_2 is therefore given by the equation $t_2 = L/c – vt_2/c$, the solution to which is $t_2 = L /(c+v)$. The total time for the light to return to the rear is

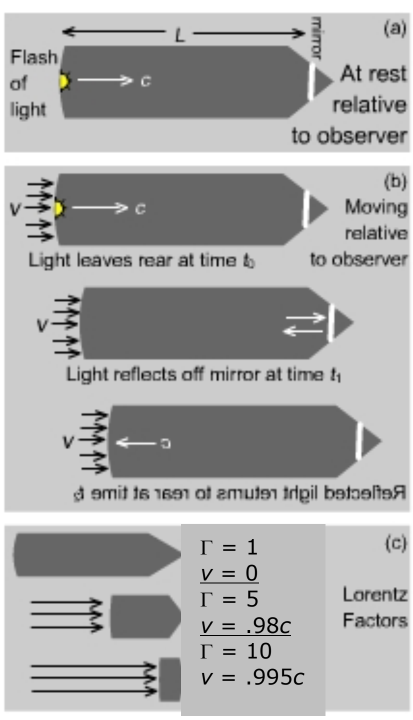

Fig. 9-4. Measurement of the length of a spaceship by an outside observer. *Top:* The spaceship is stationary and the time it takes for pulse of light to travel the distance L_{rest} from rear (left) to front (right), or vice-versa, is $t_{\text{rest}} = L/c$. *Middle:* The spaceship moves relative to the observer at horizontal speed v. The time for light to travel from rear to front is $t_1=L/(v+c)$ and from front to rear is $t_2=L/(v–c)$. The difference in time intervals corresponds to the observer measuring the length of the moving spaceship to be shorter than if $v=0$. *Bottom:* Appearance of spaceship for different values of the Lorentz factor Γ.

$$t = t_1 + t_2 = L\left(\frac{1}{c-v} + \frac{1}{c+v}\right) = L\left(\frac{c+v}{(c+v)(c-v)} + \frac{c-v}{(c+v)(c-v)}\right) = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c[1-(v/c)^2} = 2\Gamma^2 L/c\ .$$

Here we have made use of the mathematical definition of Γ (see Box 9-1).

We note that, from equation (9-1), (a) $t = \Gamma t_{\text{rest}}$, and from above, (b) $t_{\text{rest}} = 2L_{\text{rest}}/c$, so that we have (c) $t = 2\Gamma L_{\text{rest}}/c$. We have already shown that (d) $t = 2\Gamma^2 L/c$. We can set expressions (d) and (c) equal to each other: $2\Gamma^2 L/c = 2\Gamma L_{\text{rest}}/c$. If we multiply both sides by the speed of light c, and divide both sides by $2\Gamma^2$, we arrive at the final formula: $L = L_{\text{rest}}/\Gamma$, which is equation (9-2).

Note that the slowing down of time ("time dilation") and the shortening of lengths ("length contraction") in a frame moving with respect to the observer are real effects, not merely illusions or mathematical wizardry. For example, some unstable particles created in high-energy physics experiments exist for a tiny fraction of a second if created with low velocities. They then proceed to decay into lighter particles. However, the closer their initial velocities are to the speed of light, the longer they exist, as predicted in equation (9-1). While the length of an object returns to its rest value once it stops moving, its age will still be less than if it had always remained stationary. You might think that this is crazy and should lead to contradictions. Box 9-3 discusses how the apparent paradoxes are resolved.

Box 9-3. Einstein's thought experiments to clarify Relativity

At first glance, there seems to be some inconsistency in Special Relativity, since it should be symmetric. Consider an observer, whom we'll call "Speedy," on a spaceship moving at a velocity v near the speed of light relative to another observer, "Zero." If all uniform motion is relative, Speedy should measure Zero's velocity to be $-v$, *i.e.*, equal in magnitude to v but in the opposite direction. In that case, we would expect Speedy to measure lengths of things at rest in Zero's frame (e.g., a stick that Zero is holding) to be foreshortened along the direction of motion, and vice versa. So both should appear to be much thinner than normal as seen by the other. How can Zero measure lengths in Speedy's frame to be shorter while Speedy measures lengths in Zero's frame also to be less? Furthermore, how could they each measure the other's time to pass more slowly? This would seem to lead to contradictions.

Einstein explored how such issues are resolved through *gedanken* (thought) experiments, as he liked to call them. The first, dealing with time, is the "Twin Paradox." Imagine that you have a twin on the Earth and that you travel in a spaceship at a velocity of $0.98c$ to a star system that is 9.8 light-years (abbreviated "lt-yr") from the Earth. You then immediately turn around and return to the Earth. (Recall that a light-year is the distance traveled by light in one year, equal to 9.5 trillion km. The nearest star outside our solar system is about 4 lt-yr away.) In the frame of your twin on the Earth, the round-trip will take you 20 years. However, your twin will measure your time to pass 5 times more slowly than on the Earth, since the Lorentz factor for v=$0.98c$ is Γ=5.0. So, you will have aged 5 times less than this, or only 4 years. Your twin will now be 16 years older than you!

But, why don't you measure your twin's time to pass 5 times more slowly than your time as well? The reason is that there is an asymmetry: your velocity is not constant during the trip. You must first *accelerate* from a speed of zero to v, when you reach the star system you must *decelerate* back to zero, then *accelerate* in the other direction to go home, and finally you must *decelerate* again to zero when you return to the Earth. While each reference frame that moves at a constant velocity is equivalent to any other similar reference frame, accelerating reference frames are different.

Another aspect of this situation is how you measure the passage of time in another frame. The most straightforward is to observe an electronic clock in that frame. For example, the clock could send a pulse of radio waves every second as measured by a clock on the spaceship. In this case, the arrivals of the pulses at the Earth will be affected by the fact that the spaceship becomes more distant from the Earth as it approaches the star system and closer to the Earth as it returns. The rate at which the pulses are received is in fact a frequency (pulses per second, or Hz) and therefore is subject to the Doppler effect. Einstein derived a formula for the special-relativistic Doppler effect (compare with eq. 5-2):

$$\lambda_{\text{obs}} = \Gamma\left(1 + \frac{v}{c}\right)\lambda_0 \tag{9-4a}$$

or, in terms of frequency,

$$f_{\text{obs}} = \frac{f_0}{\Gamma(1+\frac{v}{c})} \tag{9-4b}$$

λ_{obs} = wavelength (in nm) and f_{obs} = frequency (in Hz) measured by the observer, λ_0 = wavelength (in nm) and f_0 = frequency (in Hz) measured at rest relative to the source of the light, Γ = Lorentz factor (no units) and v = velocity (in m/s) of the moving reference frame, c = speed of light = 3.0×10^8 m/s.

Recall from Chapter 5 that the sign of v is positive for motion away from the observer and negative for motion toward the observer. According to equation (9-3), the pulses therefore arrive faster, which means that they have a higher frequency, during the return trip than during the first half. The Γ factor in the denominator, on the other hand, is the same during the whole trip because it does not depend on the direction of the motion. Because of this factor, the faster rate of arrival during the return trip does not completely offset the slower rate during the first half of the trip. This means that, in the final analysis, the time from the beginning to the end of the trip will have passed more slowly — it will be $1/\Gamma$ times shorter — on the spaceship than on the Earth.

The second thought experiment is referred to as the "Pole-vaulter's Paradox" (see Fig. 9-5). Consider an unrealistically fast pole-vaulter with a 5-meter pole running down a field at a speed of $0.98c$ so that the Lorentz factor $\Gamma = 5.0$. To the spectators sitting on the side of the field, her pole is 5.0 times shorter, or 1 m. Now imagine that the pole-vaulter needs to run through a small building of length 2 m in the rest frame of the spectators, with two gates that open and close very quickly. The gates, initially open, close when the pole-vaulter is in the middle of the building, and then open again simultaneously as seen in the frame of the spectators. This appears normal to the spectators, except of course for the vaulter's unbelievably high speed and the shorter-than-normal pole.

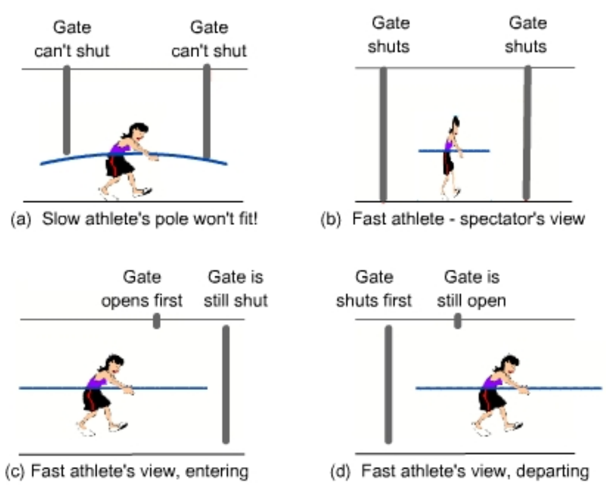

Fig. 9-5. The "Pole-vaulter's Paradox." (a) If the runner were too slow, the pole would be too long to fit inside the gates. (b) If the runner's speed is high enough, the stationary spectators see the pole (and the vaulter!) to have shorter sizes in the direction of motion than when measured at rest. Both the vaulter and the pole can fit in the building even with the gates closed. (c) & (d) The same event as viewed in the pole-vaulter's frame. The pole is too long to fit into the (length-contracted) building, but the second gate is no longer closed at the same time as the first gate. Rather, it closes and re-opens first. After the pole clears the front door, that door closes and re-opens. This thought experiment illustrates that events simultaneous in one reference frame are not necessarily simultaneous in another.

But, in the pole-vaulter's frame, her pole is still 5 m long, yet the length of the building is only 2/5.0 = 0.4 m. How then does she see her pole fit into the building? The answer is that, in her frame, the gates do not close and re-open simultaneously. The light from the second gate's closing reaches her first, so she sees that gate close and open before the front of her pole reaches it, then looks behind her and sees the first gate close (after the rear of her pole is inside the building) and then open again. In this way, the length contraction is indeed symmetric — the pole-vaulter sees very thin spectators and they see an equally thin pole-vaulter. Events that are simultaneous in one reference frame are not necessarily so in another.

One component of the relativity of simultaneity is the finite speed of light: it takes time for light to travel to us to convey the information needed to make a measurement. In everyday life, we all agree as to which events are simultaneous because the speed of light is so much faster than any other motions involved. This is often not true if we detect an event from the sound it gives off. For example, a cannon fired next to you will be heard before a cannon fired at the same time from a distance of 1 km. Similarly, astronomers might observe two stars explode in the sky, one at a distance of 1000 lt-yr and the other at a distance of 2000 lt-yr. In order to observe these two events simultaneously at the Earth, the more distant star would need actually to have exploded 1000 years earlier than the closer one. But this is just apparent simultaneity caused by the locations of the events and the finite speed of light. For motions near the speed of light, the effect on space and time described by Relativity also comes into play.

Although simultaneity of events is relative, time reversal of causally linked events is not possible according to Special Relativity. "Causally linked" means that one event causes the other to happen. For example, in no reference frame should you be able to see a fire before the match that starts it is lit.

Space Travel

It is fun to think about space travel at speeds near that of light. If you could build a spaceship that traveled at v=0.995c (Lorentz factor Γ=10), it would take you only one year according to your measurement of time to travel a distance (as measured from Earth) of 9.95 lt-yr. In your frame, all distances to things in the outside world in the forward or reverse direction would be 10 times shorter than when you were stationary relative to the Earth. So, despite the fact that the bright regions of our Galaxy extend over about 60,000 lt-yr, you could travel to the other side and back within your lifetime if you could attain a speed so close to c that the Lorentz factor was 3000 or higher.

But your trip would come at a price: your friends would all be long dead by the time you returned, and the Earth might have changed too much for you to bear. Furthermore, the cost in energy would be enormous, since the energy of your spaceship and its cargo would be Γ times its rest value (see eq. 9-5 below). An equivalent amount of energy — and therefore an enormous amount of fuel — would be necessary to accelerate the spaceship to such a high velocity. And even small objects that your spaceship hit would have a major impact, making the flight extremely dangerous.

The Four Dimensions of Space-Time

In the mathematical formulas of Special Relativity, the product of the speed of light and time, ct, appears in the same way as do the three dimensions of space. The publication of Einstein's papers on Special Relativity led Hermann Minkowski to consider time as representing another dimension, equivalent to the three dimensions of space. These four dimensions of the universe are together called space-time.

Mass and Energy

Einstein's formula that relates mass and energy, $E = mc^2$, only includes the energy contained in the mass when the object is stationary relative to the observer. The full equation, valid whether the object is moving or not, is

$$E = \Gamma mc^2. \qquad (9\text{-}5)$$

E = energy (rest-mass + kinetic, in J), m = mass (in kg), c = speed of light = 3.0×10^8 m/s.
This consists of two parts: the rest-mass energy, mc^2, and the kinetic energy, $(\Gamma-1)mc^2$. [Note: if the velocity v is much less than c, the value of $(\Gamma-1)$ is very close to $\frac{1}{2}(v/c)^2$, so that the classical expression for kinetic energy, $\frac{1}{2}mv^2$, is recovered.]

An important implication of equation (9-5) is that no object with mass can reach the speed of light. The value of the Lorentz factor at such a speed would be infinity (see Figure 9-3 and the mathematical definition of Γ in Box 9-1). This means that it would take an infinite amount of energy to accelerate an object to the speed of light. Only massless particles like photons can travel that fast.

General Relativity: How Mass Affects Space-Time

Although Einstein's theory of Special Relativity initially met with considerable skepticism among physicists, over the next several years more and more of them realized that it provides a description of natural phenomena that is both elegant and accurate. Einstein, meanwhile, was busy extending the theory to include accelerating reference frames and gravity. It took ten years of development using high-level mathematics. Finally, in 1915 Einstein completed what he called the General Theory of Relativity. Here we summarize the foundation of General Relativity and its most important implications.

Recall (see Ch. 3) Galileo's experiments showing that objects of different mass fall with the same rate of acceleration. Newton's equations can explain Galileo's result, but only if the inertial mass m in his equation $a = F/m$ is the same as the gravitational mass m in his equation $F(\text{gravity}) = -GMm/r^2$. The assumption that this is true is called the Equivalence Principle. Einstein adopted a modified version of this and used it as the basic premise upon which he constructed General Relativity. The Equivalence Principle that he assumed states that **experiments performed in a laboratory cannot determine the difference between acceleration caused by gravity and that caused by other forces**. This also implies that the results of experiments performed by an observer in free-fall in a gravitational field are the same as those performed by an observer in a gravity-free, non-accelerating environment.

After working through the mathematical consequences of these assumptions, Einstein found that space-time can be curved. It does not need to be "flat" as had been supposed. When space is flat, the shortest distance between two points — a "geodesic" — is a straight line. In curved space-time, it is not.

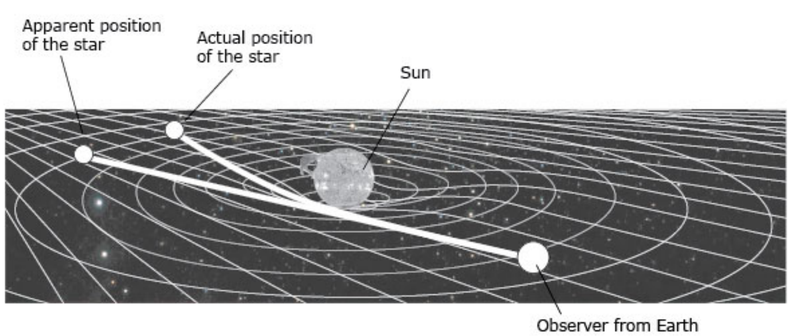

Fig. 9-6. The geometry of space-time and bending of the path of light as it passes a massive, compact object such as the Sun. Since the four dimensions of space-time cannot be represented adequately in a two-dimensional figure such as this, the drawing is meant only to show the sense of the curvature. [Source: www.answersingenesis.org]

According to General Relativity, space-time is most curved close to a massive, compact object such as a star. The geometry, shown in Figure 9-6, is similar to a hole with sloping sides. Since light is expected to follow geodesic paths, the predicted bending of space-time by a star suggested a possible test to the theory. Light traveling past a massive object should follow a curved path. The light should therefore appear to an observer on the other side of the massive object to come from a different direction than it would in the absence of the massive object. This implies that the positions of stars near the Sun in the sky during a solar eclipse — when the sky is dark enough to see stars through a telescope — should appear to shift slightly compared with their usual positions.

In 1919, astronomer Arthur Eddington organized an expedition to observe a solar eclipse and found that the positions of the stars near the Sun were indeed shifted by the amount predicted by Einstein. The publicity surrounding this event made Einstein a celebrity. More modern tests of this prediction of General Relativity have been made using radio waves from space probes when they are on the other side of the Sun from the Earth, and in each case the measurements agree with the theoretical predictions.

A glass lens also bends light, which is the principle by which a magnifying glass works. Can a massive object also act as a lens and magnify objects that lie behind it as viewed by a distant observer? The answer is "yes!" A number of such gravitational lenses have been seen. The most striking ones occur when the light from a remote galaxy passes through another galaxy or cluster of galaxies before the light reaches the Earth. Multiple magnified images, arcs, and rings result because of the different paths the light can take and still be bent into our line of sight by the gravitational curvature of space-time.

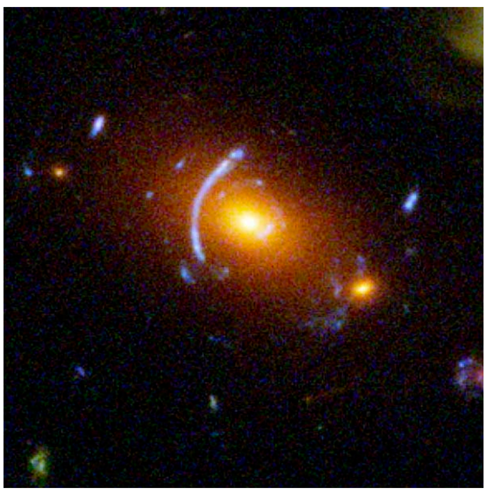

Fig. 9-7. Example of a gravitational lens. This image from the Hubble Space Telescope shows a bright galaxy (reddish color) whose gravity focuses the light from a much more distant galaxy (with a bluish-white color). The long arc, as well as other, less dramatic features in the image, are caused by the gravitational lens. This bending of the path of the light was predicted by Einstein's General Relativity. [Source: stsci.edu]

According to General Relativity, gravity has a geometrical interpretation through the bending of space-time. One can then picture the orbits of the planets as being analogous to balls tracing out elliptical paths in a frictionless, 4-dimensional roulette wheel created by the Sun's gravity.

Gravity affects time as well as space. Consider a photon emitted from a site close to a massive object. It will lose energy as it escapes the gravitational force of the object, just as an object thrown upward on the Earth loses kinetic energy as it progresses toward the peak of its trajectory. For light, energy is directly proportional to frequency (see Ch. 5), so the loss of energy means a shift to lower frequency and therefore longer wavelength. This is called the gravitational redshift. Since time is the inverse of frequency, if frequency is decreased, then the passage of time becomes more stretched out. (See the discussion in Box 9-3). An observer far away from a massive object measures time to pass more slowly than normal for phenomena that occur very close to the massive object.

General Relativity predicts that gravitational waves should be emitted when, for example, two very massive, very compact objects — such as collapsed cores of former stars called "neutron stars" (see Ch. 13) — orbit each other. As do electromagnetic waves, gravitational waves travel at the speed of light, even through a vacuum. Joseph Taylor and Russell Hulse observed such a binary neutron star system in the 1970's. They found that the size of the semi-major axis of the orbit decreases with time at a rate that is predicted by the equations of General Relativity. The orbital decay occurs because energy is lost from the system as it emits gravitational waves. Instruments designed to observe gravitational waves directly are now in operation, and several events have been observed. As will be discussed in Chapter 13, the observed spectrum of the waves and how the spectrum changes with time matches the theoretical expectations of the gravitational waves emitted when two black holes or (for one of the events) two neutron stars merging.

Summary

The speed of light c is the speed limit of the universe. No information or energy can move faster than light and no matter can even reach c. After Michelson measured the speed of light, he and Morley determined that c does not depend on the relative motion between the source of the light and the observer. Einstein adopted this, along with the requirement that all observers measure the same results of experiments that take place in their own rest frame. He used these assumptions as the basis of Special Relativity, which describes measurements of phenomena that occur in non-accelerating reference frames that are in motion relative to one another.

Einstein found that the measurements of such fundamental quantities as length and time are not absolute. Instead, they depend on the velocity of the object or system relative to the observer. For example, certain particles that are very short-lived if they are stationary have much longer lives if they move at a velocity very close to the speed of light relative to the observer. This is not an illusion: a measurement represents reality. It only seems strange to us because we are not accustomed to observing objects that move at speeds near that of light. In fact, the predictions of Special Relativity have been verified many times to extremely high precision.

An important parameter of Special Relativity is the Lorentz factor Γ, which corresponds to how many times shorter an object is measured to be than its rest length and how many times more slowly time passes in a moving reference frame as viewed by an outside observer. Einstein also formulated his famous equation $E=mc^2$ that defines the rest-mass energy. This means that mass and energy are really just two manifestations of a single property of matter, mass-energy. Scientists later used this formula to explain the energy sources of stars and also to build nuclear weapons and reactors.

After deriving Special Relativity, Einstein developed General Relativity, which considers accelerating reference frames and gravity. His interpretation of gravity as bending of space-time is qualitatively different from Newton's "action at a distance" description, yet the two produce nearly identical results except near very massive objects. The curvature of space-time causes light to follow a bent path around a massive object such as the Sun. Galaxies can even act as gravitational lenses, causing the appearance of multiple images of objects that lie at large distances beyond them. A remote observer measures time intervals of phenomena near a massive object to be longer than it is for similar events in his/her rest frame. Also, light emitted from a site very close to a compact, massive object is redshifted (shifted to longer wavelengths). This bending of light, slowing down of time, and other effects of General Relativity have all been observed, with the magnitudes of the effects agreeing with the predictions of the theory.

Glossary

Reference frame: An imaginary location ("platform") from which measurements are made.

Rest frame: A reference frame in which there is no motion of the object or person in question.

Relative velocity: The velocity measured from the rest frame of an observer. For speeds much less than that of light, the relative velocity equals the velocity of the object minus the velocity of the observer. For speeds close to c, equation (9-3) must be used.

Relativity: The branch of physics that deals with the effects of moving reference frames on the observation of events and measurement of physical properties of objects.

Everyday ("Galilean") relativity: Relationship between velocities in different reference frames that are moving at relative velocities much less than the speed of light.

Special Relativity: Einstein's theory of the effect of relative motion at a constant velocity on the observation of events and measurement of physical properties of objects. Important when the relative velocities are close to the speed of light.

General Relativity: Einstein's theory of gravity and accelerating reference frames. Describes gravity as curvature of space-time caused by the presence of mass.

Michelson-Morley experiment: Important observation that the speed of light does not change if there is relative motion between the observer and the source of the light.

Lorentz factor (symbol: Γ): A parameter whose value corresponds to the importance of the effects of Special Relativity. At speeds less than about $0.2c$, Γ is very close to 1. As the speed approaches c, Γ increases. See Box 9-1 and Fig. 9-3.

Time dilation: Effect described by Special Relativity: time passes more slowly for events observed in reference frames that are moving near the speed of light relative to the observer. See Box 9-1 and eq. (9-1).

Length contraction: Lengths and distances in the forward and reverse directions are fore-shortened when the phenomenon being observed is moving near the speed of light relative to the observer. See Box 9-2 and eq. (9-2).

Gedanken **(thought) experiment:** The use of physical principles and logic to predict the outcome of an experiment that is impractical to carry out in real life. Einstein used these to search for and resolve logical contradications in theoretical descriptions of nature.

Causality: The logical ordering of events such that all observers see phenomena occur after the events that trigger them.

Twin paradox: Gedanken experiment that illustrates time dilation and that only reference frames in uniform relative motion (constant speed and direction) are equivalent in Special Relativity.

Pole-vaulter's paradox: Gedanken experiment that illustrates length contraction and demonstrates that two events that are simultaneous in one reference frame are not necessarily simultaneous in another.

Space-time: The framework of the 4-dimensional macroscopic universe, consisting of three dimensions in space and one in time.

Equivalence Principle: Assumption that the gravitational mass equals the inertial mass. This needs to be valid for the acceleration from gravity to be independent of the mass of the falling body (see Ch. 4). Einstein's version is that no measurement can discern a difference between acceleration caused by gravity and that caused by other forces.

Geodesic: The shortest distance between two points. In flat space, a line is a geodesic. On the surface of a sphere, it is a section of a great circle. Light follows a geodesic path.

Gravitational redshift: Doppler effect on light emitted from a location near a massive object. The light loses energy, which corresponds to its frequency becoming lower, wavelength longer.

Gravitational waves: Phenomenon predicted by Einstein's theory of General Relativity. For example, ultra-dense concentrations of two or more massive objects in motion should emit these types of waves.

Gravitational lens: Phenomenon in which massive objects bend the path of light in such a way as to focus the light, as in a conventional lens. The observer sees multiple images and/or arcs of bright objects in the background whose light passes by the massive objects.

Questions for Discussion

A. Suppose that you had the technology to build your own spaceship that can travel very close to (but always slightly less than) the speed of light. Would you want to take a tour of our neighborhood in the Milky Way Galaxy? You could do so in a reasonable amount of time if your spaceship could travel at a Lorentz factor $\Gamma = 5$ or greater. What would the consequences be?

B. Does our common notion of reality remain intact if two observers in motion relative to each other measure time and length differently? Does this mean that all reality is relative and therefore subjective rather than objective?

C. Why can a material object never reach the speed of light? Can you think of any way to overcome this obstacle?

D. You would be very comfortable in a spaceship that accelerated at 1 $g = 10$ m/s^2, since you would feel the same acceleration as gravity provides on the surface of the Earth. Approximately how long would it take for such a spaceship to get close to the speed of light? You can do a rough calculation by approximating 1 year as 30 million (3×10^7) s and using the formula $v = at$ despite the fact that it would cease to be valid as your spaceship got very close to the speed of light. Is your answer shorter or longer than you expected?

E. Can you think of a reason why the Equivalence Principle should be valid? Does this call for a theory that can explain it?

F. What are the differences between Einstein's theory of General Theory and Newton's Universal Law of Gravitation? In particular, how does each explain how the force of gravity is transmitted? Compare with the explanation offered by the Standard Model of particle physics. Can all three theories be correct?

Sample Problems in Relativity

1. Imagine that you are traveling in a spaceship toward a star that, measured from the Earth, lies at a distance of 98 lt-yr. Your spaceship travels toward the star at a Lorentz factor $\Gamma=5$ (speed of 0.98c). The length of the spaceship measured at rest is 50 m.

a. How long will it take to travel from the Earth to the star as measured in the Earth's frame?

Answer: Since both the distance and the time are measured in the Earth's frame, we can calculate the time as just the distance divided by the speed:

$t = d/v = (98 \text{ lt-yr})/(0.98c) = \underline{100 \text{ yr}}$.

Note that when lt-yr is used as the unit for distance and the speed is in terms of c, time is in units of years.

b. How long will it take to travel from the Earth to the star as measured in the spaceship's frame?

Answer: The observer's rest frame is now that of the spaceship. Eq. (9-1) gives
$t = \Gamma t_{\text{rest}}$ and we need t_{rest}. So,

$t_{\text{rest}} = \text{t}/\Gamma = (100 \text{ yr})/5 = \underline{20 \text{ yr}}$.

c. What is the rest-mass energy in J of a 50 kg human in the human's rest frame?

Answer: Eq. (8-2) (or eq. 9-5 with $\Gamma = 1$) gives

$E_{\text{rest}} = mc^2 = (50 \text{ kg})(3.00\text{x}10^8 \text{ m/s})^2 = \underline{4.5\text{x}10^{18} \text{ J}}$.

d. What is the energy in J of a 50 kg human in the spaceship as measured from the Earth?

Answer: Since the spaceship is moving relative to the Earth-based observer, we use eq. (9-5):

$E = \Gamma mc^2$
$= (5)(50 \text{ kg})(3.00\text{x}10^8 \text{ m/s})^2 = \underline{2.2\text{x}10^{19} \text{ J}}$.

Note that you should only use two significant digits because this is the accuracy of the data.

e. What is the length of the spaceship in the Earth's frame?

Answer: Since the spaceship is moving relative to the Earth-based observer, eq. (9-2) gives

$L = L_{\text{rest}}/\Gamma = (50 \text{ m})/5 = \underline{10 \text{ m}}$.

f. What is the distance d between the Earth and the star in the spaceship's frame?

Answer: Distance has the same relationship as length, so we can again use eq. (9-2), substituting d for L. Here the "rest frame" is the Earth's frame, since it is at rest relative to the star whose distance is given. For a measurement inside the spaceship, we have

$d = d_{\text{rest}}/\Gamma = (98 \text{ lt-yr})/5 = \underline{20 \text{ lt-yr}}$.

(Only two significant digits are used.) So, it makes sense that it only takes 20 yr to get to the star in the spaceship's frame!

g. The spaceship has a blue tail light that emits light at a rest wavelength of 450 nm. What wavelength will be observed from the Earth as the spaceship travels (i) toward the star and (ii) back toward the Earth?

Answer: Use eq. (9-4a). Since the spaceship is moving away, the velocity is $v = +0.98c$. Therefore,

$$\lambda_{\text{obs}} = \Gamma\left(1+\frac{v}{c}\right)\lambda_0 = 5(1+0.98)(450\text{ nm}) = \underline{4500\text{ nm}}.$$

this is in the infrared part of the spectrum (see Ch. 5).

During the return trip, the light would be blueshifted ($v = -0.98c$), so if the spaceship also has a blue headlight, the wavelength observed at the Earth would be

$$\lambda_{\text{obs}} = \Gamma\left(1+\frac{v}{c}\right)\lambda_0 = (5)(1-0.98)(450\text{ nm}) = 45\text{ nm},$$

which is in the ultraviolet part of the spectrum.

2. A spaceship has a length of 30 m when measured at rest. What length is measured by an observer relative to whom the spaceship is moving at a speed of $0.98c$ (Lorentz factor $\Gamma = 5$)?

Answer: Eq. (9-2) gives

$L = L_{\text{rest}}/\Gamma = (30\text{ m})/5 = \underline{6\text{ m}}$.

3. A muon particle has an average lifetime of 2.2 μs after it is created. If a muon is produced in a collision of particles in a laboratory and its Lorentz factor is 200, how long will it survive on average?

Answer: Eq. (9-1) gives

$t = \Gamma t_{\text{rest}} = 200(2.2\ \mu\text{s}) = \underline{440\ \mu\text{s}}$.

4. A spaceship traveling at a speed of $0.866c$ (Lorentz factor $\Gamma = 2$) relative to the Earth shoots a rocket in the forward direction at a speed of $0.980c$ (Lorentz factor $\Gamma = 5$). What is the speed and Lorentz factor of the rocket measured in the Earth's reference frame?

Answer: Eq. (9-3) gives

$$v_{\text{obs}} = \frac{v+V}{1+(Vv/c^2)} = \frac{0.866c+0.980c}{1+[(0.866c)(0.980c)/c^2]} = \frac{1.846c}{1+0.849} = \frac{1.846c}{1.849} = \underline{0.998c}.$$

From the definition of the Lorentz factor given in Box 9-1,

$$\Gamma = \frac{1}{\sqrt{1-(v^2/c^2)}} = \frac{1}{\sqrt{1-[(0.998c)^2/c^2}} = \frac{1}{\sqrt{1-0.996}} = \frac{1}{\sqrt{0.004}} = 16.$$

Homework Questions

1. A spaceship initially at rest is propelled to a Lorentz factor $\Gamma = 7$ (speed of $0.98974c$). The mass of the spaceship is 20,000 kg; neglect the mass of the fuel.

a. How much energy (in J) would be required to do this? [*Hint:* Use equation (9-5).]

b. How much mass in fuel would be needed to create this much energy at the highest possible efficiency, 100% conversion of rest-mass into energy? [*Hint:* Use the equation $E = mc^2$ and solve for m.]

c. Was our neglect of the mass of the fuel correct in this case, or is much more mass needed in fuel than in the structure of the spaceship? That is, can space travel near the speed of light be fuel-efficient?

2. A spaceship moves through the Galaxy at a Lorentz factor $\Gamma = 7$ (speed of $0.98974c$).

a. How much energy would be involved in a head-on collision of the spaceship with a small space rock of mass 1 kg? [Note that, because velocity is relative, as measured by the spaceship, the rock hits the spaceship at a Lorentz factor $\Gamma = 7$.]

b. Compare this with the energy of a hydrogen bomb, about $1\text{x}10^{17}$ joules, by dividing your answer to part (a) by this number. Is space travel at speeds close to that of light hazardous?

3. A spaceship traveling at a speed of $0.9682c$ (Lorentz factor $\Gamma = 4$) is passing by the Earth so that at one moment its path is perpendicular to your line of sight toward it. You are standing on the Earth looking up at the spaceship at that moment. [Note: The direction of the spaceship's motion avoids complications related to the change in time for light from the spaceship to travel to you on the Earth. Otherwise, it is not important to the calculation.]

a. How long do you observe the spaceship to be if its length at rest is 20 m?

b. A traveler on the spaceship radios to you "hello", which in her rest frame takes 3 seconds. How long do you hear it take for her to say "hello"?

4. Two spaceships approach each other, each with a speed of $0.975000c$ (Lorentz factor of 4.50035) relative to a stationary point between them.

a. What is the relative velocity of the spaceships, i.e., the velocity of one as measured by the other? [*Hint:* keep 6 significant digits throughout; rounding will lead to an error.]

b. What is the Lorentz factor that corresponds to this relative velocity?

5. A spaceship traveling at a speed of $0.99000c$ away from the Earth ejects a shuttle-craft in the rear direction at a velocity of $0.98000c$ relative to the spaceship.

a. What is the velocity of the shuttle-craft relative to the Earth? [Hint: keep 5 significant digits throughout; rounding will lead to an error. Note that the velocities are in opposite directions, so one is positive and the other negative.]

b. What is the Lorentz factor of the shuttle-craft corresponding to the velocity calculated in part (a)?

6. A spaceship traveling at a speed of 0.553c (Lorentz factor $\Gamma = 1.2$) is heading directly toward the Earth. Space traffic controllers on the Earth shine a laser beam composed of red photons with a wavelength of 650 nm toward the spaceship to signal it to stop.

a. What is the wavelength of the laser beam measured in the frame of the spaceship?

b. What color will the laser beam have as viewed from the spaceship? [Consult Chapter 5, page 5-7 for converting wavelength to color.] Will it still have the characteristic red color of stop signs?

Astronomical Puzzle (Group Exercise)

7. An object in our Galaxy has a double set of emission lines whose wavelengths vary periodically. For example, the Hβ emission line of hydrogen has a rest wavelength of 486.1 nm. Two Hβ lines appear in the spectrum of the object. The graph (shown below) of wavelength vs. time for each line resembles a sine curve, with the longest wavelength of one line occurring at the same time as the shortest wavelength of the other line.

a. Determine the likely physical principle that can cause the changing wavelengths.

b. Devise some hypotheses that might be able to explain this phenomenon.

c. Evaluate each hypothesis logically. Is it self-consistent (i.e., does it avoid logical contradictions?) Does it pass the "Occam's Razor" criterion that hypotheses should not be overly complex?

d. For each hypothesis, make some predictions as to what further observations — e.g., images of the object — should show.

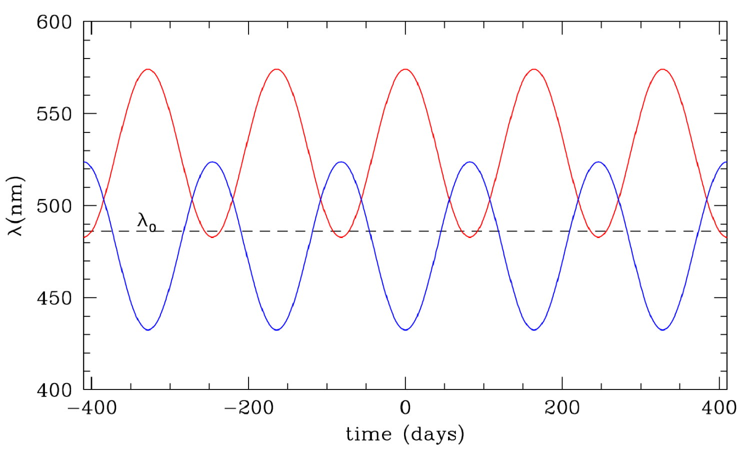

Figure for problem 7. The graph plots the wavelength of the hydrogen Hβ emission line from a cosmic object. There are two such lines — corresponding to the red and blue curves — observed at any given time. The rest wavelength of the Hβ line is shown as a horizontal dashed line.