Chapter 3: The Copernican Revolution

The Renaissance period brought a new artistic and intellectual spirit to Europe. The invention of the printing press and the manufacturing of high-quality paper – based on Chinese technology following Marco Polo's expeditions – made books widely available rather than just the prized possessions of university libraries and monasteries. Among the many cultural movements was a renewed interest in the Sun, the source of most light and heat, and therefore the heavenly body necessary for the survival of life on the Earth. The stage was thus set for a reconsideration of humanity's geocentric view of the universe.

The Sun-Centered Universe of Copernicus

Leading the way in the development of these new ideas was Nicolaus Copernicus (1473-1543) of Poland. In 1512, he published *Commentariolus*, a brief piece stating that Ptolemy's model was inaccurate and that a simpler, Sun-centered (heliocentric) model would provide a more elegant solution. By this time, it was no secret that predictions of celestial events (*e.g.*, eclipses and planetary positions) based on Ptolemy's tables were often inaccurate by hours or days. Copernicus, however, was fearful of the expected negative reaction to the heliocentric hypothesis. He therefore delayed publication of his major work, *De Revolutionibus* ("On Revolutions") until just before his death. Although a number of scholars considered the new model to be worthy of consideration, the book was renounced by both the Catholic and Protestant churches. Martin Luther declared, "This fool wishes to reverse the entire science of astronomy; but sacred scripture tells us that Joshua ordered the Sun and not the Earth to stand still" (T. Ferris, *Coming of Age in the Milky Way*, p. 67). Later, in 1616, the Catholic Church banned reading *De Revolutionibus* until 1620, when the published version was "corrected" through the insertion of a preface that it was only a mathematical model and did not describe reality. But Copernicus' model had two *scientific* problems as well:
1. its predictions were even more inaccurate than those of Ptolemy's model; and
2. the stars needed to be much more distant than had been thought in order to avoid observable parallaxes (see Ch. 2).

Despite its perceived shortcomings, Copernicus's model stimulated later scholars such as Kepler to develop more sophisticated theories of the cosmos based on the idea that the Earth orbits the Sun.

The Birth of High-Precision Observational Astronomy

Probably the most colorful figure among the Renaissance astronomers was Tycho Brahe of Denmark. Blessed with generous financial support from King Frederick II, whom his father had saved from drowning, Tycho built the castle-observatory Uraniborg on the island of Hven, near Elsinor Castle of *Hamlet* fame. He spared no expense, equipping the observatory with the best available scientific equipment, a paper mill and printing press, an intercom system, and even flush toilets. Tycho was quite stout and wore a gold-silver nose to replace his natural one, which was mutilated in a duel.

In 1560, Tycho observed a partial solar eclipse and was impressed that the tables of Ptolemy had predicted the correct day, but not the correct time. In 1563 he noted that the occurrence of a conjunction (closeness in the sky) of Jupiter and Saturn was off by several days from the prediction. He decided to dedicate his life to precise observations in the hopes that better data would lead to a more accurate model of the heavens. Indeed, Tycho made a number of important contributions that were to lay the groundwork for the theoretical breakthroughs of the 1600s. For example, he and his staff mapped the positions of thousands of stars so that the motions of the planets and Moon could be followed with high precision.

Tycho was able to establish the distance to the Moon by using triangulation – basically the parallax or difference in apparent position relative to the stars – as measured simultaneously by assistants placed at different latitudes. Tycho observed several comets and a suddenly bright star where no star was previously visible. We now know that this was a supernova, or exploding star. Tycho demonstrated by their lack of parallax that the comets and supernova were not only outside the atmosphere, but even beyond the orbit of the Moon. This contradicted the belief inherited from the ancient Greeks that transient celestial objects are relatively unimportant atmospheric phenomena.

Tycho, however, rejected Copernicus' heliocentric theory on the grounds that its predictions were inaccurate and no parallax of stars was observed to an accuracy of 1/30 of the Moon's angular diameter. (See the box on Fundamental Observations of the Naked-Eye Cosmos in Ch. 2 for a definition of angular diameter.) He supported this argument with his measurement of an angular size of stars equal to 1/15 the size of the Moon. This implied that the stars could not be so far away that they would show no parallax unless they were much larger than the Sun. He was mistaken, however: the largest angular size of a star besides the Sun is about 1 millionth the angular size of the Moon. Tycho devised his own model in which all the planets orbit the Sun, which itself orbits the stationary Earth. However, this model also proved inaccurate, and Tycho died – supposedly from a burst bladder from drinking too much during a royal dinner party – before being able to improve upon it.

Kepler' Laws of Planetary Motion

Unlike the flamboyant Tycho, the person who figured out the actual motions of the planets – German-born Johannes Kepler (1571-1630) – was a melancholy man who had to overcome a number of life's misfortunes. For example, his aunt was burned at the stake for witchcraft. His mother was similarly charged, saved only by the intervention of the respectable Johannes, who studied Lutheran theology and served as a provincial court mathematician in Austria.

In the course of his studies, Kepler read Copernicus's book, which convinced him that the heliocentric hypothesis was correct. He wrote to Tycho, who invited him to join his astronomical research group in Prague in 1600. Tycho, who was attempting to devise his own accurate model, only gave Kepler his data on Mars. These observations seemed nearly hopeless to reproduce by any model, given Mars's complicated changes in apparent speed as well as pronounced retrograde motion when it is on the opposite side of the sky from the Sun. Kepler's interpretation did not come easily. He was a methodical theorist who used trial and error to eliminate possibilities. (Most of the other great theorists have been more intuitive, using symmetries and "beauty" to guide them, as mentioned in Ch. 1.) After an exhaustive number of attempts to create a successful model, he decided to try a geometrical approach to determine the shape of Mars's orbit *as viewed from the Sun*, as would be relevant if the heliocentric hypothesis were valid. Once he traced out Mars's orbit from a series of these positions, it was obvious that the shape of the orbit is an oval. After further inspection he found that it was the most symmetric type of oval: an ellipse. At last, Kepler had found the grand pattern of the heavens: *the planetary orbits are ellipses, not circles*!

In the process of calculating Mars's orbit, Kepler found that the speed of Mars changes such that it moves faster when it is closer to the Sun. He was able to determine a mathematical law to describe the behavior of the velocity. In 1609, Kepler published the book *New Astronomy*, in which he presented his 1st and 2nd laws of planetary motion:

1. **A planet's orbit is an ellipse with the Sun at one focus.** (See Box 3-1 for the properties of an ellipse.)

2. **An imaginary line between the Sun and a planet sweeps out equal areas in equal time intervals** (see Figure 3-1).

What remained was a mathematical description of how the period – the time it takes to complete a single orbit – of a planet's orbit depends on its distance from the Sun. Using the semi-major axis A as the main parameter measuring distance from the Sun, Kepler published his 3rd law in 1619:

3. **The square of the period P of a planet's orbit is proportional to the cube of the semi-major axis A.**

It is easiest to express Kepler's 3rd Law as an equation if we use years (yr) for the period P. For the semi-major axis A, we use astronomical units (AU; the semi-major axis of the Earth's orbit around the Sun, which is 150 million km, is 1 AU). For planets orbiting around the Sun, the equation is

$$P^2 = A^3 \tag{3-1}$$

P = period (time to complete one orbit, in yr), A = semi-major axis of orbit (in AU).

For planets orbiting another star of mass M solar masses, the equation becomes

$$P^2 = A^3/\mathrm{M} \tag{3-2}$$

P = period (time to complete one orbit, in yr), A = semi-major axis of orbit (in AU), M = mass of the star (in units of solar masses, M_s ; *e.g.*, if the mass is 2 solar masses, use $M = 2$).

(See Ch. 4 for a derivation of eq. 3-2 in standard units.)

These three "rules" allowed predictions of planetary positions much more accurate than any of the previous models. They also provided a far simpler geometry of the solar system than that of Ptolemy or even Copernicus. Kepler revealed the fundamental flaws of the ancient Greek models. The planetary motions are not circular, but elliptical, and the speed of a planet is not uniform, but increases when the planet is closer to the Sun and slows down when it is farther away. From a geometrical point of view, motions in the universe are not as symmetric as possible: an ellipse is one level less symmetric than a circle. On the other hand, they still possess sufficient symmetry that orbits are governed by quite simple – perhaps elegant – mathematical laws. Although he correctly proposed that a force (later called "gravity") causes orbits and tides, Kepler was unable to find the relevant mathematical formula. Later in the 1600s, Isaac Newton would accomplish this task, as we will discuss in Chapter 4.

Box 3-1. The properties of an ellipse

An ellipse, an example of which is given in Figure 3-1, is described by two parameters: its semi-major axis ***A*** (half of the longest line, or major axis, that can be drawn across the ellipse while passing through its center) and its semi-minor axis B (half of the shortest line across the ellipse that passes through its center; the minor axis is perpendicular to the major axis). A circle is a special case of an ellipse, in which $A=B$. An ellipse is symmetric about either the major or minor axis; *i.e.* if it were sliced in half along one of these axes, each half would be a mirror image of the other. The semi-major axis is the average geometrical distance from the different points on the boundary to the focus. For this reason, the semi-major axis of a planet's orbit is often called its "average distance from the Sun."

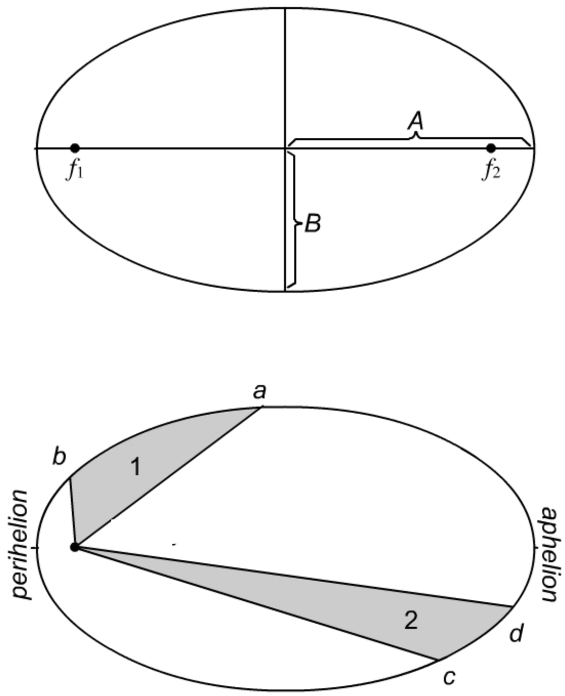


Figure 3-1. *Top*: the geometry of an ellipse. For the planetary orbits, the Sun is located at one focus (f_1); the other focus (f_2) is not important. *Bottom*: equal areas are swept out in equal times by the imaginary line connecting the Sun and a planet. Hence, it takes the same amount of time for the planet to travel from point *a* to point *b* as it does to travel the shorter distance from points *c* to *d*, if the area contained in section 1 equals that in section 2. Therefore, the planet moves faster when it is closer to the Sun.

Box 3-2. Motions in the sky as interpreted with Kepler's Laws

In the last chapter (Box 2.1 on Fundamental Observations of the Naked-Eye Cosmos), we summarized the motions in the sky that can be observed with the naked eye. Kepler's model of the solar system can explain these motions to high accuracy. The nightly circling of the stars, Moon, and planets, as well as the daily circling of the Sun, about the north celestial pole is an effect caused by the rotation of the Earth about its axis. The imaginary line of the Earth's spin axis points toward the celestial poles. The north celestial pole lies near the fairly bright star Polaris.

The apparent annual motion of the Sun along a circle that passes through the band of stars called the ecliptic (or, popularly, the zodiac) is caused by the orbit of the Earth around the Sun. The Earth's orbit is only slightly elongated – i.e., nearly a circle – so it is only about 3% closer to the Sun at perihelion (closest distance from the Sun) than at aphelion (farthest distance from the Sun). Otherwise, the Sun would appear noticeably larger in the sky near perihelion (early January) than near aphelion (early July), and the change in distance from the Sun would have an effect on the Earth's seasons. In fact, the seasons are caused by the 23.5° tilt of the Earth's axis relative to the plane containing its orbit, so the sunlight strikes the ground more obliquely – and therefore with less intensity – during winter at high latitudes, and more directly during summer. It is this tilt of the axis that causes the Sun to appear south of the celestial equator during the northern hemisphere's winter (the southern hemisphere's summer) and north of the celestial equator during the northern summer (winter south of the Earth's equator).

The motion of the Moon relative to the stars – it circles the ecliptic once every 29.5 days as seen from the Earth – is caused by the Moon's nearly circular orbit around the Earth. However, the plane containing the

Moon's orbit is inclined by about 5° to that of the Earth's orbit around the Sun, so the alignment between the Earth, Moon, and Sun needed to cause an eclipse does not occur every month. The slight ellipticity of the Moon's orbit causes some solar eclipses to last longer than others. Also, when the Moon is near apogee (farthest distance from Earth), a solar eclipse is annular, *i.e.*, the Moon eclipses most of the Sun but leaves an outer ring unblocked.

As is the case also with airplanes in the sky, the apparent speed of a planet depends on both its actual velocity and its distance from the observer on the Earth. A complication is that our observing platform, the Earth, is also in motion, orbiting around the Sun. Hence, the apparent motion of a planet is determined by both its own and the Earth's orbital motions around the Sun. Since the orbits of all the planets lie in planes close to that of the Earth's orbit, they appear to move along the constellations of the ecliptic. As the faster-moving Earth "passes" an outer planet in its orbit, the planet appears to move "backward" (westward, or retrograde) relative to the stars, just as a car being passed by your car appears to move backward relative to you even though both cars are moving forward relative to the ground. This retrograde motion of the planets is slow and therefore is most easily seen from one week to the next. The retrograde motion of an outer planet occurs when that planet is at opposition — on the opposite side of the sky from the Sun, rising in the sky at about the time of sunset.

The more rapidly orbiting inner planets have retrograde motion when they are on the same side of the Sun as the Earth, since they are then "passing" the Earth.

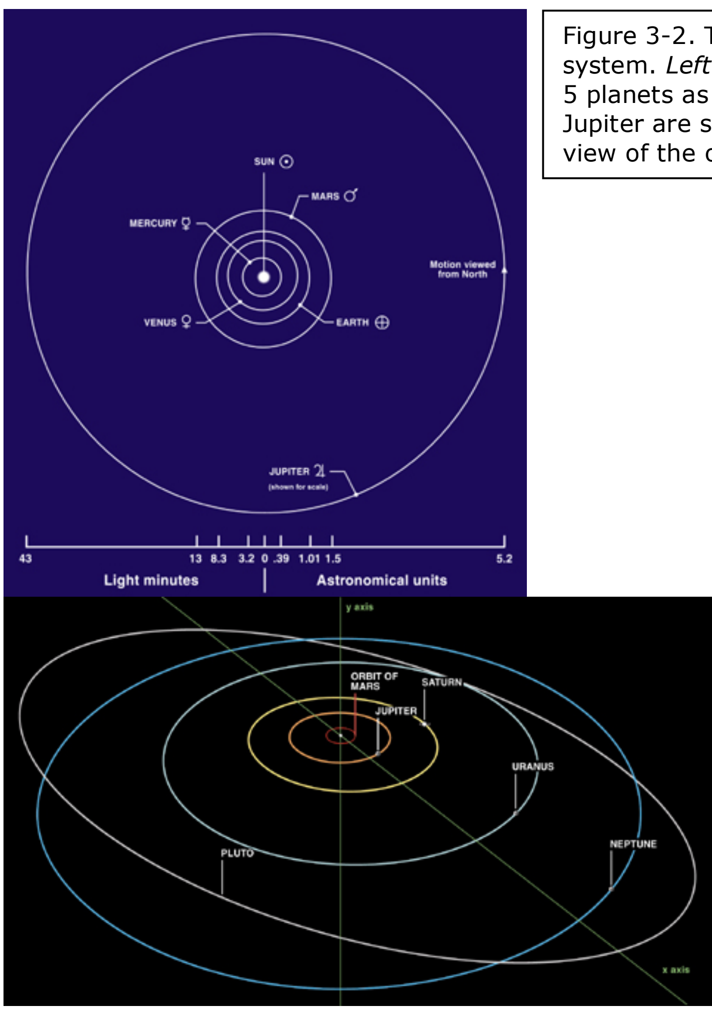
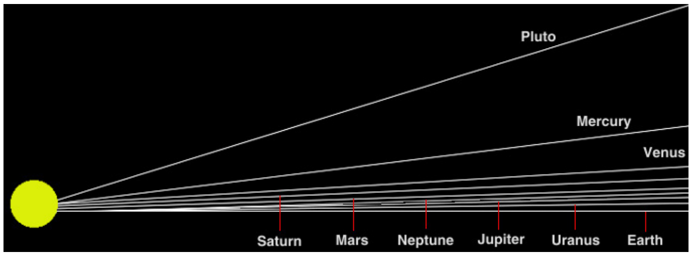


Figure 3-2. The orbits of the planets in our solar system. *Left:* innermost 5 planets. *Right:* outermost 5 planets as well as the dwarf planet Pluto. (Mars & Jupiter are shown in both diagrams.) *Bottom:* Side view of the orbits of the planets.



Top figures from nasa.gov.
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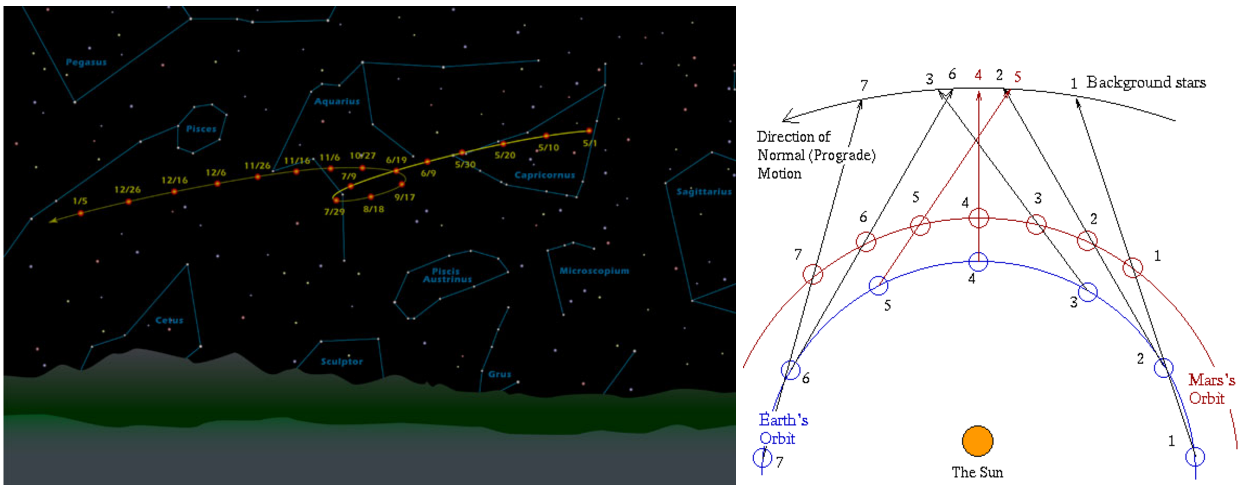

Figure 3-3. *Left:* The retrograde motion of Mars in summer 2003. When Mars rises near sunset because it is on the opposite side of the sky from the Sun, it reverses its usual eastward (toward the left on the diagram) progression among the constellations for some period. (Note: if the orbits of Mars and Earth were exactly in the same plane, the motion would be in a straight line rather than a loop.) *Right:* Correct explanation of retrograde motion: as the Earth passes Mars, Mars appears to move backward (clockwise on the diagram) as seen from Earth. [Source: http://mars.jpl.nasa.gov/allabout/nightsky/nightsky04.html]

If a planet is not in its retrograde stage, it appears to move faster *relative to the essentially fixed pattern of the constellations of stars* when it is closer to Earth and more slowly when it is on the other side of the Sun. Planets whose elliptical orbits are not nearly circular, such as Mercury and Mars, have significant variations in their actual orbital velocities, following Kepler's 2nd Law. Hence, their apparent speeds of motion as viewed from the Earth show rather complex behavior.

Galileo, Discoverer of Extraterrestrial Worlds and Modern Experimental Scientist

Galileo Galilei (1564-1642) of Italy considered scientific beliefs without observational verification to be invalid. He therefore had little patience for the teachings of the Scholastics, who insisted that the scientific views of the ancient Greeks should be accepted based on tradition and the endorsement by the Catholic Church. At the age of 25, Galileo became a professor of mathematics at the University of Pisa, but was fired at the end of his contract period because of conflicts with the Scholastic professors that composed most of the faculty. He then assumed the chair of mathematics at the University of Padua (near Venice), a post he held until 1610. Galileo made two important breakthroughs in scientific knowledge: his experiments on motion and his telescopic observations of the heavens.

Two Views of Motion: Aristotle vs. Galileo: Aristotle's view of the physics of motion, adopted by the Scholastics and the Catholic Church, was based on logic and "experience" (*i.e.*, observations) rather than experimentation. Aristotle regarded the motion of a falling object as a competition between the natural tendency to fall toward the center of the Earth and the resistance of the medium through which the object falls. He theorized that objects fall toward the center of the Earth at a constant speed, with more massive objects falling faster. He neglected as unimportant the initial period of acceleration between when the object is released and when it attains this constant speed. In the case of heavenly bodies, motion was "natural," meaning circular around the Earth and at constant speed.
Galileo devised experiments to determine whether these ideas were correct. His famous experiment of dropping two balls of similar size but different weights from the top of the Leaning Tower of Pisa and watching them hit the ground simultaneously, is considered by many historians to be a myth. Rather, Galileo based his conclusions mainly by rolling balls down inclined planes, reasoning that this is similar to falling freely. In his experiments, there was no difference in the rate of descent between more massive and less massive balls. Furthermore, he observed a continuous acceleration rather than a constant speed following a brief period of acceleration. Of course, he noticed the difference between dropping heavy balls and light objects such as a feather. Still, he reasoned that the rate of descent of an object falling through the air does not depend on its mass if the resistance of the air is neglected.

In fact, Aristotle's view of falling objects *does* provide an adequate description beyond the point at which the resistance of the medium becomes important. The approximately constant "terminal velocity" of descent – and how long it takes to reach it – depends on the mass per unit surface area. But it was Galileo's stress on the importance of acceleration and on the consideration of the ideal case of zero air resistance that led to a deeper understanding of the phenomenon of falling objects.

Galileo's Telescopic Observations: Galileo's general view of the heavens was influenced by Giordano Bruno, who proposed that space is infinite, with the stars scattered throughout it rather than on the concentric spherical shells of the Greek models. Galileo considered as correct the heliocentric model of the solar system, as well as Kepler's Laws after they were published. He also reasoned that the motions of celestial objects should be based on the same principles as are motions on the Earth.

In 1609, Galileo learned that telescopes were being made in Holland from a pair of convex lenses. He quickly understood the principle behind this and constructed his own telescope. He showed the Venetian senators boats in the harbor as seen through his telescope, which greatly impressed them. The Scholastic professors, however, refused even to look through his telescopes, claiming that what was seen through them was an illusion rather than a magnified view of reality.

Galileo proceeded to point his new instrument toward the sky, and in doing so sparked a scientific revolution. His most important results were:

1. He observed four dots of light – Moons – orbiting around Jupiter; it was like another solar system and proved that not all heavenly motion is around the Earth.

Figure 3-4. Jupiter and its four largest "Galilean" Moons, as viewed from the Earth through a telescope. From the National Optical Astronomy Observatories, http://www.noao.edu/outreach/aop/observers/jupmoon.html

2. He saw that Venus goes through phases similar to those of the Moon, and that when Venus is in its crescent phase its angular size is larger than when it is nearly full. This can only be true – and makes perfect sense – if Venus orbits around the Sun (see Fig. 3-5).

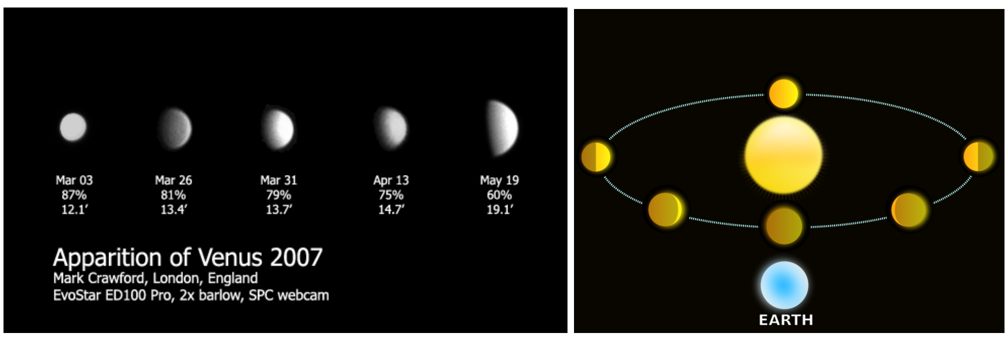

Figure 3-5. *Left:* Images of Venus with an Earth-based telescope. Note the change in apparent size, measured in arcminutes (′). From http://astro.annasach.net/images/venus-apparition.jpg. *Right:* Explanation based on the perspective from Earth: when Venus is closest to us and appears largest, we only see its dark, night side. We see more of its day side as it moves farther from us in its orbit around the Sun. [Note: this diagram is *not* drawn to scale.] [From wikipedia.com]

3. His observations revealed that on the Moon there are mountains and craters, showing that the Moon is another world, not utterly dissimilar in basic nature from the Earth.

Figure 3-6. The Moon as seen through a telescope on the Earth. Shadows near the terminator between day and night give a 3-D effect, highlighting the mountains and the craters. Galileo recognized these features, which revealed that the Moon is a geological body. [From http://www.astrosurf.com/cidadao/moon_99_02_23_north.jpg]

4. He found that the Sun has dark spots – sunspots – and therefore is not a "perfect," unblemished heavenly body as had been believed. He was able to follow the motion of the sunspots across the Sun to determine that the Sun rotates, at a rate that is a little faster than once per month.

Figure 3-7. Sunspots, while actually quite bright if observed in isolation, appear dark in contrast with the rest of the Sun.
From wikipedia.com.

5. Galileo used his telescope to resolve the Milky Way into a multitude of individual stars, thereby confirming the speculation of the Greek atomists (see Ch. 2).

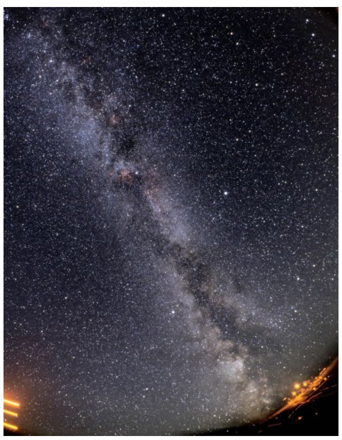

Figure 3-8. The Milky Way as seen with a camera from the Earth using a time-exposure. (The bright areas on the bottom left and right are stray lights on the ground.) Galileo resolved the cloudy-appearing structure into many individual stars.
[From: http://hubpages.com/u/101017_f520.jpg]

Galileo published his findings in *Sidereus Nuncius* (*Starry Messenger*), which quickly became very popular because it was written not in the customary Latin of scholars, but in Italian so that it could be read by any literate person in his country. In addition, the book contradicted the rigid Aristotelian beliefs of the Scholastics, who were despised by many. But the Scholastics still held great influence in the Church. In a decree handed down in 1616, Galileo was forbidden to "hold or defend" the heliocentric hypothesis. For his part, Galileo was arrogant and insisted that those who did not agree with him could not follow simple arguments; hence the feud raged on. He received a reprieve of sorts when his friend, Maffeo Barberini, became Pope Urban VIII in 1623. In 1632, the pope allowed Galileo to publish *Dialogue on the Two Great World Systems*, a fictional discussion among three philosophers: Salviati, who supports the heliocentric hypothesis, Sagredo, a common man who quickly grasps Salviati's arguments, and Simplicio, a fool who follows the Scholastic view and cannot comprehend Salviati's arguments. Galileo's critics pointed out to the pope that the Church's views were represented by the fool. Eventually, at age 70, Galileo was placed before the Inquisition and forced to recant his belief in the heliocentric hypothesis and other "heresies." He was sentenced to confinement in his villa near Florence until his death nine years later.

Galileo's conflict with the Church is the best known of many such battles between science and religion. The foundation of science is an open-ended system of explanations of nature based on hypotheses, evidence, and an open mind willing to accept the rejection of long-held theories when the evidence clearly contradicts them. This is antithetical to a religion that incorporates ideas about nature into a system of rigid beliefs. The Roman Catholic Church seems now to understand this: in 1980, Pope John

Paul II authorized an investigation into Galileo's case and, upon its conclusion, declared that Galileo had "suffered at the hands of men and institutions of the Church." He went on to say that "research performed in a truly scientific manner can never be in contrast with faith because both profane and religious realities have their origin in the same God."

Summary

The Renaissance ended the tight grip on the western world held by the science of ancient Greece. The adherence by the Catholic Church to the ideas of Aristotle created resistance to the new ideas, but their ability to explain the increasingly precise observational data eventually overcame this obstacle. In particular, the accurate positions of planets measured by Tycho allowed Kepler to develop a Sun-centered model that made accurate predictions for the positions of the Sun, Moon, and planets. The key was to model the orbits of the planets as ellipses, with the Sun at one focus and the orbital speed increasing when the planet is closer to the Sun. Kepler found a simple mathematical relationship between the semi-major axis and period of the orbit. This can be generalized to orbits around other bodies if the mass of the body is included in the equation and the units are adjusted appropriately.

At about the same time (early 1600s), Galileo showed that an object dropped toward the surface of the Earth accelerates downward at a rate that is independent of its mass. This contradicted Aristotle's claim that bodies fall at a constant speed that is higher for heavier objects. Galileo also made a telescope and pointed it toward the sky, revealing four Moons orbiting Jupiter, mountains and craters on the Moon, sunspots, and the phases of Venus. These discoveries demonstrated to those with open minds that the objects in the heavens are other worlds and that the Earth cannot be the center of it all.

These developments had the profound effect of giving scholars confidence that humans can, in fact, figure out how the universe operates. The implication was that by understanding nature humans might even be able to control it to some extent. This led to a burst of experimentation and revolutionary scientific ideas in the Enlightenment period that started with the breakthroughs of Kepler and Galileo.

Glossary

Heliocentric model (sun-centered model): Description in which the planets, including the Earth, orbit around the Sun, which is at the center of the solar system.

Kepler's Laws of Planetary Motion: Three principles that describe the orbit of an object around another object of much higher mass.

Polaris: The North Star, which lies very close to the direction in the sky toward which the Earth's axis points. As the Earth rotates, all the stars in the northern half of the sky appear to trace circles around Polaris.

Ecliptic: Band of 12 constellations through which the planets, Moon, and Sun appear to pass because the main objects in the solar system have orbits that are nearly in the same plane. Popularly referred to as the "zodiac."

Retrograde motion: Apparent "backward" (east to west) motion of a planet *relative to the stars*. Explained in the heliocentric model as a consequence of viewing the planets from the Earth, which is itself moving. Retrograde motion of a planet with an orbit larger than the Earth's occurs as the Earth passes the other planet. This occurs when that planet is on the opposite side of the sky from the Sun, so that it rises near sunset.

Perihelion: The point closest to the Sun of the orbit of a planet.

Aphelion: The point farthest from the Sun of the orbit of a planet.

Semi-major axis (symbol: A): Half of the length of an ellipse; see Fig. 3-1. The semi-major axis of Earth's orbit is 1 AU (astronomical unit, 1.5×10^{11} m).

Focus of an ellipse: A mathematical point on the major axis of an ellipse; see Box 3-1 and Figure 3-1 for details. An ellipse contains two foci.

Period of an orbit (symbol: P, units: years, yr): The time it takes for an object to complete one revolution of its orbit.
Sunspots: Relatively small areas on the Sun that appear dark because they are not as hot and bright as the rest of the visible Sun.

Questions for Discussion

A. Why did Tycho Brahe reject Ptolemy's model – because of its complex nature or its inability to make accurate predictions? Compare this with the scientific method discussed in Chapter 1. Did Tycho follow the method appropriately?

B. In what way was Copernicus's heliocentric model better than Ptolemy's? In what way was it worse (*e.g.*, why did Tycho reject it)? Does this lead to an ambiguity in how to apply the scientific method? When should scientists completely discard a model and when should they instead modify it?

C. Why is Kepler's elliptical model considered superior to the previous models? Do you consider it to be simpler or more complex than the previous models?

D. If you were a Scholastic scholar in 1609, would you have believed that what you saw in Galileo's telescope was really a magnified view of the celestial objects toward which it was pointed?

E. Did the Catholic Church have good reason to condemn the writings of Galileo? Was Galileo, in particular, partly to fault for his troubles, or was the Church completely in the wrong?

Examples of How to Solve Problems on Orbital Motions

1. An asteroid has an orbit around the Sun with a perihelion of 1.5 AU and an aphelion of 2.5 AU. Determine the period of the asteroid's orbit in years. (Here, as elsewhere, "year" corresponds to the period of the Earth's orbit.) Does it make sense that your answer is more or less than 1 yr?

Solution: Use eq. (3-1). First, determine the semi-major axis, A, which is the average of the closest and farthest distances from the Sun, *i.e.*, of the perihelion and aphelion:

$$\begin{aligned} A &= (1/2)(1.5\text{ AU} + 2.5\text{ AU}) \\ &= (1/2)(4.0\text{ AU}) \\ &= 2.0\text{ AU.} \end{aligned}$$

From eq. (3-1), $P^2 = A^3$, where P is in yr and A is in AU. [Note that this is the same as using eq. (3-2) with M=1 since the Sun is the star in the problem.] This can be solved by taking the square-root of both sides:

$P = \sqrt{A^3} = \sqrt{2.0^3}$ yr $= \sqrt{8.0}$ yr = 2.8 yr.

(Notice that not all the digits shown by the calculator are used; in general, you should limit the answer to the same number of digits as used in the data since this indicates the accuracy of the measurements.)

Yes, the answer makes qualitative sense, because the asteroid has an orbit that has a larger semi-major axis than that of the Earth, hence it must take longer to orbit the Sun.

2. An asteroid has an orbit around a star of mass 2 M_s, where "M_s" is the mass of our Sun. The asteroid is 1.5 AU from the star at its closest approach ("periastron") and 2.5 AU at its farthest distance ("apastron"). Determine the period of the asteroid's orbit in years. Does it make sense that your answer is more or less than the period of the asteroid in sample problem (1)?

Solution: Use eq. (3-2), since eq. (3-1) refers only to the Sun. First, determine the semi-major axis, A, which is the average of the closest and farthest distances from the Sun, i.e., of the perihelion and aphelion:

A = (1/2)(1.5 AU + 2.5 AU)
= (1/2)(4.0 AU)
= 2.0 AU.

From eq. (3-2), $P^2 = A^3/M$, where P is in yr, A is in AU, and M is in solar masses (M_s). This can be solved by taking the square-root of both sides:

$P = \sqrt{A^3/M} = \sqrt{2.0^3/2.0}$ yr $= \sqrt{4.0}$ yr = 2.0 yr.

This is a shorter period than the case for an asteroid with the same orbit around the Sun. Indeed, this makes sense because the more massive star causes a stronger gravitational force on the asteroid. In order to avoid falling in toward the star, the asteroid must have a higher orbital velocity to counteract gravity. The higher velocity for the same size orbit means that the period will be shorter.

Homework Problems

Notes: All units of "yr" correspond to Earth-years and "days" to Earth-days.

Show your step-by-step work for all mathematical calculations and explain your reasoning when using logic and/or inspection of the equations to determine your answer. You will not receive full credit if you merely write down the correct answer. However, you should not need a calculator if you solve the problems correctly. Recall that the solution to the equation $x^2 = y^3/z$ is $x = \sqrt{y^3/z}$ or $y = \sqrt[3]{x^2 z}$. [To compute a cubed root on most scientific calculators, you enter the number inside the root sign, then push "^" or "y^x" and then "(1/3)" followed by the equal sign. You can consult Appendix A of this book for advice on how to compare similar quantities for different objects (*e.g.*, "how many times greater…").]

1. Star system A: a single star with planets orbiting about it

a. Planet 1 has an elliptical orbit with closest distance to the star of 0.8 AU and greatest distance of 1.2 AU. Planet 2, meanwhile, has a circular orbit of radius 1.1 AU. Determine which of these two planets has a longer orbital period. (If, at any point, you use logic rather than an equation, explain your logic.)

b. Planet 4 has a circular orbit of radius 1 AU and an orbital period of 0.5 yr. Calculate the mass of the star in solar units (the mass of the Sun is given the symbol "M_s", so an answer of 2 solar masses would be written: 2 M_s).

c. Planet 5 has an elliptical orbit with a period equal to 4.0 yr. Calculate the semi-major axis of its orbit in units of AU. [You will need to use your answer to part (b). If you could not answer that, use a value M=2 M_s for the mass of the star.

d. Calculate the period of Planet 1's orbit. [See part (a) for some data regarding Planet 1's orbit.] How can you tell? [You will need the information given in part (a) and your answer to part (b); if you could not answer part (b), use a value M=2 M_s for the mass of the star.]

2. <u>Star system B</u>: a single star with planets orbiting about it

a. Planet 1 has an elliptical orbit with closest distance to the star of 0.8 AU and greatest distance of 1.2 AU. Planet 2, meanwhile, has a circular orbit of radius 0.9 AU. Determine which of these two planets has a longer orbital period. (If, at any point, you use logic rather than an equation, explain your logic.)

b. Planet 4 has a circular orbit of radius 5 AU and an orbital period of 5 yr. Calculate the mass of the star in solar units (the mass of the Sun is given the symbol "M_s", so an answer of 2 solar masses would be written: 2 M_s).

c. Planet 5 has an elliptical orbit with a period equal to 6.57 ($=\sqrt{43.2}$) yr. Calculate the semi-major axis of its orbit in units of AU. [You will need to use your answer to part (b). If you could not answer that, use a value M=3.2 M_s for the mass of the star.]

d. Calculate the period of Planet 1's orbit. [You will need the information given in part (a) and your answer to part (b); if you could not answer that, use a value M=3.2 M_s for the mass of the star.]