Chapter 5. Waves and the Electromagnetic Spectrum

Our discussion of forces and motion in Chapter 4 considered the behavior of a "body," which is some macroscopic solid object. The smallest, most fundamental form of a body is a particle. Before the 20th century, most physicists conceived of particles as tiny balls. Many of Newton's contemporaries and followers considered particles to be the fundamental buildings blocks of matter, and hence thought that formulas similar to those of Newton were sufficient for describing the dynamics of the universe.

Equally important to the description of nature are the periodic disturbances we call waves. We are all familiar with waves on water, which are ripples in water level that propagate away from the site of the original disturbance. For example, if we drop a stone into a pond, a circular wave will travel radially outward from the point where the stone hits the water. In this chapter, we will consider waves in general, sound waves and their relationship to musical notes, and then electromagnetic waves, or light. In this way, we can relate phenomena that are familiar to us to less familiar concepts that can be described with mathematics.

Basic Nature of Waves

Waves are propagating displacements of some physical property — for example, the height of the water level in a pond — from the equilibrium value that occurs when there is no disturbance. There are two basic types of waves: transverse and longitudinal. In transverse waves, the displacement occurs in a direction that is perpendicular to the direction in which the wave propagates. Waves on water are examples of transverse waves that cause the water level to oscillate. The maximum height of the water above the average surface level (or the minimum depth below this level – the oscillation is symmetric) is the amplitude of the wave. In the case of longitudinal waves, the oscillation is in the direction of propagation. This is a bit more difficult to picture; if you have ever played with the large, loose spring called a "Slinky," then you probably caused a longitudinal wave to propagate by holding both ends and pushing one end toward the other. Figure 1 shows both types of waves, although you must understand that waves are a dynamic phenomenon that is not well represented by a still-frame picture.

The maximum value of the oscillating quantity (height, in the case of water waves) occurs at the crest and the minimum value at the trough. Relative to the average height, called the **equilibrium level** (which is usually assigned a value of zero), the displacement of the crest is equal and opposite to the displacement of the trough. The wave propagates at a speed c, while the distance between successive crests (or troughs) is called the wavelength, which is given the symbol λ. Another, related quantity that is very often used is the frequency f, which is the number of wave crests that pass by a given point each second. The units of frequency are Hertz (abbreviated "Hz"); this is a measure of cycles per second, where one cycle is one complete oscillation from one crest to the next. Since a cycle has no units, a Hertz is an inverse second (1/s). As expected, the wavelength has units of length (meters). The number of wave crests that pass by a point each second obviously depends on the speed of the wave and on the wavelength, or distance between crests: the shorter the wavelength or the faster the speed is, the higher the frequency will be. The equation relating the three quantities is quite simple:

$$\lambda = \frac{c}{f} \tag{5-1}$$

λ = wavelength (in m), c = speed of wave (in m/s), f = frequency (in Hz, equivalent to 1/s).

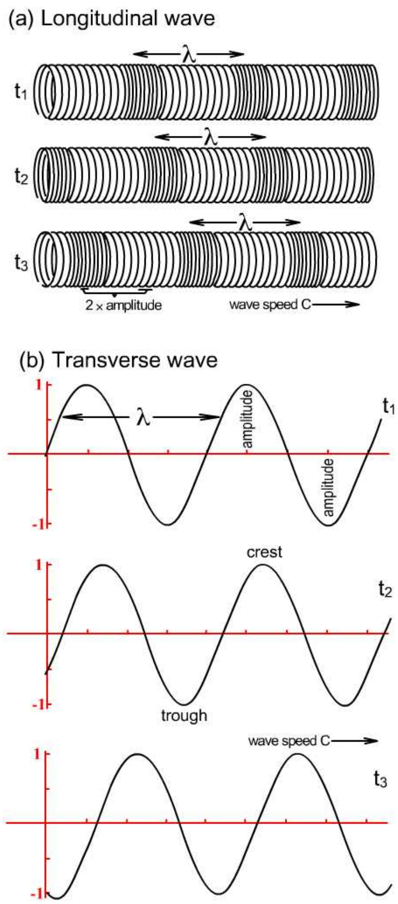

Figure 5-1. Sketch of the two types of periodic waves that are propagating to the right. Each wave is shown at three times to illustrate the motion. *Top:* longitudinal wave. The displacement is in the same direction as the propagation of the wave. This causes sections of compression sandwiched between sections of lower density (rarefaction). Sound waves are of this type. *Bottom:* transverse wave. The displacement is perpendicular to the direction of propagation of the wave. Surface waves on water and light waves are of this type.

We will therefore use the terms "wavelength" and "frequency" interchangeably; you should keep in mind that as one increases, the other decreases. The speed of propagation of a wave c depends on the nature of the medium. For example, sound waves travel through air at a speed that depends on the temperature of the atmosphere.

The Doppler Effect

If the source of waves is moving relative to the observer, the observer will measure a different number of wave crests that pass by him/her each second. Therefore, the frequency will change. It will be higher if the source is moving toward the observer and lower if it is moving away. Furthermore, the number of crests that pass by every second (the frequency) will be proportional to the speed of the source. This phenomenon is called the Doppler effect, or "Doppler shift." As long as the relative velocity v between the source of the waves and the observer is much less than the propagation speed c, the observed wavelength is given by

$$\lambda_{\text{obs}} = \lambda_0 (1 + \frac{v}{c}) \qquad (5\text{-}2)$$

λ_{obs} = observed wavelength (in m), λ_0 = rest wavelength (in m, measured when there is no motion), v = speed of source of waves (in m/s), c = speed of wave (in m/s).

It is interesting – and important – to note that the speed of the wave depends only on the physical properties of the medium through which it propagates, not on the motion of the source of the waves. Rather, the wavelength (frequency) of the wave changes. The wavelength becomes longer (frequency is lower) if the source and observer are moving apart. The wavelength is shorter (frequency is higher) if the two are becoming closer. Another way of picturing the Doppler effect is given in Figure 5-2, which shows why the measured wavelength of a wave depends on the motion of the source.

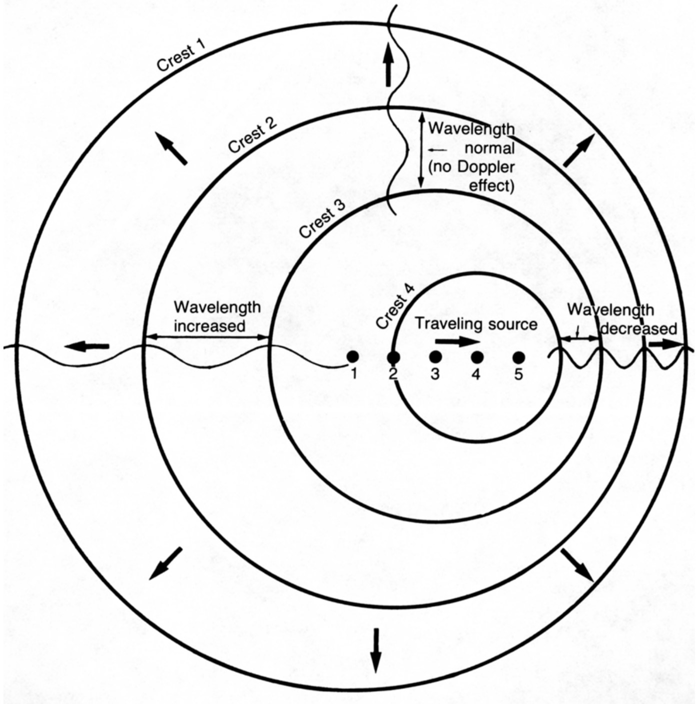

Figure 5-2. Illustration of the Doppler effect for waves. The circles represent crests of the wave. The wave spreads out radially, and each crest is centered on the position where the source of the waves was located when the crest first emerged from the source. When the viewing direction is perpendicular to the direction of motion (observer at the top or bottom of the diagram), the observed wavelength λ_{obs} equals the rest wavelength λ_0. When the source is approaching the observer (observer is to the right of the diagram), the crests are measured to be closer together, so $\lambda_{obs} < \lambda_0$. When the source is receding from the observer, the waves are measured to be farther apart, so $\lambda_{obs} > \lambda_0$. Source: www.bigear.org/CSMO/Images/CS04/cs04p25l.gif

Interference (Superposition) of Waves

When two waves pass by each other, they interact by adding their (positive or negative) displacements. This phenomenon is termed interference. While addition may seem simple, it gives rise to quite complex patterns even when only two waves of different frequencies interact. An example is given in Figure 5-3.

Since it is a property unique to waves, interference provides an excellent means for determining whether a phenomenon involves waves or particles (in the sense that particles are thought of as tiny, solid objects). We will see in Chapter 8 that this technique has indeed led to surprising results when the properties of the sub-microscopic world have been explored.

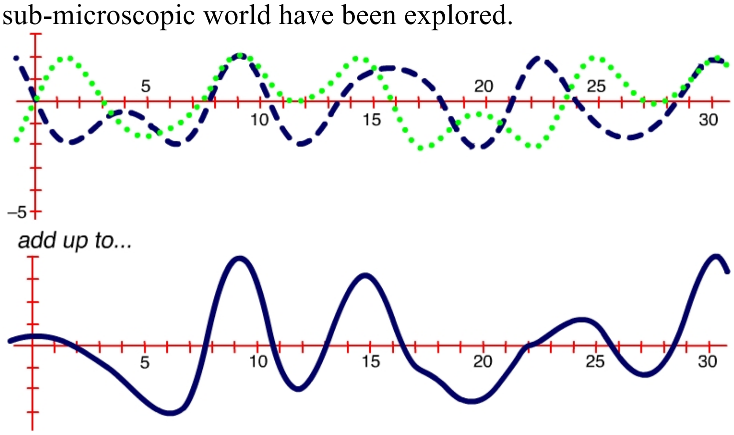

Figure 5-3. Interference of waves. When two waves intersect (*top*), their displacements at each point simply add to produce a resultant wave called an "interference pattern" (*bottom*).

Diffraction

Another important behavior is revealed when a wave strikes a barrier (such as a wall) that has a hole or slit in it: A piece of the wave passes through to create a new wave on the other side, centered on the hole or slit. Although the amplitude of the wave that emerges on the other side is lower than that of the original wave, the wavelength remains the same as that of the original wave. This phenomenon, called diffraction, is shown in Figure 5-4. As is the case for interference, diffraction is strictly a property of waves, and therefore can be used to determine whether waves or particles are involved in a phenomenon.

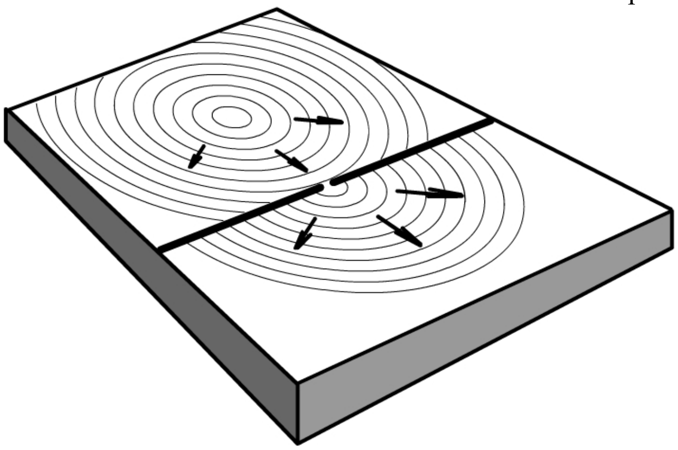

Figure 5-4. Diffraction. A piece of a wave *(upper left)* that strikes a barrier with a slit or hole passes through to the other side. The new wave *(lower right)* has the same wavelength as the original wave, but its amplitude is diminished.

Standing Waves

Consider what happens when a wave is reflected such that the reflected wave interferes with the incident wave. This is easiest to see if we only consider a small section (2-3 wavelengths) of the wave, as in Fig. 5-5. If the reflection occurs at a point where the displacement of the incident wave is zero, then by the time the reflected wave forms a trough, this trough is at the same position as the trough of the incident wave, so the interference is constructive.

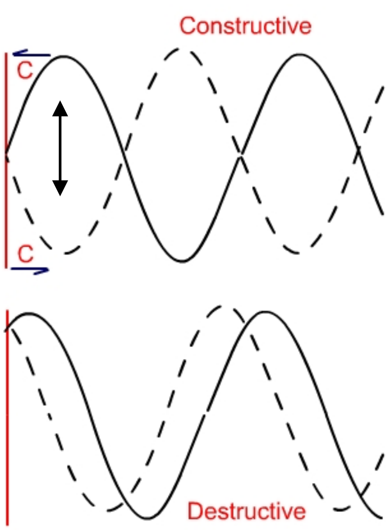

Figure 5-5. When a wave reflects off a barrier, the reflected wave interferes with the incident wave. *Top:* If the equilibrium point occurs at the barrier, the reflected and incident waves will be "in phase" so that their crests coincide, as do their troughs. In the illustration, the crests of the incident wave (solid curve) move downward to become troughs during the same time that the trough of the reflected wave (dashed curve) forms at the same distance from the barrier. This "constructive interference" amplifies the wave. *Bottom:* If the reflected and incident waves are not in phase, they will partially cancel each other.

An interesting phenomenon occurs when a wave is constrained to travel between two points from which it reflects. The reflected waves traveling in opposite directions interfere with each other. There are special wavelengths, called "harmonics," for which the troughs and crests of the waves traveling in one direction line up with those of the waves traveling in the opposite direction. This causes the wave to be reinforced. In such cases, the displacement of the incident wave is zero at the points of reflection, *which can only occur if the distance between the two points of reflection (i.e., between the boundaries) equals half the wavelength times an integer*. For other wavelengths, the waves are partially canceled by the superposition of each reflection (because they are not "in phase" — i.e., their crests and troughs do not line up with each other), so that the wave is essentially destroyed ("damped") after many reflections.

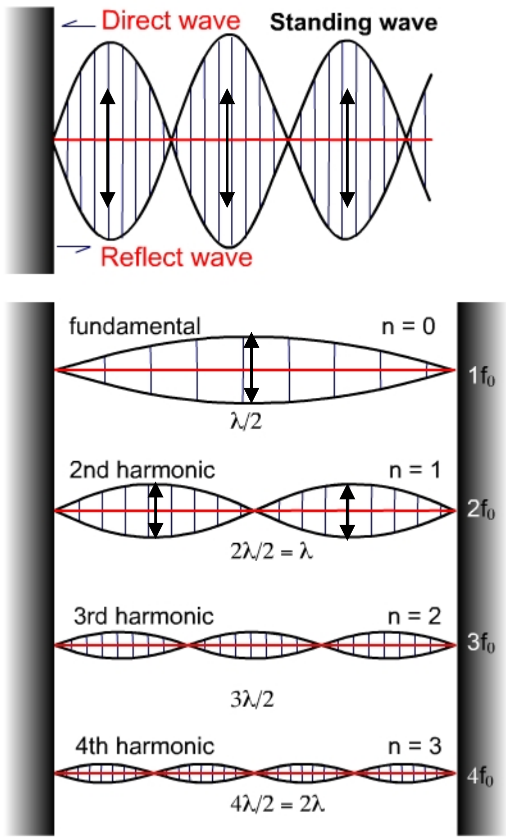

Figure 5-6. Standing waves set up when a wave is trapped between two reflective barriers. The interference between the waves traveling in opposite directions is constructive only for waves with wavelengths equal to $2L/(n+1)$, where L is the distance between the barriers and n is either zero or a positive integer. An example is a string of a musical instrument like a guitar. When the string is plucked, vibrations with many wavelengths occur. Only those with wavelengths that obey the above equation last long, though. The highest amplitude is the n=0 wave, called the "fundamental." The other surviving waves are called "harmonics." (The fundamental is also called the first harmonic.)

When the waves traveling in opposite directions are in phase, there are positions (besides the points of reflection), called nodes, where the displacement remains at zero. The number of nodes n is equal to the number of half-wavelengths that fit between the points of reflection, minus one. If L is the distance between the two reflection points, then $\lambda = 2L/(n+1)$. The superimposed (*i.e.*, combined) waves oscillate over a range of amplitudes that is greatest halfway between the nodes, while at the nodes there is no oscillation at all. Because of the presence of stationary nodes, such waves are called standing waves (or "stationary waves"). Figure 5-6 illustrates some standing wave patterns.

Sound and Music

Sound is composed of longitudinal waves caused by small disturbances in pressure. The effect of sound waves is to cause matter to vibrate from the fluctuations in pressure that the sound waves represent. The frequency of sound is sensed by humans (through vibrations of the ear drums) as pitch, with a high pitch corresponding to a high frequency. The speed of propagation of sound waves (the "speed of sound") depends on the temperature and composition of the medium through which it travels. For sound in the air, as we normally hear, the value is typically about 340 m/s (765 miles/hr). Human ears can sense a range of frequencies; above this range (some of which dogs can hear, which is the basis of a dog whistle) lie the frequencies of "ultrasound." (Very low frequency sound waves can be detected by vibrations of walls and floors that they cause.)

The Doppler effect (see the subsection above and Fig. 5-2) is quite commonly heard in the case of sound waves. For example, when a car, truck, or train sounds its horn, the pitch is higher if it is approaching you, and then dips lower after it passes you by.

Standing waves hold great importance in music. As illustrated in Figure 5-6, when a string held at high tension between two posts is plucked (or air is blown into a chamber), vibrations occur that correspond to standing waves. These consist of the fundamental tone ($n = 0$) plus higher harmonics ($n > 0$). It is the harmonics that give such musical tones their rich sounds.

The frequencies of musical notes follow a mathematical pattern. As a first example, take as the fundamental tone the pitch middle *C*, which corresponds to a frequency of 261.6 Hz. In the case of sound waves, doubling of the frequency (equivalent to cutting the wavelength in half) corresponds to an increase of the tone by one octave. Cutting the wavelength in half (*e.g.*, by shortening the string to half its size while maintaining the same tension) is the same as doubling the frequency to 523 Hz. This is the pitch high *C*, which is an octave higher than middle *C*. If we double this again to 1046 Hz, we have a *C* note that is two octaves above middle *C*.

If we add the frequency of the fundamental to that of high *C*, we get 262+523 Hz = 785 Hz, which is *G* above high *C*. If we cut this in half, to 392 Hz, we get *G* above middle *C*. If we add 262 Hz to the frequency of the very high *C* tone at 1046 Hz, we get a very high *E* note at 1318 Hz. We can cut this in half to 659 Hz to get high *E*, and in half again to 330 Hz to get *E* above middle *C*. By simply adding frequencies and multiplying or dividing the results by 2, we have therefore constructed all the elements of a *C* chord. In general, the frequency of each note of the octave (the "chromatic scale") is $2^{1/12} = 1.0595$ times higher than next lower note.

The ratios of the frequencies of the *C* chord are therefore (*E*:*G*:*C*) 1318:785:261.6, which can be reduced to 5:3:1. It is therefore the whole-number ratios of frequencies (or their higher harmonics) that, when played together, sound harmonious. The ancient Greek Pythagoras discovered this about 2500 years ago.

Light and the Electromagnetic Spectrum

Measurement of light is by far the most effective method for humans to observe and study the universe. But what is light? As determined in the 2nd half of the 1800's by the theoretical Scottish physicist James Clerk Maxwell, **light is an electromagnetic wave** (also called **electromagnetic radiation**). That is, it is a disturbance of electric and magnetic fields, just as sound is a disturbance in pressure. (Electric and magnetic fields represent the presence of forces caused by stationary and moving electrically charged particles.) However, unlike air pressure, electric and magnetic fields can exist in a vacuum. Because of this, electromagnetic waves can propagate even through a vacuum. In light waves, the electric and magnetic field direction are always perpendicular to each other, and each field oscillates as the wave propagates through space at the speed of light, which is given the symbol *c*. In a vacuum, the speed of light is 3×10^8 m/s or 300,000 km/s — an extremely high speed: a beam of light flashed from the Earth to a mirror on the Moon would take less than 3 s to return to the Earth. In fact, in Chapter 9 we will find that the speed of light is the fastest possible speed in the universe.

When the human eye sees light, the mind perceives a color. Color corresponds to the wavelength of visible light:

Red light ranges from a wavelength of 630 to 700 nm (1 nm = 1 nanometer = 1 billionth of a meter)
Orange from about 610 to 630 nm
Yellow from about 560 to 610 nm
Green from about 500 to 560 nm
Blue from about 430 to 500 nm
Violet from about 380 to 430 nm.

Together, these wavelengths compose the visible (or optical) spectrum, where a spectrum corresponds to light ordered according to its wavelengths. A rainbow is the most commonly seen example of a visible spectrum. This occurs when raindrops bend light at different wavelengths by different angles, thereby spreading them out. Just as with sound, there are many wavelengths that are possible but cannot be detected by our eyes:

Infrared (IR) — Wavelengths somewhat longer than red part of the spectrum. Intense IR radiation is sensed by humans as heat, like that felt when you hold your hand over a stove top that has just been turned off so that it no longer glows red. The infrared part of the spectrum extends from about 700 nm to about 0.1 mm.

Radio — Longer wavelengths than IR. The radio range is often subdivided into various wavelength regions ("bands") such as **microwave** (wavelengths close to 1 cm) and AM (very long wavelengths — close to 300 m).

Ultraviolet (UV) — Wavelengths somewhat shorter than violet, ranging from about 380 to 10 nm. Our skin is sensitive to UV light: after about an hour of exposure to intense UV light (as from the Sun), it will burn and/or become darker.

X-rays — Even shorter wavelengths, from about 10 nm to 0.01 nm. X-rays can penetrate many solid and liquid materials — *e.g.*, skin and bodily fluids, but not bone, which makes them very useful for imaging the inside of the body.

γ-rays (gamma-rays) — Wavelengths shorter than about 0.01 nm.

The entire electromagnetic spectrum is illustrated in Figure 5-7. As can be seen, visible light covers only a minute portion of the spectrum. Why do human eyes see such a small range of wavelengths? The reason is related to the fact that the atmosphere of the Earth allows mostly visible light from the Sun to reach the surface — most of the ultraviolet and infrared light and essentially all X-ray and γ-ray light, is blocked by the Earth's atmosphere. (This is not the entire story, since the sensitivity of the eye does not match perfectly the spectrum of sunlight we receive.) Radio waves over a wide range of wavelengths can penetrate the atmosphere, but only a small amount of the energy radiated by the Sun is emitted as radio waves. As a result, the ability to detect visible light is the most practical for a creature living on the surface of the Earth. However, for rattlesnakes, which are nocturnal, infrared detection (through the pits below their eyes) is also very useful, since they hunt animals such as mice that are warmer then their surroundings and therefore glow in infrared light (see below).

Light Acts as a Particle as well as a Wave: There is something different about the nature of light compared with most other waves: Light also acts as a particle. In fact, Newton, who thought of light as a particle, and Huygens, who thought of light as a wave, debated over its nature. It ends up that both of them were right! (We will discuss more about this in Chapter 7.) As proposed by Albert Einstein in 1905, following the work on the relation between heat and light by German physicist Max Planck in 1900, light comes in packets called photons. Each photon has an energy proportional to its frequency (inversely proportional to its wavelength):

$$E = hf = \frac{hc}{\lambda} \tag{5-3}$$

E = energy (in J, or joules, equivalent to kg m^2/s^2), h = Planck's constant = 6.63×10^{-34} J s, f = frequency (in Hz, equivalent to 1/s), c = speed of light = 3.0×10^8 m/s, λ = wavelength (in m).

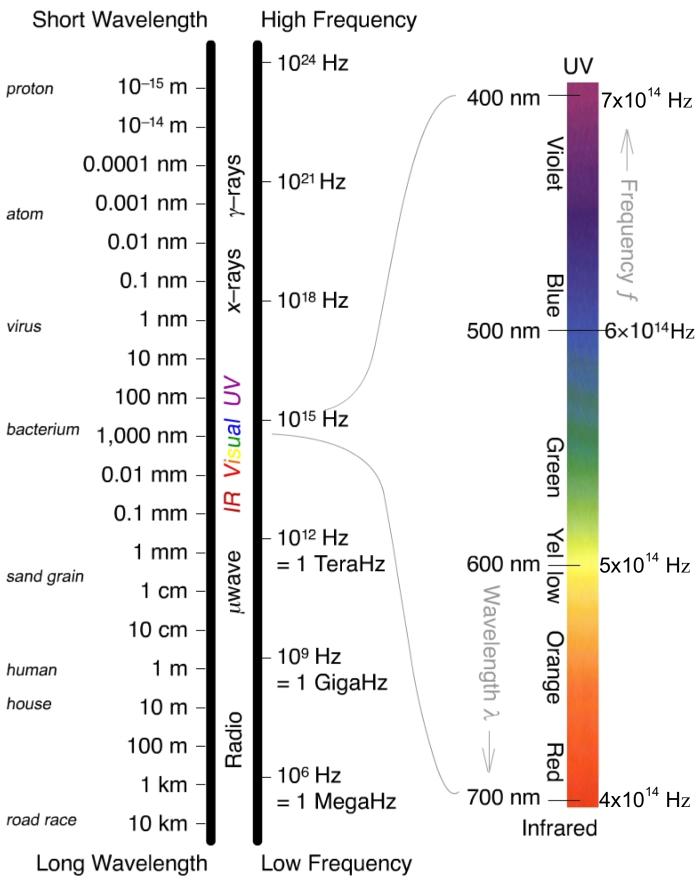

Figure 5-7. The electromagnetic spectrum. At far left are things that have similar sizes as the corresponding wavelength. Our eyes can detect only the small, visible portion of the spectrum, shown on the rightmost part of the figure. (Note: the "visual" region in the left side of the figure actually only extends from about 380 to 700 nm, while the IR sections extends from about 700 nm to 0.1 mm.

A green photon with a frequency of $6x10^{14}$ Hz (wavelength of 500 nm) therefore has an energy of only $4x10^{-19}$ J. A γ-ray of frequency $3x10^{26}$ Hz, on the other hand, has an energy of $2x10^{-7}$ J, which is substantial and can destroy cells in a human body (as can exposure to an intense beam of X-rays). At the other end of the spectrum, all the radio waves collected from celestial bodies by radio telescopes since they were first constructed in the 1940's do not amount to enough energy to melt a snowflake!

A photon also has momentum, equal to

$$p = \frac{hf}{c} = \frac{h}{\lambda}. \qquad (5\text{-}4)$$

p = momentum (in kg m/s), h = Planck's constant = 6.63×10^{-34} J s, f = frequency (in Hz, equivalent to 1/s), c = speed of light = 3.0×10^{8} m/s, λ = wavelength (in m).

This formula will prove useful in Chapter 7, where we introduce the concept of particle waves.

Electromagnetic Radiation from Opaque Objects ("Blackbody" Radiation)

What causes an object to emit light? This depends on whether the object is transparent or opaque, *i.e.*, whether you can see through it or not. The burner on your stove, the filament of a light bulb, a star, and even you are examples of opaque objects. In the latter part of the 1800's, the light emitted from an opaque object, called a blackbody (although it of course is not black if it emits light!), was studied extensively.

The luminosity L, or the amount of energy radiated per unit time (measured in watts, or W, where 1 W = 1 J/s), was found to be proportional to the temperature raised to the 4th power times the area of the surface: $T^4 \times$(*surface area*). For a sphere with radius R — a star, for example — the surface area is $4\pi R^2$, so this formula then becomes

$$L = 4\pi\sigma T^4 R^2 \tag{5-5}$$

L = luminosity (in watts, abbreviated W, equivalent to J/s), $\sigma = 5.67\text{x}10^{-8}$ W/(m^2 K^4) is the Stefan-Boltzmann constant, T = temperature (in absolute units called Kelvins, abbreviated "K"; 0 K is called "absolute zero"; adding 1 K is the same as adding 1°C), R = radius of object (in m).

The intensities of opaque objects of different temperatures are graphed in Figure 5-8 as a function of wavelength (or frequency). This is called a spectrum. The spectrum of the light emitted by an opaque object is continuous, which means that its brightness varies smoothly as the wavelength/frequency is changed. The brightness peaks at a wavelength that is inversely proportional to the temperature at its surface:

$$\lambda_{peak} = (2.88\text{x}10^6 \text{ K})/T \text{ nm} \tag{5-6}$$

λ = wavelength (1 nm = 1×10^{-9} m), T = temperature (in K).

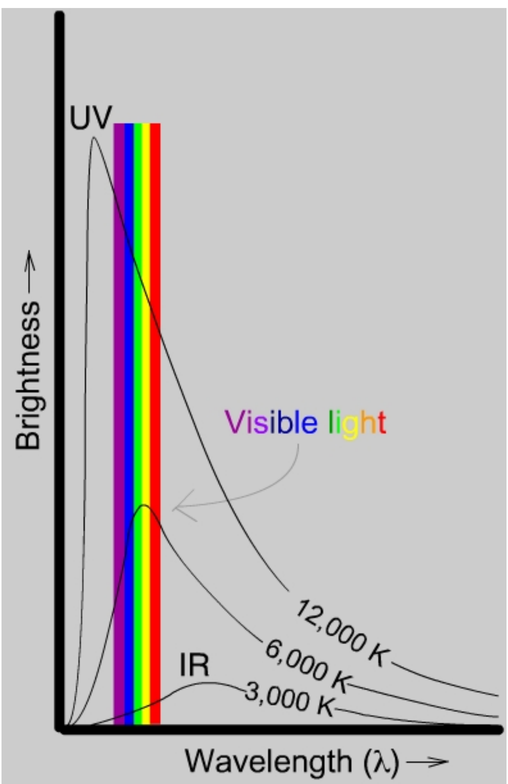

Figure 5-8. Electromagnetic spectra of blackbodies (opaque objects) at various temperatures. All of the opaque objects have the same surface area. Only objects with temperatures greater than about 1000 K emit much visible light. Note that the brightness of hotter objects is higher at all wavelengths.

Color

Although the Sun is a ball of gas, we can define its surface as the highest altitude that is still opaque to visible light. (The astronomical term for this "surface" is the "photosphere.") At this altitude, the temperature is 5800 K, so its blackbody radiation has a peak wavelength of about 500 nm. The color of light at this wavelength is green, so obviously the color of the object is not exactly that of its peak brightness. Instead, the color depends on (1) the blend of the different wavelengths of light, which is determined solely by the temperature of the surface of the glowing object, (2) the absorption and scattering — reflection in random directions — of light by the atmosphere (if one is observing an object in the sky), and (3) how our eyes and brains perceive color. For example, we see a mixture of pure red and pure green light as yellow, even though there is no light at yellow wavelengths emitted at all! In a laboratory, an opaque object with a temperature of about 1000 to 2000 K appears red to the eye, while one with a temperature above about 5000 K appears blue-white or blue.

Our eyes are not very sensitive to violet light and not at all to ultraviolet light, hence the hot objects appear blue rather than violet. (Most stars appear to be white because they are not bright enough to be detected by the color sensors in our eyes, only by the more sensitive black-and-white sensors.)

Note that the color of a hot opaque object does not depend on its size or composition: A "cool" star observed from space (where there is no atmosphere) with a temperature of 1000 K appears red. It radiates much more infrared light, but we cannot see this. An electric stove top glows with the same color when heated to the same temperature. Objects cooler than about 1000 K — humans, for example, have a temperature of 310 K — do not glow at visible wavelengths. Instead, they emit almost exclusively infrared light.

The observed brightness B of a hot, opaque object such as a star depends on its temperature T and radius R — which together determine its luminosity L — and its distance d:

$$B = \frac{L}{4\pi d^2} = \frac{4\pi\sigma T^4 R^2}{4\pi d^2} = \frac{\sigma T^4 R^2}{d^2} \qquad (5\text{-}7)$$

B = brightness (in W/m^2), L = luminosity (in W), $\sigma = 5.67\text{x}10^{-8}$ W/(m^2 K^4) is the Stefan-Boltzmann constant, T = temperature (in K), R = radius of object (in m).

Transparent objects do not emit a continuous spectrum. Rather, their spectra are composed of lines at specific wavelengths. These are discussed in Chapter 7.

The Nature of Light

Light is a form of pure energy. That is, photons have no mass in the conventional sense, only energy. The properties of light can be explained as a combination of wave phenomena and particle-like behavior. James Clerk Maxwell provided a mathematical description of light as electromagnetic waves – propagating disturbances in electric and magnetic fields in space. Max Planck found that he could explain the spectrum of blackbody radiation only if the energy transferred from heat to light is quantized in units of energy called photons. The energy of a photon is equal to a constant — now called Planck's constant — times the frequency (see eq. 5-3). The modern description of light therefore involves a wave-particle duality that was not at first understood. Chapter 7 discusses the implications of this concept.

Summary

Many of the phenomena that we experience involve waves. A periodic wave like that of sound (longitudinal pressure waves) or light (transverse electromagnetic waves) is characterized by its amplitude, its velocity of propagation c, and its wavelength λ or, alternatively, its frequency f. The wave-length is shortened (higher frequency) if the source of the waves is approaching the observer (or, equivalently, if the observer is approaching the source); it is lengthened (lower frequency) if the source is receding from the observer. This is the Doppler effect. When waves of the same type pass through the same space, they interfere with each other, resulting in an interference pattern. When passing through a hole in a barrier, diffraction occurs, with a new wave emerging on the other side. The new wave is centered on the hole and has the same wavelength as the original wave. Standing waves, which are characterized by the number of stationary nodes, occur when a wave is constrained to lie between two end points, as in a string of a musical instrument. Only standing waves with certain wavelengths survive very long (see Fig. 5-6).

The frequency of a sound wave corresponds to its pitch. The musical scale contains a well-defined pattern of frequencies. For example, the frequency of a given note (*e.g.*, high *C*) is twice that of the same note played one octave lower. The wavelength (or frequency) of visible light corresponds to its color, with red the longest (lowest frequency) and violet the shortest (highest frequency). However, the electromagnetic spectrum extends well beyond what the eye can see, ranging from radio to infrared (to visible) to ultraviolet to X-rays and -rays.

An opaque object, generally referred to as a blackbody, emits a continuous spectrum of electromagnetic waves, with the highest intensity occurring at a frequency that is proportional to the temperature of the surface of the object. Visible stars, which are essentially blackbodies, have surface temperatures ranging from about 1500 K (red) to 50,000 K (blue), with the Sun at 5800 K (yellow) being intermediate. The brightness of a blackbody is proportional to its surface temperature raised to the 4th power times the square of its radius and divided by the square of its distance from the observer.

Light is a form of pure energy that has both a wave-like and particle-like nature. Light travels in packets called photons. Chapter 7 will discuss this in more detail.

Glossary

Wave: A disturbance in which the displacement of a physical quantity (*e.g.*, air pressure or height of water) from its equilibrium value changes from negative to positive. The waves discussed in this book are periodic, which means that the displacement oscillates regularly from positive to negative values.

Transverse Wave: An oscillating disturbance in which the displacement is perpendicular to the direction of propagation of the disturbance.

Longitudinal Wave: An oscillating disturbance in which the displacement is parallel to the direction of propagation of the disturbance.

Equilibrium level: The average value of a wave, where the displacement is zero.

Crest: The point of greatest positive displacement of a wave from the equilibrium level.

Trough: The point of greatest negative displacement of a wave from the equilibrium level.

Amplitude of a wave: The maximum displacement from the equilibrium level.

Wavelength (Symbol: λ): The distance between two successive crests or troughs of a wave. Units: meters (m) or nanometers (nm = 10^{-9} m).

Frequency (Symbol: f): The number of wave crests (or troughs) that pass by a given point each second. Measured in Hertz (Hz). Note that higher frequencies correspond to shorter wavelengths (see eq. 5-1).

Doppler Effect/Doppler Shift: The change in wavelength (or frequency) caused by relative motion between the source of waves and the observer. See eq. (5-2).

Interference: Phenomenon that occurs when two waves cross each other. At each point, the displacement of each wave adds to that of the other to create a superposed wave, or "interference pattern."

Diffraction: Phenomenon that occurs when part of a wave passes through a small hole or slit in a barrier. A new wave, centered on the hole, emerges on the opposite side of the barrier. This new wave has the same wavelength as the incident wave, although its amplitude is lower.

Standing Wave: Pattern that can occur when a wave interferes with its own reflection. The pattern contains (stationary) nodes where the displacement is zero.

Node: A stationary point on a standing wave where the displacement stays at zero (*i.e.*, the equilibrium position is maintained). The points of reflection are not counted as nodes.

Harmonics: Standing waves within which one or more nodes exist. Wavelength is equal to twice the distance L between the reflection points divided by 1 plus the number of nodes n: $\lambda = 2L/(n+1)$. The standing wave corresponding to $n = 0$ is called the first harmonic, $n = 1$ is the 2nd harmonic, and so forth.

Fundamental frequency (wavelength): The frequency (wavelength) of a standing wave that has no nodes ($n = 0$). Also referred to as the first harmonic.

Pressure: Force per unit area. Units: Newtons per square meter (N/m^2).

Sound: A longitudinal wave caused by small disturbances in the pressure of a medium such as air.

Speed of Sound: Velocity at which sound waves propagate; depends on temperature and the composition of the medium.

Pitch: The frequency of a sound wave. Sound waves with longer wavelengths have lower pitch.

Electromagnetic Wave/Radiation: A propagating disturbance in electric and magnetic fields. Visible light is an example.

Field (commonly "Force Field"): A way of describing the presence of a force and how the magnitude and direction of the force depends on position. Examples: electric field, magnetic field, gravitational field.

Speed of Light (Symbol: c): The velocity at which electromagnetic waves propagate through a vacuum, equal to $3.00 \text{x} 10^8$ m/s (300,000 km/s).

Visible Light: Electromagnetic waves to which human eyes are sensitive.

Color: How humans perceive different wavelengths (frequencies) of visible light.

Spectrum: A representation of the amplitude of a group of waves as a function of wavelength (frequency). For electromagnetic waves, a spectrum indicates how brightness changes with wavelength. A commonly seen example for visible light is a rainbow.

Continuous Spectrum (See also "spectrum"): The case when the brightness varies smoothly with wavelength. Contrasts with a line spectrum (see Ch. 7).

Photon (often denoted by the symbol γ): Unit of light that can be thought of as a packet of waves (see Ch. 7). The energy and momentum of a photon depend on frequency (wavelength); see eqs. (5-3) and (5-4).
Radio Waves: Electromagnetic waves with the longest wavelengths (lowest frequencies).

Microwaves: Relatively high-frequency radio waves, with wavelengths ranging from about 1 mm to a few centimeters.

Infrared (or IR) Light: Electromagnetic waves with wavelengths longer than visible light but shorter than radio waves.

Ultraviolet (or UV) Light: Electromagnetic waves with wavelengths somewhat shorter than visible light.

X-rays: Electromagnetic waves with wavelengths shorter than ultraviolet light and much shorter than visible light.

γ-rays (gamma-rays): Electromagnetic waves with the shortest wavelengths and highest frequencies. γ-ray photons have the highest energies of all types of electromagnetic waves.

Blackbody: An opaque object. Any blackbody that has a temperature higher than absolute zero (0 K) emits a continuous spectrum of electromagnetic waves. See eqs. (5-5 to 5-7) for relations between the radiation of a blackbody and its physical properties.

Opaque: Adjective meaning “cannot be seen through.” An opaque body absorbs light that it intercepts, but also emits electromagnetic waves of its own, with greatest intensity at a wavelength that depends inversely on the body’s temperature; see eq. (5-6).

Luminosity: Energy per unit time emitted by an object. It is an intrinsic property that does *not* depend on the distance between the object and the observer. Measured in watts (W, the same as J/s).

Brightness: The energy of light (usually over a specific range of wavelengths) per unit area per unit time that is received by an observer. (Note: this is the common meaning of “brightness;” scientists use the word “flux” instead.) The brightness of an object depends on the luminosity as well as on the distance between the object and the observer (see eq. 5-7).

Temperature (Symbol: T): A quantity used to measure the thermal energy (heat) of an object or medium. Most conveniently measured in Kelvins (K). At a temperature of 0 K, the thermal energy equals zero.

Questions for Discussion

A. If a tree falls in a forest, what types of waves are emitted? Could those waves have a physical effect on anything else, even if no person were close enough to hear, see, or feel the tree falling?

B. Does the fact that the frequency of sound or light from a moving source depends on the direction of the observer mean that different observers experience different realities? Why or why not?

C. Hearing a voice through a small opening of a door is an example of diffraction. When you hear such a voice, is the pitch the same as in the room of the speaker? How about the volume (amplitude)? Compare with the description of diffraction in this chapter.

D. The notes of a major chord have frequencies that form a 5:3:1 ratio. The entire musical scale of a piano is based on similar relationships between frequencies. Have you heard any music that is considered "beautiful" by some people that does not follow this pattern?

E. Can you determine the approximate temperature of the surface of a star just by looking at it? How would you do this? What could cause you to get the wrong answer?

F. Why is it strange that light should have the characteristics of both a wave and a particle?

Examples of How to Solve Problems on Waves and Light

1. Calculate the wavelength in cm of microwaves that have a frequency of $1.5\text{x}10^{10}$ Hz.

Answer: Use formula (5-1), use the speed of light for the wave speed, $c = 3.0\text{x}10^8$ m/s, and solve algebraically for wavelength:

$\lambda = \frac{c}{f} = \frac{3.0\times10^{8}\text{ m/s}}{1.5\times10^{10}\text{ Hz}}$ = <u>0.020 m</u>. (Note that the unit Hz is the same as 1/s.)

2. An astronaut in a spaceship moving away from Earth at a velocity of 0.1c shines laser light back toward the Earth. The light has a rest wavelength of 600 nm. What is the wavelength as measured by an observer on Earth?

Answer: Use formula (5-2): $\lambda_{\text{obs}} = \lambda_0(1 + \frac{v}{c})$.

We are given $v = 0.1c$ (positive because motion is away and = 600 nm. Because the wave is light, the wave speed is the speed of light, $c = 3.0\text{x}10^8$ m/s. So, we have

$$\lambda_{\text{obs}} = \lambda_0\left(1 + \frac{v}{c}\right) = (600\text{ nm})\left(1 + \frac{0.1\,c}{c}\right) = (600\text{ nm})(1+0.1) = (600\text{ nm})(1.1) = \underline{660\text{ nm}}.$$

3. Star A has a surface temperature (T) that is three times higher than that of star B. The two stars have the same radius (R). Compare their luminosities (L). [Note: The preferred form of a comparison is to determine how many <u>times</u> greater or smaller the quantity is for one object than for the other.]

<u>Solution</u>: The relevant equation is (5-4):

$$L = 4\pi\sigma T^4R^2.$$

Here, σ is a constant. While you could plug in the value of the constant and try to calculate L for each star, this would fail because you are not given R – you only know that it is the same for each star. The easiest way to solve this problem is to **form an algebraic ratio for both the right- and left-hand sides of the equation:**

$$\frac{L_A}{L_B} = \frac{4\pi\sigma T_{\text{A}}^4 R_{\text{A}}^2}{4\pi\sigma T_{\text{B}}^4 R_{\text{B}}^2}.$$

In other words, divide the entire equation for star A by the equation for star B. The answer, as given on the left-hand side, is the ratio of the luminosity of star A to that of star B, *which is precisely what you need for your final answer to the question!* (This will not always be the case: Sometimes you need to do some algebra to get the ratio of the quantity of interest on the left-hand side.)

Now the task is much easier: You can cancel the $4\pi\sigma$ term since it is divided by itself. Also, since you are given that $R_A = R_B$ in this problem, you can cancel these as well to get

$$\frac{L_\mathrm{A}}{L_\mathrm{B}} = \frac{T_\mathrm{A}^4}{T_\mathrm{B}^4} = \left(\frac{T_\mathrm{A}}{T_\mathrm{B}}\right)^4 = 3^4 = 81,$$

where the numbers correspond to the information given that T_A is 3 times T_B. So, the answer, stated in words, is that <u>the luminosity of star A is 81 times that of star B</u>.

4. Star X has the same brightness (B) as star Y, but the temperature (T) of star X is twice that of star Y. Furthermore, the distance d_X to star X is two times that of star Y (d_Y). Compare the radius (R) of star A with that of star B.

<u>Solution</u>: The relevant equation is (5-7):

$$B = \frac{\sigma T^4 R^2}{d^2}.$$

Since we are interested in comparing radii, we should first solve this equation for R, then form an algebraic ratio before plugging in the numbers. As a first step, we multiply both sides of the equation by $[d^2/(\sigma T^4)]$ and switch the left- and right-hand sides:

$$R^2 = \frac{Bd^2}{\sigma T^4}.$$

We now form an algebraic ratio of this equation:

$$\frac{R_\mathrm{X}^2}{R_\mathrm{Y}^2} = \frac{Bd_\mathrm{X}^2/(\sigma T_\mathrm{X}^4)}{Bd_\mathrm{Y}^2/(\sigma T_\mathrm{Y}^4)} = \frac{B_\mathrm{X} d_\mathrm{X}^2(\sigma T_\mathrm{Y}^4)}{B_\mathrm{Y} d_\mathrm{Y}^2(\sigma T_\mathrm{X}^4)}.$$

[Note that, for complex divisions as on the right-hand side, the term (σT_X^4) divided into the numerator can become part of the overall denominator, while the term (σT_Y^4) divided into the denominator can become part of the overall numerator.] Since σ is a constant and the brightness is the same for both stars, these two quantities cancel out. So, we can simplify the equation to the form

$$\frac{R_\mathrm{X}^2}{R_\mathrm{Y}^2} = \frac{d_\mathrm{X}^2 T_\mathrm{Y}^4}{d_\mathrm{Y}^2 T_\mathrm{X}^4}.$$

We need to simplify further by taking the square-root of both sides to get

$$\frac{R_\mathrm{X}}{R_\mathrm{Y}} = \sqrt{\frac{d_\mathrm{X}^2 T_\mathrm{Y}^4}{d_\mathrm{Y}^2 T_\mathrm{X}^4}} = \frac{d_\mathrm{X} T_\mathrm{Y}^2}{d_\mathrm{Y} T_\mathrm{X}^2} = \left(\frac{d_\mathrm{X}}{d_\mathrm{Y}}\right)\left(\frac{T_\mathrm{Y}^2}{T_\mathrm{X}^2}\right) = (2)\left(\frac{1}{2}\right)^2 = \frac{1}{2} = \underline{0.5.}$$

So, we find that the radius of star X must be half that of star Y.

5. A star has a "surface" temperature of 6000 K. Calculate the peak wavelength of its brightness vs. wavelength curve. What color does this peak wavelength correspond to?

<u>Solution</u>: The relevant equation is (5-6): $\lambda_\mathrm{peak} = (2.88\mathrm{x}10^6\ \mathrm{K})/T$ nm. We therefore have

$\lambda_\mathrm{peak} = (2.88\mathrm{x}10^6\ \mathrm{K})/(6000\ \mathrm{K})$ nm = <u>480 nm</u>.

According to the correspondence between wavelength and color given on page 5-7, the color of this wavelength is <u>blue</u>.

Homework Problems

1. Calculate the wavelength (in meters) of a radio wave that has a frequency of 150 MHz (1.5×10^8 Hz, or 150,000,000 Hz). Compare the wavelength (a) to the typical height of a human (1.7 m) and (b) to a foot (0.3 m) by calculating the ratio of the wavelength to the size of a human or a foot.

2. A red photon has a wavelength of 660 nm. How fast must a star be moving toward the Earth for a red photon emitted by the star to change its color to an orange photon of wavelength 620 nm? You may, if you wish, express your answer either in m/s or in terms of the speed of light c (*e.g.*, "$0.021c$"). [You'll need to do a little bit of algebra. Keep in mind that, by convention, the velocity is negative if the motion is toward the observer. Assume that the speed is less than $0.2c$, then check whether this assumption is correct.]

3. (a) Use eq. (5-2), plus a bit of algebra, to determine how fast a train would need to move away from you in order for the observed wavelength to be 1.05 times the rest wavelength. (Note that receding velocities are positive.) This corresponds to half a musical tone higher than if the train were stationary. Express your answer in terms of the speed of sound c_s, (*e.g.*, "$0.9c_s$").
(b) Now use the typical value $c_s = 340$ m/s to compute the velocity in m/s.
(c) Multiply your answer in part (b) by 3.6 to convert to km/hr. Is this a reasonable speed for a train?

4. Star A has a surface temperature 2 times higher than the Sun and a radius that is half that of the Sun. How many times more luminous or less luminous is star A compared with the Sun? (That is, what is the luminosity of the star divided by that of the Sun?)

5. Star X has a surface temperature 2 times higher than the Sun and a radius 0.01 times that of the Sun. Star Y is 100 times farther away from the Earth than is star X; it has a surface temperature 4 times hotter than the Sun and a radius $\sqrt{2}$ times that of the Sun. Which star is brighter as observed from the Earth and by how many times brighter is it than the other star? (That is, what is the ratio of its brightness to that of the fainter star?)

6. A star has a "surface" temperature of 5000 K.
a. Calculate the peak wavelength of its brightness vs. wavelength curve.
b. What color does this wavelength correspond to? (Note: if the wavelength is in the infrared or ultraviolet range, state "IR" or "UV" as the color.)
c. Now imagine that the star is moving toward us at a velocity of $0.1c$, where c is the speed of light. What peak wavelength would we observe in this case?
d. What color does the peak wavelength obtained in part (c) correspond to? (See note in part b.)

7. A star has a "surface" temperature of 10,000 K.
a. Calculate the peak wavelength of its brightness vs. wavelength curve.
b. What color does this wavelength correspond to? (Note: if the wavelength is in the infrared or ultraviolet range, state "IR" or "UV" as the color.)
c. Now imagine that the star is moving away from us at $0.2c$, where c is the speed of light. What peak wavelength would we observe in this case?
d. What color does the peak wavelength obtained in part (c) correspond to? (See note in part b.)