

Instructions for Team Project on Active Galactic Nuclei

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Background

Active galactic nuclei (abbreviated “AGNs”) are the centers of galaxies with super-massive black holes that are actively accreting a large amount of gas from their surroundings. The black hole masses range from a bit less than a million to billions of solar masses. The accretion rate of gas, α , is measure in solar masses per year. The luminosity of the accretion disk of ionized gas that is orbiting – and gradually falling into – the black hole is roughly

$$L_{\text{disk}} \approx 6 \times 10^{38} \alpha \text{ watts.}$$

There is a limit to the luminosity, though, called the **Eddington limit**: If the luminosity of the disk were higher than the Eddington limit, the force from the light moving outward from the disk and hitting the gas would overcome the gravity and push the gas away, destroying the disk. For a black hole of mass M_{BH} measured in solar masses, the Eddington luminosity limit is

$$L_{\text{disk}} < L_{\text{eddington}} \approx 1.3 \times 10^{31} M_{\text{BH}} \text{ watts.}$$

Between 5 and 10% of AGNs have a pair of powerful jets of high-energy particles and magnetic fields, one in each direction along the rotation poles of the accretion disk. (The plane of the disk defines the equator of the system.) Astrophysicists think that the jets are formed by magnetic fields that get twisted by the rotation of the disk and the black hole (which, if it also rotates, drags space-time around it). The material in the jets – mostly electrons, protons, and possibly positrons (anti-electrons) – flows away from the black hole at a speed as high as 0.9998 times the speed of light c (so that the Lorentz factor Γ is as high as about 50). This high speed causes some extreme observed effects If one of the jets points almost directly at us (within several degrees):

Apparent superluminal (faster than light) motions

Bright regions (“blobs”) in the jet appear to move at a speed that can be as high as $v_{\text{app}} \approx \Gamma c$.

Relativistic beaming of the light

The light from the jet is beamed into a small angle (about $1/\Gamma$ radians), so that the luminosity of the jet appears to be about Γ^3 times higher than is actually the case.

This beaming of the light from the jets makes them extremely bright “beacons” that can be observed with telescopes even if they are billions of parsecs away. In fact, they are the brightest objects in the sky outside the band of the Milky Way at microwave and γ -ray wavelengths, at which stars are very faint. AGNs with a jet pointing at us are called **blazars**.

Speeding up of changes in brightness

Although, according to Special Relativity, time in a moving object is observed to be slower than at rest, if the object is moving toward us at near-light speed, the distance keeps getting smaller, so that light emitted later takes less time to get to us than light emitted earlier. This effect causes us to observe **events happen Γ times faster than if they were at rest** relative to us.

The class will be divided into 8 teams. Each team will need a coordinator. The coordinator will organize the team; during the presentation to the entire class, the coordinator will introduce the team, coordinate their work, and present the team's conclusions. The other members of the group will be data analysts. The data analysts will form **three sub-groups of 2-3 students each. Each sub-group will analyze one of three types of data: spectrum, multiple microwave images from different times, and variations of brightness. Each sub-group will have a presenter** of that sub-group's results. So, **each team will have four presenters in total.**

The goal of this exercise is to analyze data from some world-class telescopes to determine a number of interesting properties of blazars, with each team analyzing data for one blazar.

Spectrum (in file spectrum_blazarX.pdf, where X is your team's blazar – A, B, C, ...)

All of the AGNs that the teams will analyze are **blazars**, with an ultra-fast jet pointing within several degrees of our direction. They are also **quasars**, which are the most luminous AGNs, so much so that they completely outshine their "host" galaxies. The nearest blazar/quasar is 560 megaparsecs (abbreviated "Mpc," meaning millions of parsecs). This means that the redshift of every object that the class will study is so high that the lines in the spectrum are shifted to much longer wavelengths than the rest wavelengths. This confused astronomers when they first started to measure the spectra of quasars in the early 1960s. But their confusion was short-lived because the Doppler effect, which causes the redshifts, preserves the ratios of the wavelengths of the lines in the spectrum. They only needed to calculate the ratios to realize that the lines were those of familiar elements, but shifted to much longer wavelengths.

One of the characteristics of AGNs is that their spectra contain emission lines from gas clouds that are ionized by UV photons emitted by the hot accretion disk not too far (of order 10-100 Schwarzschild radii) from the black hole. The table below lists the most prominent emission lines in the spectra of quasars. The ions are identified by both the chemistry system (e.g., C⁺²) and the astronomers' system (e.g., C III).

Table of Strongest Emission Lines in AGN Spectra and the Ratios of their Wavelengths

Atom or Ion	λ_0 (nm)	$\lambda / \lambda(\text{H}\alpha)$	$\lambda / \lambda(\text{H}\beta)$	$\lambda / \lambda(\text{H}\gamma)$	$\lambda / \lambda(\text{Mg}^+)$	$\lambda / \lambda(\text{C}^{+2})$	$\lambda / \lambda(\text{C}^{+3})$	$\lambda / \lambda(\text{Ly}\alpha)$
Neutral H (H α)	656.3	—	1.35	1.51	2.35	3.44	4.24	5.40
Neutral H (H β)	486.1	0.74	—	1.12	1.74	2.55	3.14	4.00
Neutral H (H γ)	434.0	0.66	0.89	—	1.55	2.27	2.80	3.57
Mg ⁺ [Mg II]	279.8	0.43	0.58	0.64	—	1.47	1.81	2.30
C ⁺² [C III]	190.9	0.29	0.39	0.44	0.68	—	1.23	1.57
C ⁺³ [C IV]	154.9	0.24	0.32	0.36	0.55	0.81	—	1.27
Neutral H (Ly α)	121.6	0.19	0.25	0.28	0.43	0.64	0.79	—

Each team will be supplied with the visible-light spectrum of a blazar obtained with the Lowell Discovery Telescope (LDT) in Arizona, which is operated by Lowell Observatory. (Boston U. is a partner on the LDT.) All members of the spectrum sub-group should examine the visible spectrum of their blazar to determine which emission lines are in the visible spectrum by

calculating the ratios of the wavelengths of the emission lines (which are marked by arrows on the spectrum) and comparing them with the ratios in the table. Be careful: some of the ratios are close to each other, so the measurements of the wavelengths need to be as precise as possible. Each member of the spectrum sub-group should identify the lines in the spectrum independently, then compare their results. If there is a discrepancy, they should discuss their procedures until they come to agreement.

After agreeing on the lines that are present in the spectrum, the sub-group should calculate the ratio of the observed wavelength to the rest wavelength λ_0 of each line, then average the values of the ratios (which all should be close to each other). Then the sub-group should calculate the redshift $z: z = (\lambda/\lambda_0) - 1$, where λ is the wavelength as measured on the spectrum. You should expect the redshift of your blazar to be between 0.03 and 2.5. If it is outside this range, check your calculations. (You might want to check with the instructor about your value.)

Since there are usually no other clues to a blazar's distance, astronomers use the Hubble Law and the redshift to determine the distance. Because the universe's rate of expansion has changed with time, the Hubble Law is not as simple as that derived by Hubble from galaxies that are no more than a few hundred Mpc away. So, they use the modified Hubble Law obtained from observations of Type 1a supernovae from more distant galaxies. The relationship between redshift and distance is complex, but astronomer Ned Wright has provided a distance calculator at URL <http://www.astro.ucla.edu/~wright/CosmoCalc.html>. Go to the website, where you will find a window on the left that appears as:

69.6	H ₀
0.286	Omega _M
3	z
Open	Flat
0.714	Omega _{vac}
General	

but you want to enter your own values, e.g.:

70	H ₀
0.286	Omega _M
1.00	z
Open	Flat
0.714	Omega _{vac}
General	

So, erase the "69.6" and type "70" since we use a Hubble constant of 70 (km/s)/Mpc. Erase the "3" and type in the redshift that you have determined for your blazar. In the example on the right above, a redshift of 1.00 was typed in. You can leave the other numbers as they are. Then click on the "Flat" button to do calculations for a universe with a flat geometry, which is what astronomers think is the case. You will then see the numbers in the window on the right change. For the example of $z=1.00$, the window on the right now appears as:

For $H_0 = 70$, $\Omega_M = 0.286$, $\Omega_{vac} = 0.714$, $z = 1.000$

- It is now 13.642 Gyr since the Big Bang.
- The age at redshift z was 5.869 Gyr.
- The [light travel time](#) was 7.773 Gyr.
- The [comoving radial distance](#), which goes into Hubble's law, is 3331.4 Mpc or 10.866 Gly.
- The comoving volume within redshift z is 154.873 Gpc³.
- The [angular size distance \$D_A\$](#) is 1665.7 Mpc or 5.4329 Gly.
- This gives a scale of 8.076 kpc/".
- The [luminosity distance \$D_L\$](#) is 6662.8 Mpc or 21.731 Gly.

The critical information that you should record here is:

- a. The age of the universe at redshift z (how old the universe was when the light we now observe left the blazar); call this “age-universe”, symbol t_u , measured in Gyr (10^9 years)
- b. The luminosity distance d_L ; this is defined such that the luminosity $L = 4\pi d_L^2 B$, where B is the brightness
- c. The scale that converts arcseconds (”) to kiloparsecs, which is the same as the scale that converts milli-arcseconds (mas) to parsecs (pc), which the image sub-group will need; give the symbol s (in pc/mas) to this scale; in the example $z=1.00$, $s = 8.076$ pc/mas.

Report to your team which emission lines you have identified and their observed wavelengths, and the values of z , t_u , d_L , and s .

For one of the lines that has a “clean” profile (meaning that it looks similar to a bell curve), **measure the width, W , of the line halfway up the line** (this is called the “full width at half maximum”). Do this in mm, then multiply by the number of nm per mm, which you can determine from the scale on the graph. **Report this to your team as W** (in nm). Also, measure on the spectrum the brightness (in watts/m² – note that the scale is the number on the graph times a small number like 10^{-15}) at 510 nm. If there is an emission line at that wavelength, estimate what the brightness would be if there were no line. **Report this to your team as B_{510} .**

Microwave Images (in file VLBAimages_blazarX.pdf, where X is your team’s blazar – A, B, C, ...)

The author and his collaborators use the Very Long Baseline Array (VLBA), operated by the National Radio Astronomy Observatory (NRAO), to make microwave images of the jets of blazars. The array consists of 10 radio dishes scattered around the US, including Mauna Kea, Hawaii on the western end and the US Virgin Island of St. Croix and Sargent Camp, New Hampshire (a BU facility) on the eastern end. The observations are obtained at a wavelength of 7 mm, at which the VLBA images have an ultra-high resolution of about 0.15 mas. At the distances of blazars, this corresponds to less than a parsec, which is adequate for following the motions of bright “blobs” as they move down the jet and away from the black hole.

An image of a blazar jet contains a bright spot at one end called the “core” and, often, one or more moving blobs. The image represents brightness by closed curves called “contours,” where the brightness is maximum at the center of the contours, and a larger number of concentric contours corresponds to higher brightness.

Everyone in the image sub-group will examine a set of images of the team’s blazar, made from data obtained on different dates. (The process required to make images from VLBA data is quite complex, so you are provided with the final images.) Each image includes a 4-digit number that is the “reduced Julian date” (RJD), which is the Julian date minus 2450000.5. (Astronomers use Julian date – days since noon on 1 January 4713 BCE – because calendar years can contain either 365 or 366 days. RJD=5900 is 30 May 2020.) The center of the core is marked with a “+” and the center of the blob is marked with a red “X”. There may be two blobs, in which case the 2nd one is marked with a blue “X”; in this case, half of the sub-group should analyze the red blob and the other half analyze the blue blob. For each blob, measure the following:

- a. The scale factor f of each image in mas/mm (they should be nearly, if not exactly, the same): measure the distance in mm between the 1 mas mark and the 0 mas mark; put the number of mm in your calculator and use the $1/x$ function to get f
- b. The distance r of the center of the blob from the center of the core in each image, in mm
- c. Multiply r by f to get the distance y of the center of the blob from the center of the core in each image, in mas

Make a table of your measurements, listing the values of time t (the RJD minus the RJD of the first image), f , r , and y . The sub-group should then **average the values of each measurement of f , r , and y after checking for, and correcting, major differences.**

One or more of the members of the sub-group should create a graph where the horizontal axis is t in days and the vertical axis is y (in mas). Then the sub-group needs to determine the best-fit straight line by using the least-squares method:

- i. For each day, calculate t^2 and yt . (These will be zero on the day of the first image.)
- ii. Add all the values of t , all the values of y , all the values of t^2 , and all the values of yt . Use the Greek letter Σ to denote these sums: Σt , Σy , Σt^2 , $\Sigma (yt)$.
- iii. Calculate the slope m of the best-fit line, where N is the number of images:

$$m = \frac{N \Sigma(yt) - \Sigma y \Sigma t}{N \Sigma t^2 - (\Sigma t)^2}$$

- iv. Calculate the y intercept, b , of the best-fit line

$$b = \frac{\Sigma y - m \Sigma t}{N}$$

- v. The formula of the best-fit line is then $y = mt + b$. Draw this line on your graph.

- vi. The slope is in mas/day. Convert this to mas/yr by multiplying by 365.24 days/yr. This is the “proper motion,” whose symbol is the Greek letter μ .

- vii. Calculate the RJD when the blob was coincident with the core (i.e., when y was 0), under the assumption that the proper motion was constant. From the equation for the line, set $y = 0$ so that $0 = mt_0 + b$; then $t_0 = -b/m$ (t_0 will be negative) is the time of zero blob-core separation. Now add t_0 to the RJD of the first image to get the RJD of zero separation. Refer to this as the “ejection time,” T_0 .

- viii. Report to your team coordinator the proper motion μ and the ejection time T_0 , and show the team the sub-group’s graph.

Variations (in file lightcurves_blazarX.pdf, where X is your team’s blazar – A, B, C, ...)

Astronomers call a graph of brightness vs. time a “light curve.” NASA’s Fermi Gamma-ray Space Telescope measures the γ -ray brightness of every position in the sky every 3 hours. However, it usually takes about one week to detect enough γ -rays from a blazar to measure the brightness. Visible light observations are made less regularly, since they cannot be done during the day or during cloudy weather. Blazars emit visible light from both the accretion disk and the jet, but only

the jet's light changes brightness rapidly – over days and weeks; the disk's visible-light brightness is steadier. A blazar's γ -rays come only from the jet. Both visible and γ -ray light is made by ultra-high energy electrons in the jet. So, one might expect both to vary in brightness together. But the visible light is from synchrotron radiation, whose brightness depends both on how many high-energy electrons are present and on how strong the magnetic field is. The γ -ray brightness depends on the number of high-energy electrons, but also on the number of visible-light photons from the disk, jet, and hot gas clouds that are present. So, they only sometimes vary in a similar way.

The variations sub-group will examine the γ -ray and visible light curves to determine how fast the changes in brightness occur, which can be used to calculate the largest size that the region involved in the emission can have. For example, if the changes occur in one year, the region must be no more than 1 light-year from the far side to the near side, since in a larger region the light from the far side will take longer than 1 year to reach us after the light from the near side reaches us. This limit, though, needs to be corrected for the effects caused by the host galaxy moving away from us and the jet moving toward us at nearly the speed of light.

The PDF file contains one graph of the γ -ray and visible-light brightness vs. time over 4000 days (RJD = 5000-9000; see the Microwave Images section for the definition of RJD). It also contains one or more zoomed-in light curves over shorter time intervals, with which measurements are easier to make. The horizontal lines on the γ -ray data indicate the range of time over which the brightness was measured and the vertical lines correspond to the uncertainty in the brightness. (The time range and uncertainty of the visible-light measurements are smaller than the data points on the graph. Note: γ -rays were observed every 7 days; any missing data points means that the γ -rays were not detected during that time span.) Each member of the variations sub-group should measure and record the following:

1. RJD time ranges of “outbursts” and “flares” when the γ -ray and/or visible light brightness was much higher than usual. (You should use your judgment as to what “much higher” means.) A flare is short-lived and usually occurs over a short time span (1 week or less), while an outburst lasts longer. Major outbursts are already marked on the light curves you will examine.
2. Inspect the zoomed-in light curves of outbursts for the shortest time range when the brightness decreased substantially (e.g., to $\frac{1}{2}$ of its previous value). Then estimate from the graph the brightness just before the decrease, B_{high} , the brightness after the decrease, B_{low} , and the RJD times of high (t_{high}) and low (t_{low}) brightness. (Make sure that you multiply the brightness by the small number indicated in the units, e.g., 10^{-15} .) Calculate the time-scale of variability, t_{var} , as $t_{\text{var}} = (t_{\text{low}} - t_{\text{high}}) (B_{\text{low}}/B_{\text{high}})$. Use days as the unit of t_{var} .
3. Estimate the highest brightness, B_{max} , seen in the γ -ray and in the visible light curves.
4. Determine whether the peaks of any γ -ray and visible-light flares occurred at the same time.
5. Count the number of significant flares over the entire period of each of the γ -ray and visible light curves on the first page.
6. Estimate the fraction of time that there was an outburst or flare in progress (to nearest 10%).

The sub-group should report to the team coordinator the number of significant flares, the fraction of time when there was an outburst or flare, the time ranges of outbursts, the times of peaks of flares, B_{high} , B_{low} , B_{max} , and t_{var} , all at both γ -ray and visible wavelengths.

Analysis by the Entire Team:

The team coordinator will gather all of the data reported by the sub-groups and share the information with the entire team. The spectrum sub-group should have reported the **emission lines** and their **observed wavelengths** λ , the **width** W of one of them, the values of z , **universe-age**, d_L , s , and B_{510} . The image sub-group should have reported the **proper motion** μ and the ejection time T_0 of one or more blobs, along with the **graph** of distance from the black hole, y , vs. time t , specifying the **RJD time corresponding to $t=0$** . The variations sub-group should have reported the **number of significant outbursts & flares**, the **fraction of time when there was an outburst or flare**, the **time ranges of outbursts and flares**, t_{var} , B_{high} , B_{low} , and B_{max} , all at both γ -ray and visible wavelengths.

All members of the team should perform the following calculations and communicate the results to the coordinator. The coordinator should work with team members to correct the calculations for any of the results that are inconsistent.

- A. The luminosity at 510 nm. This is given by $L_{510} = 4\pi d_L^2 (3.1 \times 10^{22} \text{ m/Mpc})^2 B_{510}$ (L_{510} is in watts).
- B. The mass of the super-massive black hole. Astronomers have found that the formula below works quite well. It is based on the idea that the emission lines come from thousands of clouds of hot gas that are orbiting the black hole. The clouds that produce a particular emission line lie at distance r from the black hole such that they receive a particular brightness from the accretion disk; so the value of $L/(4\pi r^2)$ is about the same from one AGN to another. The value of W for the line depends on the average speed of the clouds' orbits, which should be $\sqrt{GM_{BH}/r}$. This is because the width of the line comes from some clouds moving toward us (relative to the AGN), some moving away, and others moving perpendicular to our line of sight. So, different clouds have different Doppler shifts. The average orbital speed is then $(W/\lambda)c$. This leads to a formula for the black hole's mass (in solar masses, M_s):

$$M_{BH} \approx 8 \times 10^6 (300W/\lambda)^2 \sqrt{(L_{510}/1 \times 10^{37} \text{ watts})} M_s$$

- C. The Schwarzschild radius $R_{BH} = 3000 M_{BH}/M_s$ meters. Divide this by 1.5×10^{11} meters/AU to get the **radius of the black hole in astronomical units (AU)**, the semi-major axis of the Earth's orbit.
- D. The Eddington luminosity (formula on p. 1) and the ratio of L_{510} to $L_{eddington}$. If this ratio is less than 1, your blazar obeys the Eddington limit. Is this the case?
- E. The accretion rate a in solar masses per year (solve the first equation on p. 1 for a).
- F. The apparent speed v_{app} of the blob(s) whose motions were followed in the images. This is the proper motion μ multiplied by (i) the scale factor s , (ii) the number of light-years in a parsec, 3.26, and (iii) $(1+z)$; the last of these is needed because the blazar's galaxy is moving at a speed near c relative to us, so we see its time slowed down and need to correct for this. The formula is:

$$v_{app} = \mu s (3.26 \text{ lt-yr/pc}) (1+z) \text{ times the speed of light } c.$$

Note that c can be used as a unit; this is common when Relativity is involved. As is stated on p. 1, the value of v_{app} can be as high as $v_{app} \approx \Gamma c$. Blazars tend to have the highest possible apparent speeds, so we can approximate that $\Gamma \approx v_{app}/c$.

Use this value of Γ to calculate the actual speed of the blob: $v = \sqrt{1 - (1/\Gamma^2)} c$.

(Use c as the unit, i.e., do not enter the value of c . (Your value of v should be slightly less than $1c$. You should not round off the values on your calculator too much.)

- G. Determine whether the ejection time of the blob(s) is at the same time as a flare or outburst in either γ -rays or visible light (or both). If so, then perhaps the flare/outburst occurred in the blob. It could, though, just be a random coincidence. You can estimate the probability of random coincidence as the fraction of the time when a flare or outburst occurred, divided by the square-root of the number of blobs whose ejection times coincided with an outburst/flare.
- H. The maximum size, R_{\max} , of the region in the jet that caused the flare you used to determine t_{var} . To get the time-scale of brightness changes in the rest frame of the jet, first you need to correct t_{var} for motion near the speed of light by multiplying it by $\Gamma/(1+z)$. Since t_{var} is measured in days, the result will be the maximum size in light-days, which is the distance that light travels in 1 day, 2.6×10^{13} meters, or 170 astronomical units. The formula is
- $R_{\max} \approx t_{\text{var}} [\Gamma/(1+z)]$ light-days. **Multiply this by 170 AU/light-day to get R_{\max} in AU.**
- Then **divide the answer by R_{BH} in AU**, which you obtained in step C above, to get the ratio R_{\max}/R_{BH} .
- Is the region in the jet that caused the variations in brightness larger or smaller than the black hole?
- I. Did the γ -rays and visible light reach their maximum brightness at the same time during any of the outbursts or flares? If so, they probably came from the same region in the jet.
- J. The γ -ray light is produced in the jet. In this case, the brightness is much higher than it would be if the jet were not moving in our direction at a velocity near the speed of light. Calculate the maximum γ -ray luminosity L_{γ} in watts by multiplying $B_{\max,\gamma}$ by $4\pi d_L^2$ ($3.1 \times 10^{22} \text{ m/Mpc}$)². Now divide the answer by Γ^3 to estimate the actual luminosity in the jet's rest frame.

Your team will present the results of your analysis to the entire class at a time scheduled by your instructor. The team coordinator will lead the team's presentation, first calling on the presenter from each of the sub-groups, and then presenting calculations A – J. The presenters should all show the class the procedures that they followed – including any difficulties encountered – and the results that they obtained.

Feel free to consult with the instructor for guidance throughout the period of the project.