Constellation Design for Color-Shift (CSK) Keying Using Convex Optimization Methods

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Contributions

• Differentiable non-convex formulation solvable by interior point methods
  – Can specify perceived color of light source

• Efficient design heuristic for large constellations
  – Can NOT specify perceived color of light source
Color-Shift Keying (CSK)

Advantages:
• Zero flicker
• Reduced Inrush Currents

Constraints:
• $p_r + p_g + p_b = I$
• $0 \leq p_{r,g,b} \leq P_{r,g,b}$
• $\sum w_i s_i = c_{avg}$
Transmittable Set

\[ p_r + p_g + p_b = I \]

\[ P_{r,g,b} > I \]
Transmittable Set

\[ p_r + p_g + p_b = I \]

\[ P_{r,g} > I \]
\[ P_b < I \]
System Model

\[ H = \begin{bmatrix} h_{r,r} & h_{r,g} & h_{r,b} \\ h_{g,r} & h_{g,g} & h_{g,b} \\ h_{b,r} & h_{b,g} & h_{b,b} \end{bmatrix} \]

\[ \tilde{s}_i = Hs_i \]
System Model

\[ \tilde{s}_i = Hs_i \]

\[ r_i = Hs_i + n_i \]
Basis Change

\[ \phi_i = \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \frac{1}{l} T P_r \tilde{s}_i \]
Basis Change

\[ \phi_i = \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \frac{1}{I} TP_r \tilde{s}_i \]

\[ \Phi = \text{vec}(\phi_i) \]

\[ 0 \leq p_{r,g,b} \leq P_{r,g,b} \]
\[ p_r + p_g + p_b = I \]

\[ A\Phi \leq b \quad \sum w_i s_i = c_{\text{avg}} \quad \rightarrow \quad C\Phi = \overline{c_{\text{avg}}} \]
Objective

\[ \max_{\{\phi_i\}} \min_{i \neq j} \{\|\phi_i - \phi_j\|_2^2\} \]

- Non-Convex
- Discontinuous
# Optimization Methods

<table>
<thead>
<tr>
<th>Stochastic</th>
<th>Deterministic</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Checkmark] Works with any objective</td>
<td>![Checkmark] Well defined stopping criteria</td>
</tr>
<tr>
<td>![Checkmark] Avoids poor local minima</td>
<td>![Checkmark] Comparatively fast (per trial)</td>
</tr>
<tr>
<td>![X] Brute force/inefficient</td>
<td>![Checkmark] Easy to implement</td>
</tr>
<tr>
<td>![X] Converge in probability</td>
<td>![X] Sensitive to starting point</td>
</tr>
</tbody>
</table>
Interior Point Methods

• Intended for Convex Optimization
  – Finds local minima of non-convex problems

• Assumptions:
  – Linear inequality constraints ✔
  – Affine Equality Constraints ✔
  – Continuous Objective ✗
Continuous Approximation

• Minimum approximation:

\[
\min\{d_i\} \approx -\ln \left( \sum_i e^{-\beta d_i} \right) / \beta
\]

• New Objective:

\[
\max \{\phi_i\} -\ln \left( \sum_{i \neq j} e^{-\beta \|\phi_i - \phi_j\|_2^2} \right) / \beta
\]
Results: Validity

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad N = 8 \]
Results: Arbitrary Regions

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ N = 16 \]

\[ d_{min} = 0.3021 \]

\[ d_{min} = 0.2915 \]
Results: Color Balance

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad N = 8 \]
Results: Color Balance

\[ N = 8 \]
\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
Results: Large Constellations

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad N = 128 \]

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix} \quad N = 128 \]
Heuristic
Heuristic
Comparison

Normalized Minimum Distance vs Constellation Size

- No Balance
- No Balance: Hexagonal
Comparison

<table>
<thead>
<tr>
<th>Constellation Size</th>
<th>Normalized Minimum Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.5162</td>
</tr>
<tr>
<td>7</td>
<td>0.5020</td>
</tr>
<tr>
<td>8</td>
<td>0.4879</td>
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<tr>
<td>9</td>
<td>0.4738</td>
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<tr>
<td>10</td>
<td>0.4596</td>
</tr>
<tr>
<td>11</td>
<td>0.4455</td>
</tr>
</tbody>
</table>
Conclusions

• A systematic optimization approach to CSK design has been presented, which functions under:
  – Any constellation size
  – Arbitrary constraint region and color balance
• Under no color balance, an efficient heuristic based on hexagonal lattices has been demonstrated
QUESTIONS?
Detailed Constraints

\[ \phi_i = \frac{1}{l} T P_r \tilde{s}_i \]

\[ A\phi_i \leq b \]

\[ \Phi = \text{vec}(\phi_i) \]

\[ Q = \begin{bmatrix} A & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & A \end{bmatrix} \]

\[ d = \begin{bmatrix} b \\ \vdots \\ b \end{bmatrix} \]

\[ C\Phi = \overline{c}_{avg} \]
Minimum example

let $d_1 \ll d_2$ and $\beta = 1$

\[
\min_{i} \{d_1, d_2\} \approx -\ln(e^{-d_1} + e^{-d_2}) \\
\approx -\ln(e^{-d_1}) = d_1
\]
\begin{itemize}
\item \( \phi_i \leftarrow \text{random starting point} \)
\item \( \beta \leftarrow 1 \)
\item \textbf{while} \( \beta < \beta_{\text{stop}} \) \textbf{do}
\begin{align*}
\phi_{\text{opt}} & \leftarrow \arg \max_{\{\phi_i\}} - \ln \left( \sum_{i \neq j} e^{-\beta \|\phi_i - \phi_j\|^2_2} \right) / \beta \\
\beta & \leftarrow 2 \beta \\
\phi_i & \leftarrow \phi_{\text{opt}}
\end{align*}
\textbf{end while}
\end{itemize}
Results: Cross-talk

\[ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.1 \\ 0 & 0.1 & 0.8 \end{bmatrix} \]

\[ d_{\text{min}} = 0.2474 \]