**Microeconomic Theory Qualifying Exam**
June 2015

**Instructions.** You have a maximum of 4 hours and 15 minutes to complete this exam (suggested time allocation: 15 mins to review the questions and up to 4 hrs to answer them).

Answer all four questions. The questions are equally weighted.

Write on one side of the provided paper only. Start the answer to each question on a new sheet of paper and be sure to write your candidate number, question number, and page number on each sheet.

Be concise in your answers, and think before you write. Good luck!
Question 1

Consider the following nonstandard model of abstract choice: An agent has a utility function

\[ u(x, y) = xy, \]

over the set of bundles \( \mathbb{R}^2_+ \), and also an aspired level of utility \( \bar{v} > 0 \). Her choice correspondence \( C \) is defined as follows: for any feasible set \( B \subset \mathbb{R}^2_+ \), \((x, y) \in C(B) \) if and only if

\[ u(x, y) \geq \min \{ \bar{v}, \max_{(x', y') \in B} u(x', y') \}. \]

Intuitively, this agent satisfies rather than always choosing her utility maximizing alternative: she chooses any alternative that meets some underlying “aspired level of utility” \( \bar{v} \), though if no alternative meets her aspiration she maximizes her utility in the usual way. Imagine, for instance, an employer who hires any candidate that “will do”, but if she finds none then she chooses the best candidate available.

(a) Is her choice correspondence rationalizable in the usual way via maximization of some complete and transitive preference order? Explain.

(b) Restrict attention to Walrasian budget sets \( B(p, w) \) with strictly positive price vector \( p \) and wealth \( w > 0 \). Does this agent’s demand correspondence satisfy: (i) homogeneity of degree 0. (ii) Walras’ Law? Justify your answers.
Question 2

Consider the following three-person bargaining game in which players 1, 2 and 3 bargain over how to divide a surplus of size $e > 0$. Player 1 makes the first offer, denoted $x^1 = (x_1^1, x_2^1, x_3^1) \in \mathbb{R}_+^3$ where $x_1^1 + x_2^1 + x_3^1 = e$. Next, player 2 accepts or rejects. If 2 accepts, then it’s 3’s turn to accept or reject. If 3 also accepts, the game is over and the outcome is $x^1$. If either 2 or 3 rejects, the game goes to period 2 and it becomes player 2’s turn to make an offer with player 3 responding first and player 1 responding second. If 3 and 1 accept 2’s offer, then the outcome is the offer $x^2$ made by player 2. If not, the game goes to period 3 with player 3 making the offer, player 1 responding first and player 2 responding second. The game continues this way until players reach an agreement. If there is agreement on an offer of $x = (x_1, x_2, x_3)$ where this is the $t$th offer made, then player $i$’s payoff is $\delta^{t-1}x_i$; here $\delta \in (0, 1)$ is the common discount factor.

a) Find a stationary subgame perfect Nash equilibrium (SPNE) of this game. That is, an SPNE where each player $i$ makes the same offer every time he gets a chance (irrespective of history), accepts the same offers every time he is the first responder, and accepts the same offers when he is the second responder.

b) Suppose now that, before the three-person bargaining game described above is played, player 3 chooses how much effort $e \in (0, \infty)$ to exert. Player 3’s effort determines the size of the surplus over which players will bargain; in particular, the size of the surplus is equal to player 3’s effort choice $e$. The cost for player 3 of exerting effort level $e$ is $\frac{1}{2}e^2$. Players 1 and 2 observe the size of the surplus, and then the three-person bargaining game described above begins. Assume that for any chosen effort level $e > 0$, at the bargaining stage players play the equilibrium you found in part (a).

What is player 3’s optimal effort choice? How does it depend on $\delta$? How would your answer to these questions change if player 3 were the first player to make an offer at the bargaining stage?
Consider an Arrow-Debreu economy with $I$ consumers, 2 states (denoted $s$ and $s'$), 2 physical goods, and where all consumers have strongly monotone expected utility preferences and common beliefs. Initial endowments are given, for consumer $i$, by $\omega_i = (\omega_{is}, \omega_{is'})$, $\omega_{is}, \omega_{is'} \in \mathbb{R}^2_{++}$. Let $(x^*, p)$ be the unique Arrow-Debreu equilibrium in this economy. Here $x^* = (x^*_i)_{i \in I}$, $x^*_i = (x^*_{is}, x^*_{is'})$, $x^*_i, x^*_{is}, x^*_{is'} \in \mathbb{R}^2_{+}$, and $p = (p_s, p_{s'})$, where $p_s = (1, 2), p_{s'} = (1, \frac{1}{2})$.

Now consider how the allocation $x^*$ could be achieved alternatively via a Radner equilibrium. Specifically, let there be two securities, where the first is riskless and pays 1 in each state and the second pays 1 in state $s$ and 0 in state $s'$ (dividends are in units of the first physical good).

(i) Consider a particular consumer, called Alice and corresponding to $i = A$. Her consumption and endowments are given by $x^*_A = (2, \frac{1}{3}), x^*_{A'} = (\frac{1}{2}, \frac{5}{3})$, $\omega_A = (1, 1), \omega_{A'} = (1, 1)$.

Describe as fully as you can the portfolio that Alice must hold ex ante in any Radner equilibrium having $x^*$ as its allocation.

(ii) What can you say about the price of the two securities in the Radner equilibrium? Is the security price vector that you identify such that the portfolio from part (i) is self-financing?

(iii) Now add another security $r = (r_s, r_{s'}) >> 0$ to the Radner economy. What can you say about the price of this security in any Radner equilibrium for the new economy?

(iv) "The introduction of the new security will affect the equilibrium prices of the first two securities and/or the portfolios held in equilibrium." Do you agree or disagree? Explain fully.

(v) Do your answers to any of the above questions change if we allow beliefs to vary across consumers? If yes, how? If not, explain why not?
Question 4

People can be either ill or well. Utility is $s + m$, where $s = 0$ if ill and 1 if well, and $m$ is money. The probability $p$ of being ill varies uniformly across the (very large) population with size normalized to 1, i.e. $p$ is uniformly distributed on $[0, 1]$.

a) Derive the market demand function for a vaccine that will prevent the illness with probability 1.

b) What is the expected market demand function for a drug that will cure the illness with probability 1?

c) Compare the expected value of the aggregate willingness to pay for each.

d) A monopolist can develop EITHER the drug OR the vaccine. The development costs are equal (and small compared to the aggregate willingness to pay); production costs are each zero. Which does the monopolist develop, assuming

(1) She sets a single price for whichever product she chooses to produce?

(2) She uses any pricing rule she likes as long as it is conditioned on publicly available information, while both an individual’s status (ill or well) and his probability of illness are private information (information about the utility function and the distribution of $p$ is common knowledge)?

e) Is the product choice in part (d) likely to be efficient in the sense of maximizing aggregate surplus?

f) Suppose now that consumers are risk averse: utility is $u(s + m)$ where $u' > 0 > u''$. Explain how this might change your answers to (c), (d) and (e). No explicit calculation is needed for this part, but be sure to consider the two cases of mild and extreme risk aversion.