Solutions for Microeconomic Theory Qualifying Exam  
June 2015

Question 1
(a) The agent’s choices are consistent with maximizing the utility $v(x, y) = \min \{\bar{v}, u(x, y)\}$, and thus her choice correspondence is rationalizable.

(b) (i) Yes. Scaling $p$ and $w$ up or down by the same proportion leaves both $B(p, w)$ and $u(x, y)$, and hence choice, unchanged.

(ii) No. If $\max_{(x', y') \in B(p, w)} u(x', y') > \bar{v}$ then bundles in the interior of $B(p, w)$ can be chosen. Imagine the indifference curve corresponding to $\bar{v}$. Then any bundles above this indifference curve and below the budget line will be chosen.

Question 2
1. Let $y_2$ and $y_3$ be the lowest offer that a player accepts when she is in the second and third position, respectively. Let $x_1 = (e - y_2 - y_3, y_2, y_3)$ be the offer made by the proposer. In a stationary SPNE, player 2 has to accept any offer that gives her a payoff of $\delta(e - y_2 - y_3)$, since this is how much she will get if she rejects the offer. Therefore, $y_2 = \delta(e - y_2 - y_3).$

Similarly, player 3 has to accept any offer that gives her a payoff of $\delta y_2,$ since this is how much she will get if she rejects the offer. Therefore, $y_3 = \delta y_2.$ Then, $y_2$ is the solution to $y_2 = \delta(e - y_2 - \delta y_2)$: $y_2 = \frac{\delta e}{1 + \delta + \delta^2};$

$y_3 = \frac{\delta^2 e}{1 + \delta + \delta^2}.$ The payoff of player 1 is $e - y_2 - y_3 = \frac{e}{1 + \delta + \delta^2}.$

2. If player 3 puts effort $e,$ her payoff is $\frac{\delta^2 e}{1 + \delta + \delta^2} - \frac{1}{2}e^2.$ Her optimal choice of effort is $e^* = \frac{\delta^2}{1 + \delta + \delta^2}.$ Note that $e^*$ is increasing in $\delta.$ If player 3 made the first offer, her payoff from exerting effort $e$ would be $\frac{e}{1 + \delta + \delta^2} - \frac{1}{2}e^2,$ and her optimal choice of effort would be $e^{**} = \frac{1}{1 + \delta + \delta^2};$ $e^{**}$ is decreasing in $\delta.$
Question 3

A preliminary observation, consistent with Walras’ Law, is that

\[ p_s (x^*_A - \omega_{As}) + p_{s'} (x^*_A - \omega_{As'}) = \left[ 1 - \frac{4}{3} \right] + \left[ -\frac{1}{2} + \frac{\frac{15}{2}}{3} \right] = 0 \]

(i) The candidate Radner equilibrium has expected spot market prices equal to \( p_s \) and \( p_{s'} \) for the two states respectively. Define the returns matrix \( R \),

\[
R = \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix}.
\]

Then, by the spot market budget constraints for a Radner equilibrium, the portfolio \( z^*_A \in R^2 \) must imply the wealth transfers across states as described by:

\[
Rz^*_A = \begin{bmatrix}
p_s \cdot (x^*_A - \omega_{As}) \\
p_{s'} \cdot (x^*_A - \omega_{As'})
\end{bmatrix} = \begin{bmatrix}
-1/3 \\
1/3
\end{bmatrix},
\]

or

\[
z^*_A = R^{-1} \begin{bmatrix}
-1/3 \\
1/3
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
1 & -1
\end{bmatrix} \begin{bmatrix}
-1/3 \\
+1/3
\end{bmatrix} = \begin{bmatrix}
1/3 \\
-2/3
\end{bmatrix}.
\]

Market completeness implies that \( \exists \) unique portfolio. (This \( z^*_A \) is necessary if we consider only Radner equilibria with spot market price expectations equal to \( p_s \) and \( p_{s'} \). In general, there might be other Radner equilibria that yield the allocation \( x^* \), for example, if \( (x^*, p') \) is a distinct AD equilibrium. This possibility is excluded by the uniqueness assumption.)

(ii) The security price vector \( q \) is given by

\[
q_1 = 1 \cdot 1 + 1 \cdot 1 = 2, \quad q_2 = 1 \cdot 1 + 0 = 1.
\]

Then \( z^*_A \) is self-financing because

\[
q^T z^*_A = \begin{bmatrix}
2 & 1
\end{bmatrix} \begin{bmatrix}
1/3 \\
-2/3
\end{bmatrix} = 0.
\]

(iii) The new security is redundant (\( R \) is complete) and thus can be priced uniquely through no-arbitrage and the Fundamental Theorem—that is, given \( q \) above, then the new security has a unique price consistent with no-arbitrage equal to \( \mu_s r_s + \mu_{s'} r_{s'} \), where \( \mu \in R^2 \) solves

\[
q = \begin{bmatrix}
2 \\
1
\end{bmatrix} = \mu^T R = \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix} \mu \implies
\mu_s = \mu_{s'} = 1
\]
(iv) Since the new security does not affect the scope for transferring wealth across states, it does not affect optimal contingent consumption plans. Conclude that the initial Radner equilibrium, plus the no-arbitrage price for the new security, and zero demand (for every consumer) for the new security, constitute a Radner equilibrium.

(v) None of the preceding depends on the commonality of beliefs. Beliefs enter implicitly in that they must support \((x^*, p)\) as an AD equilibrium (and full support plays a role in the equivalence between AD and Radner equilibria, as well as in no-arbitrage arguments.). But given such an equilibrium, subsequent arguments rely primarily on budget constraints, and not at all on further claims about ex ante optimality where beliefs might be important.

**Question 4**

a) \(1 - P\), where \(P\) is price

b) 0 for \(P > 1\); \([0, 1/2]\) if \(P = 1\); 1/2 if \(P < 1\). In fact, without the large numbers, demand is an r.v. with range 0 to 1 for \(P=1\), but expected demand is 1/2.

c) The (expected) aggregate surpluses equal 1/2 for both products.

d) (1) drug, She charges 1 for the drug, getting 1/2, and \(\max_p p(1-p) = 1/4\) with price 1/2 for the vaccine; since this is less drug is preferred

(2) drug, since with asymmetric info the monopolist cannot extract all the surplus for the vaccine, but can for the drug with a linear pricing rule

e) yes; the surplus is the same for each product, and the monopolist delivers an efficient quantity of the drug

f) (c'): willingness to pay and aggregate surplus for the vaccine goes up because of its insurance effect, while demand for the drug is unchanged

(d'): for small levels of risk aversion the answers remain the same (continuity); for large levels (think of infinite risk aversion, where all types are willing to pay 1 for the vaccine and so the monopolist can extract the aggregate surplus of 1 by charging a price of 1), the vaccine will be preferred.

(e') There is a range with low risk aversion where the product choice is (ex-ante) inefficient, since the surplus is strictly higher for the vaccine but the firm develops the drug anyway. For high enough risk aversion the vaccine will be delivered, though as usual (except in the infinitely risk averse case) too little of it will be offered for sale unless the monopolist can perfectly price discriminate, which is not possible under the assumptions of the problem.