Credit Constraints, Firms’ Precautionary Investment, and the Business Cycle*

Ander Perez†
Universitat Pompeu Fabra and Barcelona GSE
August 28, 2014

Abstract

We study the implications of firms’ investment maturity choices in the presence of credit constraints. We show that when long term capital is irreversible and firm specific, firms that suffer credit constraints might have a preference for short term capital because it can attract more external finance and does not face the risk of costly liquidation in future events of financial distress. This mechanism limits firms’ incentives to shift resources into long-term investments when a negative temporary aggregate productivity shock hits the economy, significantly dampening the short-run negative output reaction to the shock, at the expense of strong propagation effects. Small temporary shocks to the severity of financing frictions generate large and long-lasting effects on output through their impact on the composition of investment, and the anticipation of future distress in financial markets can create a short run boom by inducing a shift towards shorter maturity investments.

Keywords: Investment maturity; Financial frictions; Business cycles; Idiosyncratic risk; Firm heterogeneity

JEL Classification Numbers: D92, E22, E32, E44, G31, G32

*I am grateful to Kosuke Aoki and Nobu Kiyotaki for very valuable discussions, and for very helpful comments Marios Angeletos, Andrea Caggese, Vicente Cunat, Wouter den Haan, Antoine Faure-Grimaud, Roman Inderst, Eirik Kristiansen (discussant), Eric Leeper, Albert Marcet, Raoul Minetti, Alex Michaelides, Morten Olsen (discussant), Ali Ozdagli, Javier Suarez, Aleh Tsyvinski (discussant), David C. Webb, Alwyn Young and seminar participants in LSE, HEC Montreal, Indiana University, Boston Fed, CEMFI, Bank of Spain, Alicante, Carlos III, Autonoma de Barcelona, IAE-CSIC, Tilburg, Cambridge, Birkbeck, Pompeu Fabra, Bank of England, the European Finance Association Meetings 2009 (Bergen), the II Bank of Italy Conference on Macro Modelling, the Society for Economic Dynamics Meetings 2010 (Montreal), the 2013 French Macro Workshop, and the 2014 CREI Asset Prices and the Business Cycle Workshop.

†Email address: ander.perez@upf.edu
1 Introduction

How do financial constraints of firms affect the dynamics of the aggregate composition of investment? And how does the behavior of the composition of investment in the presence of financing constraints influence the response of an economy to shocks? A large literature starting with Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) has studied the role of firms’ financing frictions in amplifying business cycles by focusing on how firms’ investment capacity is affected by worsening borrowing constraints in recessions or following a tightening of monetary policy. Common to most of these models is the assumption that firms can invest in only one type of project, which precludes analyzing questions regarding the composition of investment.

One of the most important dimensions of project choice, and the one we focus on, is the duration or maturity of investment projects. Firm-level empirical work has found that financial constraints generate a preference for short term investments.\footnote{Chevalier and Scharfstein (1996) show that credit constrained firms are more likely to increase price markups at the expense of longer-term market share building. Peyer and Shivdasani (2001) find that highly leveraged firms have a preference for investment projects that deliver cash flows earlier. Aghion et al. (2012) show that credit constrained firms display a more procyclical pattern of their share of R&D investment, consistent with notion that worsening credit conditions in downturns hinder investment in long term projects. The evidence in Eisfeldt and Rampini (2007) that credit constrained firms tend to purchase used, rather than new, capital, is also consistent with a bias towards projects with shorter lifespans and a more front-loaded pattern of cash flows.} At the aggregate level, the average maturity of aggregate investment falls in downturns (Dew-Becker (2012)), and particularly so in less financially developed economies (Aghion et al. (2010)). Work starting with Rajan and Zingales (1998) has shown that industries with a high degree of external finance dependence, which is associated with a long duration of investment projects, grow less in poorly developed financial systems and tend to do worse in recessions (Braun and Larrain (2005)). Taken together, this evidence suggests that credit constraints create a bias towards short maturity investments, with potential business cycle implications.

We first develop a model which produces results consistent with the evidence described above and use it to address several questions. Does the ability of firms to choose the maturity of their investment projects interact with financing constraints and idiosyncratic firm risk to dampen or amplify the short-run effects of aggregate productivity shocks, relative to an economy with financially unconstrained firms? Does it influence how the effects of productivity shocks propagate through time? How does it influence the impact of financial shocks? To answer these questions we introduce a dynamic general equilibrium model of an economy in which firms produce a homogeneous final good using two technologies which they can operate simultaneously. One uses short-term capital and produces output in the short-run, while the other uses irreversible and firm-specific long-term capital which produces output over a longer time period. Firms face the possibility every period of being forced to exit the economy and will only be able to recover a small fraction of the value of long-term capital in that event due to its irreversibility and firm-specificity. Firms also suffer the possibility of idiosyncratic operating cost shocks every period that capture unexpected operating expenditures such as additional capital maintenance costs or higher input costs. External finance is restricted to riskless debt, and firms can only pledge a fraction of their future earnings. As a result, they optimally accumulate units of the final good as a buffer against problems in obtaining credit in the future.
Credit constraints create a bias towards short-term investment through two mechanisms. Because debt is restricted to be riskless, only short-run output is pledgeable because a firm might be forced to exit at any point in time and will not be able to realize in liquidation, and pledge beforehand, the full present value of the capital’s future output. As a result, firms with a binding credit constraint have a preference for short term capital because it produces relatively more pledgeable output and can attract more external finance. This result complements previous work suggesting that the negative effects of financing constraints and investment irreversibility reinforce each other (Caggese (2007), Khan and Thomas (2013)) by providing a mechanism by which irreversibility causes a worsening of financial constraints. Furthermore, the anticipation of future credit constraints also creates a bias towards short-term investment. Long term capital runs the risk of having to be partially liquidated at a cost in the future if the firm is in financial distress after suffering several negative idiosyncratic operating cost shocks, and this causes long term capital to have a lower expected return for firms with low liquidity buffers. This precautionary behavior is in line with evidence that the expectation of future financing constraints has important implications for current firm behavior.2

Next we describe and quantify a novel mechanism that delivers short-run dampening of productivity shocks and long-run propagation. In an economy with frictionless credit markets, following a negative temporary aggregate productivity shock firms decrease their short-term investment, which offers poor returns in downturns, but barely alter their long-term investment, whose returns are less cyclical. Output contracts sharply but recovers quickly. In an economy that features credit constraints, the decrease in profits and net worth increases the share of firms with binding credit constraints. Firms’ net worth recovers slowly so credit constraints are expected to remain tight for some time. Both the currently binding credit constraints and the anticipation of future constraints mean that many firms are unable or unwilling to sustain previous levels of long term investment and instead increase the share of their investment allocated to short term capital. This shift to short term projects dampens the short run output fall. Over the first five quarters the output fall is smaller in the financially constrained economy by an accumulated amount equivalent to 2.3% of quarterly steady state output. But the large drop in long-term investment generates large propagation effects. Convex costs of adjustment in investment and the slow recovery of firms’ balance sheet strength mean that the large initial fall in the stock of long-term capital takes a long time to recover and over the subsequent thirty-five quarters the output fall in the financially constrained economy is greater than in the unconstrained economy by an accumulated amount equivalent to 6.9% of quarterly steady state output. As a result, a trade-off arises between contemporaneous amplification and long-term propagation of the effects of shocks; stronger dampening is associated with larger propagation. Idiosyncratic risk plays an important role for our results and we show that decreasing the volatility of these shocks reduces the strength of the contemporaneous dampening and intertemporal propagation of productivity shocks, as well as the amplification and propagation of financial shocks. Our

2Surveys by Graham and Harvey (2001) and Bancel and Mittoo (2004) find that Chief Financial Officers consider financial flexibility (having enough internal funds to avoid having to forego positive net present value projects in the future) to be the primary determinant of their policy decisions. Almeida, Campello and Weisbach (2011) provide a survey of empirical evidence that shows that the expectation of future financing problems significantly affects firms’ current investment policies. Caggese and Cunat (2007) find that that firms that expect credit constraints in the future are more likely to hire workers using short-term contracts.
results about short run aggregate dynamics complement a small literature that studies how distortions in the type of investments firms carry out induced by financing constraints might have implications for long-run growth and development (Acemoglu and Zilibotti (1997), Matsuyama (2007) and Aghion et al. (2010)).

Our mechanism also introduces a novel explanation for the countercyclicality of financing constraints that provides an alternative to the two main explanations offered in the literature, based on countercyclical agency costs on one hand and collateral constraints and lack of indexation of debt contracts on the other. The incentive to shift towards long-term projects in recessions because their returns are relatively acyclical decreases the amount of pledgeable output for a given level of total investment, and makes it more likely that credit constraints bind and that firms’ capital maturity structure moves away from its optimum.

The interaction of investment maturity choice and firm credit constraints also influences the reaction of an economy to financial shocks. Following an unexpected and temporary tightening of financing constraints, aggregate investment falls, but disproportionally so in long term capital, as firms in financial distress shift out of long-term investment. Even if the financial shock is short-lived, the large drop in the stock of long-term capital takes a long time to recover and propagates the effects of the financial shock for many periods after the severity of financing constraints subsides. To highlight the role of firms’ precautionary behavior in anticipation of future credit constraints, we also analyze the effect of news of future financial shocks. Firms’ precautionary behavior in reaction to the news induces a shift into short-term investment which raises current output, at the expense of a fall in long-term investment that generates large future decreases in output.

We also study the impact of investment tax credits given their direct relationship with the distortions presented in this paper. We model investment tax credits as an expensing allowance for a fraction of total investment done in a period, and find that cyclical expensing allowances that allow for larger tax credits in downturns have an important impact on the dynamics of the composition of investment following a productivity shock, and generate a reduction in short run output and an increase in medium and long run output, relative to the benchmark calibration absent cyclical expensing allowances. This result points towards one of the key policy implications of the frictions modelled in this paper, which is that policies that are targeted towards eliminating distortions that hamper long term investment should be evaluated in the long term, as they might come at the cost of a drop in short run output.

Taken together, our results suggest that firms’ investment maturity choice is an important ingredient in models that try to analyze the influence of financial frictions in firms for short-run macroeconomic dynamics.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the calibration of the model and analyzes firms’ optimal decisions in the steady state. Section 4 presents the main results of the model concerning the reaction of the economy to productivity and financial shocks. Section 5 analyzes the role of idiosyncratic firm uncertainty and investment tax credits, and section 6 concludes.

---

3 As in Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), or Christiano et al. (2010).
4 As in Kiyotaki and Moore (1997) or Iacoviello (2005).
2 Model

We introduce an infinite horizon, discrete time model of an economy populated by a continuum of financially unconstrained households and a continuum of financially constrained firms, both of measure one. Households own the firms and provide labor to them. Firms combine labor and capital, which can be short-term or long-term, to produce a homogenous final good that can be used for consumption and investment. They face idiosyncratic risk and as a result are heterogeneous in terms of their holdings of the final good and of long-term capital. A government collects corporate income taxes from the firms and distributes the revenues to households with a lump-sum transfer. We begin our description of the economy by studying the optimization problem faced by firms, then follow with a discussion of households’ optimization and the competitive equilibrium.

2.1 Firms

Firms maximize the present value of expected future dividends paid out to the households. We discuss next their production opportunities, financing choices, life cycle, and optimization problem.

Production opportunities

Firms can simultaneously operate two different technologies to produce the final consumption good. Each production process uses capital $k_{h,t+1}$ invested in period $t$ and labor $l_{h,t}$ to produce an amount of final goods $f_h(k_{h,t+1}, l_{h,t})$ in period $t + 1$, according to the production function

$$f_h(k_{h,t+1}, l_{h,t}) = \theta_{h,t+1}k_{h,t+1}^{\alpha_h}l_{h,t}^{\gamma_h},$$

where $h \in \{S = \text{short term}, L = \text{long term}\}$ refers to the type of technology, and $\theta_{h,t+1}$ is an aggregate productivity term given by

$$\theta_{h,t+1} = \theta_h \theta_{t+1},$$

in which $\theta_h$ is a constant productivity parameter of the technology type $h$ and $\theta_{t+1}$ is an aggregate productivity factor common to both short and long-term production technologies. The production functions feature decreasing returns to scale, captured by $\alpha_h + \gamma_h < 1$.

There are two differences between short and long-term technologies. First, short-term capital $k_{S,t+1}$ depreciates fully in one period, while long-term capital $k_{L,t+1}$ depreciates at a rate of $\delta$, where $0 < \delta < 1$. Accumulation of long-term capital follows the rule

$$k_{L,t+1} = (1 - \delta)k_{L,t} + i_{L,t}, \tag{1}$$

where $i_{L,t}$ represents period $t$ investment in long-term capital. Second, $\theta_S > \theta_L$, which means that a given level of investment in short term capital delivers higher per-period output than...
an identical amount of long-term investment. These two differences are meant to capture the
notion that the short-term investment provides a front-loaded pattern of output, whereas the
long term investment delivers a back-loaded one.

Positive investment in long-term capital suffers from convex costs of adjustment according
to the function \( \psi(i_{L,t}) \), which satisfies \( \psi(0) = 0, \ \psi'(\cdot) > 0 \) and \( \psi''(\cdot) > 0 \). Long-term capital
is firm-specific and partially irreversible. A firm can convert long-term capital back into the
final good and obtain \( \pi \) units (0 < \( \pi < 1 \)) of the final good per unit of terminated long-term
capital in return. If long-term capital was fully reversible, or could be used productively in other
firms and as a result had a liquid secondary market, it would not be perceived as a long-term
investment by the firm as it could undo such a liquid investment at any time at a very low cost.\(^7\)
Our modelling choice tries to capture investments to which the firm is tied to for a long period
of time, and can only undo at a cost.

Firms face the possibility of suffering an idiosyncratic operating cost shock \( s_t \) each period
with probability \( p \). The size of these shocks is proportional to the size of the firm’s operations,
measured as the total capital stock of the firm, so that

\[
s_t = \varepsilon (k_{S,t+1} + k_{L,t+1}) ,
\]

where \( \varepsilon > 0 \). The occurrence of the shock, which happens in the middle of a period after
investment is done but before output is produced, is captured by the indicator function \( 1_{s_t} \),
which takes value 1 if the shock occurs, and value 0 if it doesn’t. The additive nature of these
shocks is meant to capture that these are not shocks to output, but rather direct shocks to
firms’ operating income capturing unexpected operating expenditures, such as higher capital
maintenance or input costs. Firms need to hold enough liquidity to be able to withstand the
operating cost shock if it occurs, as if the shock is not paid, the project terminates and yields
nothing (not even the liquidation value of long-term capital). The modelling of these shocks
follows the model of corporate liquidity demand of Holmstrom and Tirole (1998). As we will
see, idiosyncratic risk is an important ingredient in our results, and considering other sources
of risk such as firm-level productivity shocks does not affect the key results of the paper.

**Firms’ flow of funds and financing**

Firms need to finance wages and investment, including adjustment costs of investment. They
can do so by using retained earnings, the proceeds from liquidating long term capital, or by
borrowing. Firms are able to borrow an amount \( b_t \) (or save an amount \(-b_t\)) using one-period
debt contracts at the equilibrium riskless interest rate \( 1 + r_{t+1} \).

---

\(^6\) Adjustment costs of investment are introduced because the dynamic stochastic general equilibrium (DSGE)
literature has found that they are an important feature in bringing the performance of these models closer to the
data (Smets and Wouters (2007)).

Even though adjustment costs are not modeled for short-term investment, this is not because of the belief
that these costs are not relevant for short-term investment. It is because in our model short-term investment
depreciates fully within one period and one can introduce them by adjusting the short-term production function
accordingly. In the interest of simplicity we choose then not to model them separately.

\(^7\) For simplicity, we capture irreversibility and illiquidity with the same parameter, \( \pi \). An interesting extension
could endogenize the resale value of capital, and possibly capture phenomena such as a negative correlation
between a firms’ desire to liquidate capital and the resale value of this capital, due to correlated shocks within
industries (Shleifer and Vishny (1992)).
A firm’s holdings of the final good, which we will also refer to as "asset holdings" or "asset position", computed at the beginning of a period is denoted $a_{f,t}$, and the dynamics of this asset position are given by

$$a_{f,t+1} = f_S(k_{S,t+1}, l_{S,t}) + f_L(k_{L,t+1}, l_{L,t}) - 1_s s_t - b_t + (1 + r_{t+1}) - tax_{t+1},$$  \(\text{(3)}\)

where borrowing $b_{t+1}$ is

$$b_{t+1} = i_{L,t} + \psi(i_{L,t}) + k_{S,t+1} + w_t(l_{S,t} + l_{L,t}) + d_t - a_{f,t} \quad \text{if } i_{L,t} \geq 0,$$

\(\text{(4)}\)

or

$$b_{t+1} = \pi i_{L,t} + k_{S,t+1} + w_t(l_{S,t} + l_{L,t}) - a_{f,t} \quad \text{if } i_{L,t} < 0.$$

\(\text{(5)}\)

We denote with $a_{f, t+1}^+$ the asset position at the beginning of period $t+1$ if the shock occurred ($1_s = 1$) during period $t$, and conversely $a_{f, t+1}^-$ the asset position in the case it did not ($1_s = 0$).

Firms pay an amount of corporate taxes $tax_t$, which is a result of a flat rate $\tau$ over positive taxable profits, calculated as

$$tax_t = \tau \left[ f_S(k_{S,t}, l_{S,t-1}) + f_L(k_{L,t}, l_{L,t-1}) - 1_s s_t - r_t b_t ight.$$  $$- \psi(i_{L,t-1}) - w_t(l_{S,t-1} + l_{L,t-1})] \quad \text{if } i_{L,t-1} \geq 0,$$

\(\text{(6)}\)

or

$$tax_t = \tau \left[ f_S(k_{S,t}, l_{S,t-1}) + f_L(k_{L,t}, l_{L,t-1}) - 1_s s_t - r_t b_t ight.$$  $$+ \pi(-i_{L,t-1}) - w_t(l_{S,t-1} + l_{L,t-1})] \quad \text{if } i_{L,t-1} < 0.$$

\(\text{(7)}\)

Taxable profits in expression (6) include revenues net of allowed expenses (the operating cost shock, adjustment costs of investment when there is positive investment, and wages), minus (plus) interest expense (interest income). When there is some liquidation of long term capital, the proceeds from disinvestment are treated as revenues for tax purposes (expression (7)). There is an asymmetry at zero profits, and firms pay no taxes if profits are negative. In the benchmark version of the model we do not consider investment tax credits, such as deduction allowances for capital depreciation or additional investment expensing allowances. We do so later in Section 5.2, in which we also study how tax policies can be used to deal with the distortions discussed in this paper.

Firms face financing constraints that limit their ability to obtain external finance using both debt and equity. Firms are unable to obtain any equity finance, which means that their dividends cannot be negative, so

$$d_t \geq 0,$$

\(\text{(8)}\)

and not being able to issue equity also means that firm assets can never be below zero, or

$$a_{f,t} \geq 0,$$

\(\text{(9)}\)

because, should the firm be forced to exit (which occurs with positive exogenous probability, as
will be discussed next) with a negative asset position, the shareholders would have to contribute additional funds to cover the negative asset position. This would violate constraint (8). One of the implications of this constraint is that a firm might have to liquidate part of its long term capital stock, or reduce short term investment, if the current investment choices will result in a negative asset position if the cost shock occurs.

Firms on the other hand are able to obtain debt financing using non-contingent debt, and the amount borrowed is constrained to be no more than a fraction $\mu_t$ of the present value of future output. Because only riskless debt is available, promised future debt repayments cannot be larger than the lowest possible realization of all future earnings. The additional assumption that firms suffer a small possibility every period of having to exit, which is discussed below, means that earnings obtained in periods after $t+1$ cannot be pledged because the firm might not exist after $t+1$. As a result, firms can only borrow up to a fraction $\mu_t$ of the present discounted value of the lowest possible realization of all future earnings, which are next period’s earnings if the firm suffers a shock, plus the liquidation value $\pi$ of undepreciated capital $(1 - \delta)k_{L,t+1}$, or

$$b_{t+1} \leq \mu_t \frac{f_S(k_{S,t+1}, l_{S,t}) + f_L(k_{L,t+1}, l_{L,t}) - s_t + \pi (1 - \delta)k_{L,t+1}}{1 + r_{t+1}}. \quad (10)$$

The assumption that only a fraction $\mu_t$ of future earnings are pledgeable can be justified, following Hart and Moore (1994) and Kiyotaki and Moore (1997), if one assumes that each project requires the input of the firm manager and that without her services revenues are a fraction $(1 - \mu_t)$ lower. Then she could threaten to withdraw her services and renegotiate her liability down to a fraction $\mu_t$ of revenues. Another justification could be along the lines of Holmstrom and Tirole (1997). One could assume that the borrower has to contribute with an unobservable effort to increase the expected return of the project and that in order for her to have the proper incentives at least a particular fraction of the expected returns need to accrue to her and cannot be pledged to lenders.\(^8\)

The crucial implication of this borrowing constraint is that long term irreversible capital $k_L$ attracts less external finance than short term capital $k_S$. If long term capital was fully reversible or easily redeployable to a similarly productive use outside the firm, then our firm could effectively pledge the full present value of the future returns from long term capital (adjusted for $\mu_t$), and short and long term capital would both attract the same amount of external finance. The additional assumption of irreversibility and firm-specificity of long term capital means that firms instead can only promise the limited liquidation value $\pi$ of each unit of long term capital. The returns of short-term investment, on the other hand, are fully pledgeable, because they all occur in period $t+1$, immediately before the firm is exposed to the risk of exit. In summary, the share of the present value of future returns that can be pledged is lower for long-term investment than for short-term investment. We provide further discussion of this idea when we develop it formally in Section 2.4.1.

\(^8\)It is also important to note that $b_{t+1}$ is debt net of asset holdings given how it is defined. Asset holdings and their returns could in principle be pledgeable, and if that is the case then firms could hold a positive balance of assets in their balance sheet and simultaneously be borrowing. If debt is not risky, however, as is the case in this model, firms have no motive to do so. See Acharya et al. (2007) for a detailed discussion of this point.
Firm entry and exit

Firms face an exogenous exit shock which occurs with probability $\eta$, which can be interpreted as a permanent negative idiosyncratic productivity shock that forces firms to exit. Firms distribute all of their asset holdings $a_{f,t}$ plus the liquidation value of their undepreciated long-term capital $\pi(1 - \delta)k_{L,t}$ as dividends before exiting. A firm that exits is immediately replaced by a new firm with no assets or long-term capital, which means that firms are created homogeneous but become endogenously heterogeneous through the effect of the idiosyncratic operating cost shock.

Firm Optimization

Firms maximize the present discounted value of dividends $d_t$ distributed to their shareholders, the households. There is no aggregate uncertainty and firms take as given the path of future aggregate variables, given by the sequence starting at time $t$ of the interest rate, wage, aggregate productivity and financing constraints $z_t = \{r_s, w_s, \theta_s, \mu_s\}_s^{t=\infty}$. A firm’s individual state variables are its asset holdings $a_{f,t}$ and its stock of long term capital $k_{L,t}$. Its value function, calculated at the beginning of period $t$, conditional on not having suffered the exogenous exit shock and thus being able to operate for another period, is given by

$$J(a_{f,t}, k_{L,t}, z_t) = \max_{d_t, k_{S,t+1}, k_{L,t+1}, l_{S,t}, l_{L,t}, b_{t+1}, d_t} \left\{ M_{t,t+1} \left[ \eta \left( p a_{f,t+1} + (1 - p) a_{f,t+1} + \pi(1 - \delta)k_{L,t+1} \right) \right] + (1 - \eta) \left( p J(a_{f,t+1}, k_{L,t+1}, z_{t+1}) + (1 - p) J(a_{f,t+1}, k_{L,t+1}, z_{t+1}) \right) \right\},$$

subject to (3), (8), (9), and (10), where $M_{t,t+1}$ is the discount factor of households that the firm uses to discount future dividends.

Firms’ optimal choices for short term capital, long term capital, labor for each production process, borrowing and dividends are given by $k_{S,t+1}(a_{f,t}, k_{L,t}, z_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, z_t)$, $l_{S,t}(a_{f,t}, k_{L,t}, z_t)$, $l_{L,t}(a_{f,t}, k_{L,t}, z_t)$, $b_{t+1}(a_{f,t}, k_{L,t}, z_t)$, and $d_t(a_{f,t}, k_{L,t}, z_t)$.

2.2 Households

We now turn to the intertemporal optimization problem faced by a representative household. It maximizes its expected lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $c_t$ is consumption and $\beta$ is the discount factor. Households are the shareholders of the firms and own a perfectly diversified portfolio of all the firms in the economy, which pays a dividend each period equal to

$$\text{div}_t = \int_{\text{no exit}} d_t(a_{f,t}, k_{L,t}, z_t) d\Gamma_t(a_f, k_L) + \int_{\text{exit}} (a_{f,t+1} + \pi k_{L,t}) d\Gamma_t(a_f, k_L)$$

where $\Gamma_t(a_f, k_L)$ is the joint distribution of asset holdings and long-term capital holdings in the population of firms. In addition to equity shares, they hold an amount of riskless bonds issued
by firms equal to $a_{t+1}$, which pay a return $1 + r_{t+1}$.

The representative household is endowed with one unit of labor, which they supply inelastically in return for a wage $w_t$. The household’s budget constraint is

$$c_t + a_{t+1} = a_t(1 + r_t) + d_{it} + w_t + T_t,$$

where $T_t$ are lump-sum transfers provided by the government and financed by the corporate taxes paid by the firms.

### 2.3 Equilibrium

Given a sequence starting at time $t$ of aggregate productivity and borrowing constraints which we denote by $x_t = \{\theta_s, \mu_s\}_{s=0}^{\infty}$, let firms’ choices for dividends, short term investment, long term capital holdings, labor for short-term production, labor for long-term production, and borrowing be given by $d_t(a_{f,t}, k_{L,t}, x_t)$, $k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)$, $l_{S,t}(a_{f,t}, k_{L,t}, x_t)$, $l_{L,t}(a_{f,t}, k_{L,t}, x_t)$, and $b_{t+1}(a_{f,t}, k_{L,t}, x_t)$, respectively, and choices by households for consumption and savings be given by $c_t(a_t, x_t)$ and $a_{t+1}(a_t, x_t)$, respectively. Let $\Gamma_t(a_f, k_L)$ denote the joint distribution of asset holdings and long-term capital holdings in the population of firms. We are now ready to define an equilibrium.

**Definition 1** An equilibrium is a sequence of interest rates $\{r_t\}$, a sequence of wages $\{w_t\}$, a sequence of transfers $\{T_t\}$, a sequence of consumption and savings policies $\{c_t(a_t, x_t)\}$ and $\{a_{t+1}(a_t, x_t)\}$, a sequence of dividends, short term investment, long term capital holdings, labor for short-term production, labor for long-term production, and borrowing given by $d_t(a_{f,t}, k_{L,t}, x_t)$, $k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)$, $l_{S,t}(a_{f,t}, k_{L,t}, x_t)$, $l_{L,t}(a_{f,t}, k_{L,t}, x_t)$, and $b_{t+1}(a_{f,t}, k_{L,t}, x_t)$, respectively, and a sequence of distributions for firms’ asset and long-term capital holdings $\{\Gamma_t(a_f, k_L)\}$, such that, given the initial distribution $\Gamma_0$ and an initial wealth of households $a_0$:

(i) $c_t(a_t, x_t)$ and $a_{t+1}(a_t, x_t)$ are optimal given $\{r_t\}$, $\{w_t\}$ and $\{T_t\}$,

(ii) $d_t(a_{f,t}, k_{L,t}, x_t)$, $k_{S,t+1}(a_{f,t}, k_{L,t}, x_t)$, $k_{L,t+1}(a_{f,t}, k_{L,t}, x_t)$, $l_{S,t}(a_{f,t}, k_{L,t}, x_t)$, $l_{L,t}(a_{f,t}, k_{L,t}, x_t)$, and $b_{t+1}(a_{f,t}, k_{L,t}, x_t)$ are optimal given $\{r_t\}$ and $\{w_t\}$,

(iii) $\Gamma_t$ is consistent with the investment and borrowing decisions of firms and with aggregate taxes collected $T_t$,

(iv) the bond market clears:

$$\int b_{t+1}(a_{f,t}, k_{L,t}, x_t) d\Gamma_t(a_f, k_L) = a_{t+1}(a_t, x_t),$$

which requires that net aggregate borrowing of the firm sector (the left-hand side of the equation) is equal to aggregate saving of the household sector (the right-hand side), and finally

(v) the labor market clears:

$$\int l_{S,t}(a_{f,t}, k_{L,t}, x_t) d\Gamma_t(a_f, k_L) + \int l_{L,t}(a_{f,t}, k_{L,t}, x_t) d\Gamma_t(a_f, k_L) = 1.$$
2.4 Characterization of Some Properties of the Equilibrium

To illustrate some of the properties of the model, it will be convenient to look at simplified versions of the model in which some important features of the equilibrium can be characterized analytically. The analysis clarifies the two reasons why credit constraints create a bias towards short term capital.

2.4.1 Firms with currently binding credit constraints effectively perceive a higher rate of interest and a relatively lower productivity of long-term capital

First, we show that a firm with a binding credit constraint effectively perceives a higher rate of interest and discounts future output more, decreasing the productivity of long term capital relative to short term capital. Intuitively, the mostly long run returns of long term capital do not relax the current binding constraint and are penalized, in contrast with the entirely short run returns of short-term capital. A much simplified version of the firm problem described above is enough to make this formal argument. We abstract from labor and short term capital, and just consider a simple production function that transforms $k_{t+1}$ units of capital into $f(k_{t+1})$ units of consumption goods the following period. The capital accumulation equation is as before and there are no costs of capital adjustment. The firm chooses capital $k_{t+1}$, borrowing $b_{t+1}$, and dividends $d_t$ to maximize the present discounted value of dividends

$$d_t = a_{f,t} - [k_{t+1} - (1 - \delta)k_t] + b_{t+1}$$

where $a_{f,t}$, the firm’s asset position at the beginning of period $t$, is

$$a_{f,t} = f(k_t) - b_t(1 + r_{t+1}).$$

There is no idiosyncratic uncertainty, and we ignore firm exit. As in the main model, firms are unable to issue any equity, which means that

$$d_t \geq 0,$$  \hspace{1cm} (15)

and they can only borrow up to a fraction $0 < \mu < 1$ of the output in $t + 1$;

$$b_{t+1}(1 + r_{t+1}) \leq \mu f(k_{t+1}),$$  \hspace{1cm} (16)

and cannot pledge any part of the output produced in period $t + 2$ or after. We assume this form of the financing constraint, which has been derived earlier in section 2.1.

Denoting with $\lambda_t$ and $\gamma_t$ respectively the shadow values of the equity constraint (15) and the debt constraint (16), the Bellman equation of the firm’s problem can be written as

$$V(a_{f,t}) = \max_{d_t, b_{t+1}, b_{t+1}} d_t(1 + \lambda_t) - \gamma_t \left[ b_{t+1} - \frac{\mu}{1 + r_{t+1}} f(k_{t+1}) \right] + \frac{1}{1 + r_{t+1}} V(a_{f,t+1}),$$

using $r_{t+1}$ as the discount factor. Assuming that these constraints are only binding in period $t$, so that the firm is unconstrained in all periods after $t$, the first order condition for the amount
of investment in period $t$ is given by

$$1 = \frac{f'(k_{t+1})}{(1 + r_{t+1})(1 + \gamma_{t})(1 + \mu_{t})} + \frac{(1 - \delta)}{(1 + \gamma_{t})(1 + r_{t+1})}$$

$$= \frac{f'(k_{t+1})}{(1 + r_{t+1})(1 + \gamma_{t})(1 + \mu_{t})} + \frac{(1 - \delta) f'(k_{t+2})}{(1 + r_{t+1})(1 + r_{t+2})(1 + \gamma_{t})} + \frac{(1 - \delta)^2 f'(k_{t+2})}{(1 + r_{t+1})(1 + r_{t+2})(1 + r_{t+3})(1 + \gamma_{t})} + \ldots$$

$$= \frac{f'(k_{t+1})}{(1 + r_{t+1})(1 + \gamma_{t})(1 + \mu_{t})} + \sum_{j=2}^{\infty} \frac{(1 - \delta)^{j-1} f'(k_{t+j})}{(1 + \gamma_{t}) \prod_{i=2}^{j} (1 + r_{t+i})}. \quad (17)$$

According to expression (17), firms will invest until the cost of one additional unit of capital, 1, is equal to the present value of its future stream of cash flows, discounted taking into account the shadow value of the current credit constraint. It becomes clear that firms effectively perceive a higher rate of interest to all maturities, through the effect of $\gamma_{t}$. It also becomes clear that the discount rate adjustment is stronger for maturities $t + 2$ and above. Whereas the discount rate for marginal output of period $t + 1$ is multiplied by the factor $(1 + \gamma_{t}) / (1 + \mu_{t})$, the discount rate for marginal output of period $t + 2$ and beyond is multiplied by the factor $(1 + \gamma_{t})$ as long as $\mu > 0$. This difference increases with the collateral value of short term returns $\mu$, and when $\mu = 1$ (short term returns are fully pledgeable) we get

$$1 = \frac{f'(k_{t+1})}{(1 + r_{t+1})} + \sum_{j=2}^{\infty} \frac{(1 - \delta)^{j-1} f'(k_{t+j})}{(1 + \gamma_{t}) \prod_{i=2}^{j} (1 + r_{t+i})}, \quad (18)$$

which means that firms in this case only apply a higher discount rate to long term ($t + 2$ and beyond) cash-flows.

The intuition behind this result is that increasing investment today only increases pledgeable output in period $t + 1$, but does not produce any pledgeable output in periods $t + 2$ and beyond and those late cash-flows do not contribute to relaxing the borrowing constraint today. This is equivalent to a steepening of the effective yield curve faced by the firm and as a result a stronger discounting of cash-flows received after period $t + 1$.

2.4.2 Firms with future anticipated binding credit constraints effectively perceive a relatively lower productivity of long-term capital

Second, we show that a firm with future anticipated binding credit constraints effectively perceives a relatively lower productivity of long-term capital. Intuitively, future binding credit constraints increase the likelihood of having to liquidate long term capital at a cost, thus decreasing its expected productivity. To make this formal point, we also consider a simplified setup and again abstract from adjustment costs of investment and labor input. We consider two production functions, a short term and a long term one, that transform $k_{h,t+1}$ units of capital into $f_{h}(k_{h,t+1})$ units of consumption goods the following period, without the need to use labor. The firm chooses capital $k_{L,t+1}$ and $k_{S,t+1}$, borrowing $b_{t+1}$, and dividends $d_{t}$ to maximize the present discounted value of dividends. There is idiosyncratic risk in the form of a profit
shock $\varepsilon_t(k_{S,t+1} + k_{L,t+1})$, where, for analytical tractability, $\varepsilon_t$ is a continuous variable with cumulative distribution function $G(\varepsilon)$ and support $[0, \varepsilon_{\text{max}}]$. Firms face the possibility of forced exit every period with probability $\eta$, in which case they liquidate their undepreciated stock of long term capital in return for $\pi < 1$ goods per unit of capital. The first order conditions in an unconstrained setup for short and long term investment are

$$1 = \frac{1}{1 + r_{t+1}} E_t \left[ f'(k_{S,t+1}) - \varepsilon_t \right],$$

for short term capital, and

$$1 = \frac{1}{1 + r_{t+1}} E_t \left[ f'(k_{S,t+1}) - \varepsilon_t + (\eta\pi + (1 - \eta))(1 - \delta) \right]$$

for long term capital. Expectations are taken over the realization of $\varepsilon_t$. Expression (20) shows that the valuation of long term capital in period $t + 1$ is equal to 1, unless the firm is forced to exit, in which case it becomes $\pi < 1$.

Turning to a context with financing constraints, consider that firms are unable to issue any equity, which means that

$$d_t \geq 0,$$  

(21)

and that they can only borrow up to a fraction $0 < \mu < 1$ of output in $t + 1$,

$$b_{t+1}(1 + r_{t+1}) \leq \mu [f(k_{L,t+1}) + f(k_{S,t+1})],$$

(22)

and cannot pledge any part of the output produced in period $t + 2$ or after.\(^9\)

Firms might have to liquidate part of their long term capital stock if the current investment level will result in a negative asset position if the cost shock occurs. The amount of long term capital that has to be liquidated is captured by variable $\sigma_t(k_{L,t}, a_{f,t})$.

Denote with $\lambda_t$ and $\gamma_t$ respectively the shadow values of the equity constraint (21) and the debt constraint (22), and assume that these constraints are binding in every period. The first order conditions for short and long term investment are respectively:

$$1 + \lambda_t - \gamma_t \mu f'(k_{S,t+1}) = \frac{1}{1 + r_{t+1}} E_t \left[ (f'(k_{S,t+1}) - \varepsilon_t)(1 + \lambda_{t+1}) \right],$$

(23)

$$1 + \lambda_t - \gamma_t \mu f'(k_{L,t+1}) = \frac{1}{1 + r_{t+1}} E_t \left[ (f'(k_{L,t+1}) - \varepsilon_t)(1 + \lambda_{t+1}) \right.$

$$\left. + (\eta\pi + (1 - \eta)(\sigma'_{k_{L,t+1}}\pi + (1 - \sigma'_{k_{L,t+1}}))(1 - \delta)(1 + \lambda_{t+1}) \right]$$

(24)

Expression (24) shows that in the presence of financing constraints, the expected value of one unit of capital when the firm does not suffer the exit shock is less than 1, whereas in an unconstrained scenario the valuation of the undepreciated capital is always 1 if the firm is in operation (expression (20)). In particular, in a fraction $E_t \left( \sigma'_{k_{L,t+1}} \right)$ of the states in which the

\(^{9}\)For simplicity, and without loss of generality, we ignore the possibility of pledging undepreciated long term capital and the impact of the cost shock on the borrowing constraint.
firm can continue to operate, it will need to liquidate the additional unit of capital invested at a low valuation of $\pi < 1$, to avoid the possibility of ending with a negative asset position the following period. As a result firms with future anticipated binding credit constraints effectively perceive a relatively lower productivity of long-term capital.

3 Calibration, Firm Optimal Policies and Steady State Analysis

3.1 Mapping Short and Long-Term Capital to their Real-World Counterparts

The long-term technology $f_L(k_{L,t+1}, l_{L,t})$ in our model produces most of its returns in the long-run and utilizes capital that is partially irreversible and illiquid, features which are meant to capture the notion that it is an investment that the firm is tied to for a long period of time and can only undo at a cost. The types of investment that fall into this category are research and development (R&D), firm-specific structures, and other long term intangible capital items such as firm-specific human capital, firm branding, or growth options. Structures such as commercial or industrial property that are easily redeployable, such as urban offices, have a high resale value and investing in such an asset is effectively perceived by the firm as a short term investment given that it is not tied to it for a long period of time, despite the fact that it is irreversible and long-term.

One implication of our model is that these types of long-term capital produce low collateral and attract a low amount of debt finance and instead are financed mostly by retained earnings, which can be thought of as internal equity. This is in line with existing empirical evidence. Brown, Fazzari, and Petersen (2009) document that U.S. firms finance most of their R&D expenditures out of retained earnings and equity issues, an observation in line with the conclusion in Hall (2002) that R&D-intensive firms feature much lower leverage on average than less R&D intensive firms. Gatchev, Spindt and Tarhan (2009) document that, in addition to R&D, also marketing expenses and product development are mostly financed out of retained earnings and equity. Fama and French (2005) document that small, high-growth firms use more equity to cover their financing shortfalls, which can be interpreted as financing the option value of future growth opportunities with equity.

Further evidence consistent with the notion that long-term assets have less collateral value than short term assets are the findings by the empirical literature that started with Rajan and Zingales (1998) that industries that rely more heavily on external finance grow faster in countries with better developed financial systems. Braun and Larrain (2005) show that these industries do worse in recessions. Their measure of reliance on external finance, defined as capital expenditures minus cash flow from operations divided by capital expenditures, is tightly associated to the maturity of investment; these authors cite the gestation period of a project as one of the key factors behind the degree of reliance on external finance. The evidence that firms

---

10 Irreversibility refers to the firm’s limited ability to reconvert the long-term capital into consumption goods in the short run within the firm. But if the long term capital was easily redeployable and could be used productively by other firms, irreversibility would not be a problem. For this reason, the long term capital is also assumed to have a high degree of firm-specificity and as a result a low redeployability and resale value to other firms.
that operate long-term assets are more affected by a deterioration of credit market conditions is consistent with the notion that long-term assets find it more difficult to attract external finance.

On the other hand, the short-term technology \( f_S(k_{S,t+1}, l_{S,t}) \) in our theory is characterized by a short maturity of its returns.\(^{11}\) The types of investment that fall into this category are inventory investment, investment in machinery and equipment, and structures with high resale value.

One implication of our theory is that these types of capital are mostly financed with debt, which has been shown empirically to be the case. This is perhaps most clear in the case of leases, which can be interpreted as collateralized debt financing in which the debtor can very easily repossess the leased asset in case of default.\(^{12}\) The structure of lease contracts, designed to facilitate reposssession and redeployment of the leased asset, suggests that they are most useful in the case of assets which are not highly firm specific and can easily find alternative uses. Eisfeldt and Rampini (2009) report that a big share of machinery, equipment, buildings and other structures are financed with leases. Inventory investment and other assets with short maturities under one year attract substantial debt finance in the form of trade credit and bank credit lines (Petersen and Rajan (1997), Sufi (2009)). Finally, investment in commercial real estate is primarily financed with mortgage loans (Benmelech, Garmaise, and Moskowitz (2005)). Furthermore, these authors find, consistent with the results in this paper, that higher asset redepolyability leads to larger loans with longer maturities.

### 3.2 Calibration

We calibrate the economy at the quarterly frequency, and the chosen values for the parameters are shown in Table 1. We set some parameter values directly based on existing microeconomic and macroeconomic evidence, and calibrate the remaining parameters so that key aggregate variables from the simulated steady state of the model match their empirical counterparts in recent U.S. history. A summary of all the empirical moments used to calibrate parameters can be found in Table 2.

The utility function for households is chosen to be iso-elastic of the form

\[
u(C_t) = \frac{C_t^{1-\varphi} - 1}{1 - \varphi},
\]

with a constant relative risk aversion coefficient, \( \varphi \), of 2. The intertemporal preference rate \( \beta \) is set at 0.99 to match an annualized average interest rate of approximately 4 percent.

The calibration of parameters concerning the firm sector can be divided into three categories: those that affect their production technologies, those that control firms’ financing constraints, and those that determine firm entry and exit.

The firm technology parameters \( (\gamma_S, \alpha_S, \gamma_L, \alpha_L, \theta_S \text{ and } \theta_L) \) are calibrated as follows. We draw on the existing empirical evidence on estimates of the degree of returns to scale at the

\(^{11}\)We could also have modelled a third type of capital, long-term capital with a high degree redeployability or of reversibility, which would have behaved similarly to short term capital in terms of how financing constraints affect it.

\(^{12}\)In practice, the lessor retains the property of the asset and transfers the right to use the property to the lessee. Assets subject to operating leases do not show in the firms’ balance sheets, unlike assets subject to capital leases, in which there is some transfer of ownership of the asset.
firm level and set $\gamma_S + \alpha_S = \gamma_L + \alpha_L = 0.9$, in line with Thomas (2002) and Restuccia and Rogerson (2008). To calibrate the precise values assigned to the labor and capital shares of the short term and long term technologies, $\{\gamma_S, \alpha_S\}$ and $\{\gamma_L, \alpha_L\}$ respectively, we draw on the estimates in Corrado, Hulten and Sichel (2009) of the income shares of labor, tangible capital, and intangible capital in the U.S. over the period 1993-2005, which are 60%, 25% and 15%, respectively. Their empirical definition of intangible capital is broadly consistent with our own definition of long term irreversible and firm specific capital. It includes research and development, business investment in computer software, and spending on strategic planning, redesigning existing products, investments to gain market share, and long-term investments in brand names. Given that these empirical estimates are at the aggregate level, the larger aggregate income share of tangible capital (which maps into short term capital in our model) can be due to a larger productivity $\theta_S$ of sectors that utilize tangible capital or due to a larger input share of capital $\alpha_S$ in those sectors. There are no such estimates available that could allow us to make this distinction, and we choose to assume that capital shares are the same for both technologies, so that, after applying the adjustment for decreasing returns to scale, we obtain $\gamma_S = \gamma_L = 0.54$ and $\alpha_S = \alpha_L = 0.36$. We then set the difference between productivity parameters $\theta_S$ and $\theta_L$ so that the aggregate income shares of tangible and intangible capital are those estimated by Corrado, Hulten and Sichel (2009). We set the level of these productivity parameters to match the median firm size growth rate, which is driven by firm profitability. We calculate the median total asset growth for Compustat firms to be around 5% annually, or approximately 1.23% quarterly. We check that our results are robust to alternative choices in which $\alpha_S > \alpha_L$ and $\theta_S = \theta_L$ and results do not vary in a quantitatively significant way. As an additional disciplining device, we make sure that we roughly match the aggregate amount of long-term investment as a share of total investment in the data. We calculate the ratio of long-term to total investment for the U.S. using data from the Bureau of Economic Analysis (BEA). The BEA provides data on gross private domestic fixed investment by type. We consider long-term investment to include research and development (R&D), and possibly also certain types of investment in non-residential structures, in software development or in industrial equipment. Short-term investment is considered to include inventory investment, equipment investment, and residential fixed investment. Long-term investment as a share of total investment using a restrictive definition that only considers R&D investment has been on average close to 11% over the period 2000-2013, while, at the other extreme, if we also consider non-residential structures, software development and industrial equipment, this measure has been on average 47% during the same time period. Our simulated share of long term investment falls within this range, close to the lower bound.

The parameters driving the degree of idiosyncratic risk, which are the probability $p$ of suffering an operating cost shock, and the size $\varepsilon$ of the shock per unit of capital, are jointly calibrated to roughly match three moments in the data: (i) the ratio between the median size

---

13 These estimates are in line with values used by other studies, such as Hall and Mairesse (1995) and McGrattan and Prescott (2012) who impute an intangible capital input share of 0.19 and 0.076 respectively. The estimates for the labor share are also in line with values used in previous studies such as King and Rebelo (1999) and Thomas (2002).

14 As discussed in section 3.1, fixed residential investment, even if possibly very long-term in nature, is a short-term investment from the perspective of a firm that has the ability to sell it easily in a liquid resale market.
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi )</td>
<td>Coefficient of relative risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Households' discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td>( \alpha_S )</td>
<td>Capital share in short-term technology</td>
<td>0.36</td>
</tr>
<tr>
<td>( \alpha_L )</td>
<td>Capital share in long-term technology</td>
<td>0.36</td>
</tr>
<tr>
<td>( \gamma_S )</td>
<td>Labor share in short-term technology</td>
<td>0.54</td>
</tr>
<tr>
<td>( \gamma_L )</td>
<td>Labor share in long-term technology</td>
<td>0.54</td>
</tr>
<tr>
<td>( \theta_{SS} )</td>
<td>Aggregate productivity factor (steady state)</td>
<td>1</td>
</tr>
<tr>
<td>( \theta_S )</td>
<td>Productivity factor of short-term technology</td>
<td>0.7</td>
</tr>
<tr>
<td>( \theta_L )</td>
<td>Productivity factor of long-term technology</td>
<td>0.5</td>
</tr>
<tr>
<td>( p )</td>
<td>Probability of a negative operating cost shock</td>
<td>0.25</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Size of operating cost shock per unit of total capital</td>
<td>3</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation of long-term capital</td>
<td>0.025</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Multiplicative parameter of convex adjustment cost function</td>
<td>4</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Exponential parameter of convex adjustment cost function</td>
<td>2</td>
</tr>
<tr>
<td>( \mu_{SS} )</td>
<td>Pledgeability of earnings</td>
<td>0.8</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Corporate tax rate</td>
<td>0.35</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Exogenous probability of firm exit</td>
<td>0.025</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Recovery value of long-term capital</td>
<td>0.40</td>
</tr>
</tbody>
</table>

of young firms and the median size of the whole population of firms, (ii) the standard deviation of the ratio of profits to sales, (iii) the probability of negative profits. Using the Capital IQ-Compustat database for U.S. listed firms during 2002-2011 we calculate the ratio between the median total assets of young stock-exchange listed firms (those with less than 1 year following their IPO) and the median asset holdings of the whole population of listed firms, which is 0.51. Using this same data, we get a standard deviation of the ratio of profits to sales of 0.35 and a probability of negative profits of 25%, both calculated at the annual frequency. Our definition of profits is equal to after-tax profits, as defined in Section 2.1.

The depreciation rate of long-term capital is set at \( \delta = 0.025 \), which is a standard value for quarterly Real Business Cycle (RBC) models. The adjustment cost function for long-term investment, \( \psi(i_{L,t}) \), is assumed to adopt the functional form

\[
\psi(i_{L,t}) = \chi^\zeta i_{L,t},
\]

with values \( \zeta = 2 \) and \( \chi = 4 \) set to generate aggregate adjustment costs that are roughly equal to 0.2% of aggregate investment, in the lower end of the estimates discussed in studies of adjustment costs of capital such as Gourio and Kashyap (2007), and also to be broadly in line with the firm-level standard deviation of investment rates (investment over capital) observed in the data. Cooper and Haltiwanger (2006) show using Longitudinal Research Database data between 1972 and 1988 that the standard deviation of the investment rate is 0.306. The recovery rate of long term capital, \( \pi \), is set to match the observed recovery rates of corporate bonds in cases of default (Chen (2010)), estimated at around 40%, which is a proxy for the recovery value of the firm’s assets. The corporate tax rate \( \tau \) is set at the U.S. level of 35%.

The steady state value for the parameter regulating borrowing constraints (\( \mu_{SS} \)) is set to
Table 2: Empirical Moments Matched in the Calibration

<table>
<thead>
<tr>
<th>Empirical Moment</th>
<th>Source</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Targets to calibrate $\alpha_S, \alpha_L, \theta_L, \theta_S, p$ and $\varepsilon$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Income share of intangible capital</td>
<td>(1)</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>- Income share of tangible capital</td>
<td>(1)</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>- Long-term investment over total investment</td>
<td>(2)</td>
<td>0.11 – 0.47</td>
<td>0.15</td>
</tr>
<tr>
<td>- Median assets of young firms over median assets of all firms</td>
<td>(3)</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>- Probability of negative profits (annual)</td>
<td>(3)</td>
<td>0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>- Standard deviation of the ratio of profits to sales (annual)</td>
<td>(3)</td>
<td>0.35</td>
<td>0.27</td>
</tr>
<tr>
<td>- Median quarterly total asset growth</td>
<td>(3)</td>
<td>1.23%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Targets to calibrate $\zeta$ and $\chi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Adjustment costs as share of aggregate investment</td>
<td>(4)</td>
<td>0.2% – 6.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>- Standard deviation of investment over capital (annual)</td>
<td>(5)</td>
<td>0.306</td>
<td>0.175</td>
</tr>
<tr>
<td>Targets to calibrate $\mu$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Average net leverage ratio</td>
<td>(6)</td>
<td>0.079</td>
<td>0.068</td>
</tr>
<tr>
<td>Targets to calibrate $\eta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Job destruction rate (annual)</td>
<td>(7)</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Additional Moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Share of firms that pay out dividends or repurchase shares</td>
<td>(8)</td>
<td>44%</td>
<td>23%</td>
</tr>
<tr>
<td>- Share of credit constrained firms</td>
<td>(8)</td>
<td>50%</td>
<td>48%</td>
</tr>
</tbody>
</table>


match the average net leverage ratio for Compustat publicly listed firms, which is 7.9% according to Bates et al. (2009) using data from 1980 to 2006. They calculate net leverage as the ratio of total debt minus cash holdings to the book value of total assets. Net leverage in our model is calculated as the current value of debt $b_{t+1}$ minus holdings of the asset over the total value of book assets, assuming assets are marked to market and valued according to the current firm value $J(a_{f,t}, k_{L,t}, z_{SS})$, where $z_{SS}$ is the steady state defined by $\{r_{SS}, w_{SS}, \theta_{SS}, \mu_{SS}\}$. In addition, we check that the model produces reasonable values of the share of credit constrained firms and of dividend paying firms. Farre-Mensa and Ljungqvist (2014) provide estimates of the fraction of financially constrained firms in the U.S. They find that roughly three quarters of privately held firms are financially constrained. Within the sample of publicly listed firms, they report different estimates of the share of financially constrained firms that range between 10% and 45%. Given these estimates, we target a share of financially constrained firms to be around 50% in our sample. Our measure of financially constrained firms in the model includes those firms whose borrowing constraint is binding, and those whose equity issuance constraint is binding. They also estimate the share of dividend paying firms to be around 44% in their

\[15\text{Net leverage and leverage are the same in our model because firms do not simultaneously hold the asset and borrow. For this reason, the relevant counterpart in the data is net leverage.}\]
sample, and we obtain a share which is somewhat lower.

Finally, we set the size of the exogenous firm exit shock $\eta = 0.025$ to match the U.S. empirical level of 10 percent job destruction per year, following Bilbiie et al. (2012). Exiting firms are immediately replaced by new firms with no holdings of assets or long-term capital.

3.3 Firm Policies and Firm Heterogeneity in the Steady State

We now take a closer look at firm policies in the steady state equilibrium that arises from the calibration described above. We solve the model numerically using value function iteration and simulation techniques. A detailed explanation of the solution method for the steady state equilibrium can be found in the Appendix.

Figure 1 displays long-term capital as a share of total capital in the space defined by asset and long-term capital holdings of the firm. Long-term capital decreases significantly as a share of total capital as asset holdings of the firm decrease. Firms with very low asset holdings are credit constrained and have a bias towards short-term capital because long-term capital produces less pledgeable output per unit of investment than short-term investment. But also the anticipation of future credit constraints creates a bias towards short-term investment. Firms with low asset holdings but which are currently unconstrained also have a bias towards short term capital. Idiosyncratic uncertainty interacts with financing constraints to favor short term investments because long term capital runs the risk of having to be liquidated at a cost in the future if the firm runs into financial distress, meaning that long term capital has lower expected returns for firms with low liquid buffers, even if they are currently unconstrained.

Figure 2 displays the distribution of asset and long-term capital holdings, the two dimensions along which firms differ. Firms are created identical with no liquid assets or long term capital, and slowly build up the stock of both. The optimal asset holdings are determined by the trade-off between firms’ desire to self insure against the possibility of future negative operating cost shocks that weaken their financial position, and the desire to pay out dividends given that the after-tax return to cash holdings, $(1 - \tau)(1 + r_{t+1})$ is lower than the discount rate firms use,
Figure 2: FIRM HETEROGENEITY. This figure displays the distribution of asset holdings $a_{f,t}$ and long-term capital holdings $k_{L,t}$ in the steady state equilibrium.

which is the equilibrium interest rate $(1 + r_{t+1})$.$^{16}$ This optimal level is around 1 for most firms, although the precise value varies cross-sectionally as a function of the firm-level stock of long-term capital. Similarly to asset holdings, firms have an optimal holding of long term capital, and for most firms this value is around 0.2, but some firms with particularly high asset holdings might be willing to hold a larger stock given that they face a lower risk of falling into financial distress and having to liquidate part of their capital at a cost.

The speed at which the firm is able to accumulate assets is determined by several factors which are calibrated to match their observed counterparts. First, the main drivers of asset accumulation are the returns on cash holdings and firm operating profitability. Second, the exogenous firm exit possibility limits firms’ ability to accumulate assets over time. Finally, firms have an incentive to pay out dividends when they reach a certain size, because of the tax penalty on cash savings discussed above.

4 Amplification and Propagation of Productivity and Financial Shocks

4.1 Productivity Shocks

In this section we analyze the transitional dynamics following an unexpected, temporary and persistent negative shock to aggregate productivity $\theta_t$. We solve the model numerically using value function iteration and simulation techniques. A detailed explanation of the solution method for the transitional dynamics can be found in the Appendix.

$^{16}$The dynamic trade-off between the tax penalty on returns to cash holdings and the reduction in expected future financing costs conferred by holding cash has been addressed by Riddick and Whited (2009).
Figure 3: Responses of key aggregate variables to a 2% negative shock to aggregate productivity $\theta_t$. Responses are calculated as percent deviations of each variable from their steady state values, except for the productivity shock and the interest rate, for which the actual values are plotted, and the share of long term investment over total investment, for which the absolute deviations from their steady state values are plotted.

We introduce an unanticipated shock in period $s$ to aggregate productivity $\theta_t$ equivalent to $-2\%$ of its steady state value ($\theta_{SS} = 1$). We assume that following the shock, $\theta_t$ evolves according to

$$\log \theta_t = \rho \log \theta_{t-1}$$

where $\rho = 0.9$. Firms and households learn about the productivity shock in advance of making their optimal choices in period $t = s$. As a result, firms are always able to repay any borrowing they undertook in period $t = s - 1$. They have a perfect foresight about the evolution of $\theta_t$ from $t = s$ on.

To highlight the effect of firm financing frictions, we will compare the reaction of two different economies to this productivity shock. One economy will be the benchmark economy described in sections 2 and 3 (the financial frictions economy), in which firms suffer from financing constraints. The other economy will be one in which firms do not suffer from financing constraints (the unconstrained economy). In the unconstrained economy firms can pledge all of their future output to their lenders, and firms can access equity markets, which in the model corresponds to negative dividends. The calibration is otherwise kept unchanged for both versions of the model.

The responses of nine key aggregate variables to the productivity shock are displayed in Figure 3. The responses are calculated as percent deviations of each variable from their steady state value, except for the productivity shock and the interest rate, for which the actual values are plotted, and the share of long term investment over total investment, for which the absolute deviation in percentage points from its steady state value is plotted.
In the unconstrained economy short-term investment drops moderately following the shock, but long-term investment is not very responsive, because a temporary negative productivity shock decreases short term investment returns more than long-term investment returns, encouraging a shift towards long-term investment. Long-term investment as a share of total investment increases above its steady value for approximately 10 quarters, peaking at a share 0.75 percentage points above its steady state value, and decreases slowly back to its steady state value. The decrease in output tracks the evolution of the productivity parameter $\theta$ closely, and recovers back to its steady state value as $\theta$ approaches its steady state value of 1. There is little amplification or persistence in the economy with financially unconstrained firms.

Turning to the benchmark economy that features financial frictions for firms, the decrease in profits and net worth increases the share of firms with binding credit constraints. Firms' net worth recovers slowly so credit constraints are expected to remain tight for some time. Both the currently binding credit constraints and the anticipation of future constraints mean that many firms are unable or unwilling to sustain previous levels of long term investment and increase the share of their investment allocated to short term capital. Long-term investment falls by around 1.8% relative to its steady state value at its lowest point in quarter 4, while short term investment falls less and later, by around 0.8% relative to its steady state value at its lowest point in quarter 5. As a result, long-term investment as a share of total investment decreases below its steady value, bottoming at a share 0.75 percentage points below its steady state value. The large fall in long-term investment translates into a large drop in the stock of long-term capital. This stock takes many periods to recover due to the combination of costs of adjustment in investment and the loss of net worth in the firm sector and as a result an increased reliance in external finance, which is subject to frictions. Firms' investment dynamics translate into a dampening of the effects of a negative productivity shock in the short-run, and a strong propagation in the medium and long-run. Output drops more and sooner in the unconstrained case; it bottoms at $-1.6\%$ relative to the steady state level in the fourth quarter following the shock, while the output drop in that quarter when firms face financing constraints is $-0.9\%$, relative to their respective steady states. In the medium and long run, the large initial drop in long-term capital and its slow recovery means that the propagation of the effects of the shock is greater in the economy with financing constraints.$^{17}$

The mechanism introduced in this paper affects the contemporaneous amplification and the intertemporal propagation of responses to shocks in opposite directions, so it is useful to cal-

\[ \text{\footnotesize Notes:} \]

$^{17}$General equilibrium forces could induce financially unconstrained firms to compensate for constrained firms' lower investment and hiring. In particular, it could be that following the negative productivity shock, financially constrained firms' decreased ability to borrow and hire would lower both the interest rate $r_t$ and the equilibrium wage $w_t$, so that financially unconstrained firms would react by investing (in both the long and short term technologies) and hiring more, reducing the relevance of financial constraints. In our simulations, the interest rate $r_t$ (after a small initial drop), rises above its steady state value for more than 20 quarters, peaking at 1.5% in the financial constraints economy, around 50 basis points above its steady state value. This increase is due primarily to the large decrease in household saving, and means that in equilibrium interest rates do not encourage unconstrained firms to invest more. Furthermore, the decrease in rates back to their steady state level after period 4 means that, after that period, the downward sloping yield curve encourages a relative shift towards long-term investment in the medium-term. A final effect of interest rates is that higher interest rates tighten borrowing constraints by lowering the present value of future pledgeable output and induce a precautionary behavior in firms that encourages them to shift even more strongly into short-term investment. Wages (not shown in the figure) do fall slightly, although not enough so that unconstrained firms make up for the lower investment and hiring of constrained firms.
Figure 4: Credit constraints following a 2% negative shock to aggregate productivity $\theta_t$. Low (high) asset holdings firms are those whose asset holdings are in the lowest (upper) tercile. The share of long term investment over total investment is calculated as absolute deviations from its steady state value.

calculate the integral of the deviations in output to gauge which effect dominates in determining total output loss in response to a negative shock. Total output loss over the 40 quarters following the shock is -15.2% of one quarter’s output in the steady state in the financial frictions economy, and -10.6% in the unconstrained economy, a difference of 4.6%. This difference can be decomposed into a dampening over the first five quarters in the financially constrained economy by an accumulated amount equivalent to 2.3% of quarterly steady state output, and an additional propagation over the subsequent thirty-five quarters in the financially constrained economy relative to the unconstrained economy by an accumulated amount equivalent to -6.9% of quarterly steady state output. If we discount future output using the households’ quarterly discount rate $\beta = 0.99$, the accumulated difference over the 40 quarters falls to 3.3%. Clearly, the negative long-term effects of financial constraints dominate any positive short term effects, in terms of aggregate output.

4.1.1 Countercyclicality of financing constraints

It is worth inspecting more closely the dynamics of the severity of firm financing constraints and of firm heterogeneity. There are three mechanisms that influence how tight borrowing constraints are along the transitional dynamics. One is the decrease in firms’ asset holdings following the shock. Another is the increase in interest rates which decreases the present value of pledgeable output and tightens borrowing constraints. A third, novel, source of countercyclicality of borrowing constraints arises from the coexistence of short and long-term investment opportunities. A temporary negative shock to productivity encourages a shift towards investing in long-term capital because its returns are relatively more acyclical. Long-term capital produces less pledgeable output per unit of investment, all else equal, so this shift towards long-term investment results in tighter borrowing constraints.

Figure 4 shows the combined effects of these mechanisms. The share of borrowing constrained agents increases from 48% to around 51% and the increase is persistent. This increase occurs despite the fact that due to the negative productivity shock there is a decrease in demand for external funds to finance investment. If we compare the behavior of high asset holdings and low asset holdings firms, a classification that broadly captures financially constrained and un-
Figure 5: Responses of key aggregate variables to a 10% and a 20% negative shock to financial constraints $\lambda_t$. Responses are calculated as percent deviations of each variable from their steady state values, except for the productivity shock and the interest rate, for which the actual values are plotted, and the share of long term investment over total investment, for which the absolute deviations from their steady state values are plotted.

constrained firms, respectively, we can observe that low asset holdings firms react by strongly shifting their investment towards the short-term project while high asset holdings firms only respond to the changes in the differential rates of return of short and long term productivities and shift into long term projects.

4.2 Financial Shocks

In this section we analyze the transitional dynamics following an unexpected, temporary and persistent negative shock to the severity of financing frictions. In particular, we introduce an unexpected shock to $\mu_t$, the parameter that determines the share of next period’s output that can be pledged to lenders today. We compare the impact of shocks of two different sizes, equal respectively to 10% and 20% in absolute terms. In other words, $\mu_t$ decreases in $t = 1$ unexpectedly from $\mu_{t=0} = 0.9$ to $\mu_{t=1} = 0.8$ in one case, and to $\mu_{t=1} = 0.7$ in the other, and we assume that following the shock, for $t > 1$, $\mu_t$ evolves according to

$$\log \mu_t = \rho \log \mu_{t-1}$$

where $\rho = 0.6$.\footnote{The value for the quarterly persistence of the financial shock is taken from Hall (2011), who estimates the persistence of the increase in spreads during the financial crisis of 2008-2009.} As with the technology shock, agents are assumed to learn about the shock on the period in which it occurs and have a perfect foresight about the evolution of $\mu_t$ from then on. The aggregate productivity factor $\theta_t$ is assumed to remain constant at its steady state value $\theta_{SS}$. The resulting transitional dynamics are displayed in Figure 5.

A tightening of financing constraints results in lower total investment in both cases. A 10%
(20%) absolute decrease in the pledgeability of future output decreases total investment by around 10% (35%) at the trough, relative to total steady state investment. The impact of the financial shock is large, and increases disproportionately with the size of the shock. On top of the impact on the total quantity of investment, there are important compositional effects as well. The fall in long-term investment in both cases is larger than the fall of short-term investment, and the difference increases more than proportionally in the size of the financial shock. A 10% financial shock is associated with a decrease in investment in long-term capital as a share of total investment of around 4 percentage points (from 36% to 32%), while in the case of a 20% shock, this shift out of long-term investment is of around 12 percentage points (from 36% to 24%). Two forces are at play. First, given that on the margin long-term investment produces less pledgeable output than short-term investment, a tightening of credit constraints shifts investment towards the short-term project. Second, the tightening of credit constraints increases the likelihood that the firm will have to liquidate part of its long term capital following negative operating cost shocks, lowering the expected return of long term investment. The evolution of the distribution of firms’ asset holdings enhances both effects. As firms become poorer they need to finance a bigger share of their investment with external finance, which biases their investment towards short term investment. Also, as they become poorer they are more likely to be financially constrained in the future and to have to liquidate part of their long term capital.

The combination of lower wages and dividends and the desire of households to smooth consumption translates into higher interest rates for much of the transition period, following an initial brief decrease, which further decreases the present value of firms’ future pledgeable output and tightens constraints. Output drops by close to 2% four quarters after the impact of the 10% negative financial shock, and by close to 5% in the case of a 20% financial shock. The relative difference in the output reaction grows with time due the compositional effect. The large decrease in long-term investment in the case of a large shock means that the capital stock decreases strongly and recovers only slowly, due to the combined effect of convex costs of adjustment and credit constraints. While the output fall 4 quarters following the shock is roughly 2.5 times stronger in the large shock case, it is more than 5 times stronger 10 quarters after the shock, and remains 3 times stronger 40 quarters after the shock.

4.2.1 Anticipated Future Financial Shocks and Precautionary Behavior

Credit constraints interact with the choice of maturity of investment projects not just through currently binding credit constraints, but also through the possibility of future binding constraints. To isolate the precautionary behavior induced by future credit constraints, we conduct an exercise in which the firm learns about a future permanent shock to output pledgeability factor $\mu_t$ that will occur 9 quarters ahead, and which will decrease $\mu$ from 0.9 to 0.6. The results of simulating such an exercise can be found in Figure 6.

Firms react to the knowledge of a future tightening of credit constraints by shortening the maturity profile of their portfolio of investment. In fact, short term investment is above its steady state value for 4 quarters after the news of the shock, and starts decreasing sharply after the 6th quarter. Long-term investment on the other hand decreases gradually over the 8 quarters between the news of the shock and the moment the shock hits, being close to 40%
below its steady value immediately before the shock hits. As a result, output actually increases above its steady state value for 5 quarters, peaking at 0.75% above the steady state level of output. As the economy approaches the period in which credit constraints become tighter, output falls sharply due to both the accumulated decrease in long-term capital and the decrease in short term investment. This result points at the strong precautionary effect of the possibility of future binding constraints.

5 Robustness and Extensions

5.1 The Role of Idiosyncratic Risk for Firms

In this section we study how the degree and the nature of idiosyncratic risk affect the results. Idiosyncratic uncertainty in our model, in the form of a shock to operating costs, plays two roles. It is at the source of the second mechanism (the precautionary mechanism) through which financial frictions distort investment choices in favor of short-term projects, and it is necessary to be able to replicate the empirical cross-sectional distribution of firm size. If we eliminate idiosyncratic uncertainty, the result that firms with currently binding credit constraints allocate a smaller share of total investment to long term capital would still be present, but the second mechanism disappears, as the likelihood that the firm might suffer a negative shock that generates the need to liquidate long term capital would be absent. In terms of replicating the empirical distribution of firm size, a model with no idiosyncratic uncertainty would generate
Table 3: Effects of Variations in the Volatility of the Idiosyncratic Cost Shock

<table>
<thead>
<tr>
<th>Standard deviation of (profits/sales)</th>
<th>Initial dampening(^1)</th>
<th>Long-run propagation(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35 (benchmark)</td>
<td>2.3%</td>
<td>-6.9%</td>
</tr>
<tr>
<td>0.20</td>
<td>1.9%</td>
<td>-5.8%</td>
</tr>
<tr>
<td>0.10</td>
<td>1.5%</td>
<td>-4.9%</td>
</tr>
</tbody>
</table>

\(^1\) Accumulated difference between the relative output fall in the financially constrained economy and the financially unconstrained economy, in the initial quarters after the shock during which the output fall in the financially constrained economy is less strong than the one in the unconstrained economy, relative to their respective steady states.  
\(^2\) Accumulated difference between the relative output fall in the financially constrained economy and the financially unconstrained economy in quarters after the shock during which the output fall in the financially constrained economy is stronger than the one in the unconstrained economy (up to quarter 40).

To evaluate the importance of idiosyncratic risk and the precautionary channel, we conduct an exercise in which we perform a mean preserving decrease in the volatility of the operating cost shock. The results are reported in Table 3 below. We observe that the initial dampening and the medium run propagation of productivity shocks in the financial frictions economy relative to the financially unconstrained one weaken, suggesting that idiosyncratic risk and the precautionary behavior that it induces in firms play an important role.

Idiosyncratic uncertainty for firms could be of a different nature, with implications for our results in some cases. If we consider that risk arises from productivity shocks, in the form of a stochastic and firm-specific productivity parameter \( \theta_{i,t} \), where the subscript \( i \) refers to firm ‘i’, short run dynamics of the composition of investment change. In our simulations for this exercise we have considered iid idiosyncratic productivity shocks, to avoid adding an additional state variable to the firm problem. Productivity risk of an iid nature makes short run investment riskier, given that long term investment returns are subject to several independent productivity shocks throughout the lifetime of the project, while short run returns are only subject to a single shock in the short run. As a result, firms’ precautionary behavior induced by future credit constraints generates a smaller, though still strong, bias towards short term investment. The bias arising from currently binding credit constraints remains the same. We find, using the same calibration targets as in our benchmark model, that our dampening and propagation results are similar, although moderately weaker.

5.2 Corporate Income Taxation as a Policy to Encourage Long-Term Investment

We now study the impact of an additional corporate tax feature. The U.S. corporate tax code has featured during several periods in the postwar era two main instruments for subsidizing capital expenditures; investment tax credits and allowed depreciation in excess of economic depreciation. We focus on the first feature, investment tax credits, given its more direct relationship with the distortions presented in this paper. We model investment tax credits as an expensing allowance for the \( \tau_d \) fraction of total investment, short or long term, done in that
period. We first analyze this tax policy in the form of a permanent allowance, and secondly as a countercyclical allowance designed to alleviate the inefficiencies associated with the large decrease in long-term investment in downturns.\footnote{Another important feature of the corporate income tax system is the ability for losses to be carried back to offset previous taxes, or to be carried forward to offset future taxes. We choose not to model this feature because it is less directly concerned with the distortions addressed in this paper, and because in practice there are many limitations on these provisions such that there is still a large degree of asymmetry of the corporate tax system around zero profits, a feature which we capture.}

The first policy measure, constant expensing allowances, is calibrated so that the credits as a share of total investment are around 2%, in line with recent U.S. evidence (McGrattan and Prescott (2005)). Such a policy decreases the distortion towards short term investment of financially constrained firms by decreasing the current burden of long term investment, which attracts less external finance than short term investment, but does not alter the qualitative or quantitative results concerning the response to shocks significantly.

The second policy measure, cyclical expensing allowances, are modelled as a variable expensing allowance for the \( \tau_{d,t} \) fraction of total investment that depends on aggregate productivity \( \theta_t \) in the following way:

\[
\tau_{d,t} = \tau_{d,SS} + \kappa (\theta_{SS} - \theta_t),
\]

where \( \kappa > 0 \) is calibrated so that the credits as a share of total investment during the first 5 quarters following the shock are roughly the same as at their peak in the U.S., which was in the late 1970's and early 1980's, when they were around 6% (McGrattan and Prescott (2005)). This policy is meant to capture the recent U.S. policy of temporary and partial expensing allowances on business equipment investment, one of the key countercyclical fiscal policies put in place in the recent crisis in the U.S. (Edge and Rudd (2011)). Introducing this policy has a noticeable impact on the dynamics of the composition of investment. This policy generates a reduction in short run output and an increase in medium and long run output, relative to the benchmark calibration absent cyclical expensing allowances. Initial dampening is decreased from 2.3% in our benchmark simulations, to 1.8%. Long-run propagation is brought down from 6.9% in our benchmark simulations to 5.3%. This result points towards one of the key policy implications of the frictions modelled in this paper, which is that policies that are targeted towards eliminating distortions that hamper long term investment should be evaluated in the long term, as they might come at the cost of a drop in short run output.
6 Conclusion

This paper introduces a model in which credit constrained firms can choose between investment projects that differ in their maturity. When firms can only borrow using non state contingent debt and face the possibility of forced exit with costly liquidation of long term irreversible capital, they have a preference for short term capital because it can attract relatively more external finance. The presence of idiosyncratic risk further increases the preference for short term capital because it introduces the likelihood that long term capital might have to be liquidated early if the financial condition of the firm deteriorates in the future. In effect, credit constraints increase the rate at which long-term returns are discounted and also decrease those returns in future states of financial distress.

We use this framework to explore how the consideration of firms’ maturity choice problem in the presence of financing frictions might affect the role of such frictions in amplifying or dampening shocks to the macroeconomy. A novel dampening mechanism of technology shocks is identified, which is shown to be quantitatively large. On the other hand, this framework is able to account for the empirically documented cyclical variation in the composition of investment, a feature which most existing models studying the macroeconomic implications of financial constraints cannot account for.

One limit of the analysis is that it only considers the impact of aggregate shocks that are unexpected. Introducing aggregate uncertainty in productivity or financing frictions could influence the results obtained in this paper, particularly if firms are able to write financial contracts based on the outcomes of these aggregate shocks. But in reality we observe that firms typically borrow using debt instruments which are not contingent on aggregate variables, and we conjecture that making this assumption would preserve most of the results. New insights could be derived in a setting with aggregate uncertainty, such as the ex-ante distortions caused by the anticipation of aggregate shocks. Firms for example might react in advance to the possibility of negative aggregate or financial shocks by delaying dividend payments even further or by decreasing their exposure to the possibility of long term capital liquidation by allocating a bigger share of their investment to short term capital. The magnitude of the results about short run aggregate dynamics would possibly weaken as result of these precautionary measures. From a computational standpoint, however, a framework with aggregate shocks and two-dimensional firm heterogeneity becomes very demanding, and whereas a body of methodological research has provided feasible and accurate approximations for a subset of environments featuring one-dimensional heterogeneity in the household sector and aggregate uncertainty, similar work has yet to appear in more general contexts that could be applicable to an environment like the one described in this paper.

This paper highlights the importance for models designed to study the macroeconomic implications of credit constraints of considering the type of investment firms carry out, in addition to the amount. While this paper focuses on a particular set of dimensions of project heterogeneity, several other important dimensions remain to be analyzed in the context of a general equilibrium heterogeneous agent framework like the one presented in this paper, such as minimum investment size, degree of investment irreversibility, or volatility of returns. Models that incorporate these or other dimensions of heterogeneity could be fruitful avenues of research.
A Appendix: Numerical Solution

A.1 Steady State

In the steady state, equilibrium interest rate $r_{ss}$ will be equal to the households’ rate of time preference given that they are financially unconstrained, so

$$r_{ss} = \frac{1}{\beta} - 1.$$  

Firms’ optimization in the steady state is solved taking $r_{ss}$ and a guess for steady state wage $w_{SS}$ as given, and assuming aggregate productivity $\theta_t$ and financing frictions $\mu_t$ are constant at their respective steady state levels. Value function iteration is used, and the firm’s idiosyncratic state space is discretized. Along the dimension of asset holdings $a_{f,t}$ the grid contains 20 points starting at $a_{f,t} = 0$, and given that important non-linearities arise in the value function for low values of asset holdings, the grid’s density decreases exponentially with asset holdings. Along the dimension of long-term capital holdings $k_{L,t}$ the value function is moderately concave for most of the region, due to the presence of convex costs of adjustment, and for this reason 40 equally-spaced gridpoints are chosen, again starting at $k_{L,t} = 0$.

The steady state distribution of firm asset and long-term capital holdings is calculated using the firms’ optimal policies by performing 600 iterations of a firm sector composed of 250,000 firms. From the converged distribution we obtain the steady state aggregate output, aggregate dividends, aggregate net corporate borrowing, aggregate tax revenues, and a labor-market clearing wage. In equilibrium, firms with large asset holdings save and those with low asset holdings borrow. It is the case for all combinations of parameter values considered that net borrowing of the firm sector is positive. This routine is iterated until a converged solution for the wage $w_{SS}$ is obtained.

In the steady state equilibrium households’ consumption must be equal to the sum of wages, dividends, transfers and the return on savings:

$$C_{ss} = r_{ss} A_{ss} 1 + r_{ss} + W_{ss} + D_{ss},$$

where aggregate savings $A_{ss}$ are equal to aggregate net corporate borrowing.

A.2 Transitional Dynamics Following Temporary Shocks

The solution method to obtain the transitional dynamics proceeds as follows. The process is broadly similar for technology and financial shocks, and the few differences are pointed out. We choose 150 as the number of periods needed after the technology shock for the equilibrium to converge back to the steady state. We use 80 periods for financial shocks, which are less persistent.

1. We guess a path for $r_t$, $w_t$ and for $C_t$ for all the transition periods.
2. We start backwards and, taking the $t+1$ value function of firms as given, calculate optimal time $t$ policies of firms and the resulting value function given (i) the path of the shock, and (ii) the conjectured paths for $\{r_t\}$ and $\{w_t\}$. We do this for every period, using the updated $t+1$ value function when calculating the time $t$ policies and value function.

3. Starting from the steady state distribution of firms, we simulate the entire firm sector (250,000 firms) for the duration of the transitional dynamics using the optimal policies calculated in the previous step. For each period, we calculate aggregate dividends $D_t$, aggregate tax revenues $T_t$, aggregate net corporate borrowing (which we will equate to $A_{t+1}$) and labor demand. Next, for each period we obtain the $r_{t+1}$ that clears the bond market. We do so by first calculating the household consumption $C^{eq}_t$ that would be consistent with a level of household savings equal to net corporate borrowing

$$C^{eq}_t = A_t + W_t + D_t - A^{eq}_{t+1}/(1 + r_{t+1}),$$

taking $r_t$ from our guess and $A_t$ from the previous period, and $W_t$, $D_t$ and $A^{eq}_{t+1}$ from the simulation of firms. Finally, we calculate the equilibrium interest rate that clears the bond market

$$r^{eq} = \frac{1}{\beta} \left( \frac{C^{eq}_t}{C_{t+1}} \right)^{-\gamma} - 1,$$

taking $C_{t+1}$ as given from the conjectured path. The wage $w_t$ on the other hand is adjusted upwards(downwards) if demand exceeds(falls short) of supply, by a factor proportional to the imbalance.

4. Update the conjectured path for $r_t$, $w_t$ and $C_t$ using a small updating parameter for the maximum relative variation in these three variables (0.02 in our case), and repeat 4-5 until convergence.
References


