Real business cycles
and classical monetary economics

text reading: Gali, Chapter 2
slides based on those of Jordi Gali
Questions

1. How does real activity fluctuate over time if all individuals are optimizing, markets clear, and prices are fully flexible?

2. With conditions of monetary equilibrium, what is the behavior of the price level and the nominal interest rate?

Notation: upper case letters generally denote economic variables, while the corresponding lower case variable is a logarithm. (M,m=\log(m)). This is one of two common notations in macro with the other involving upper case letters for nominal variables and lower case variables for real ones.
Assumptions in Gali chapter

- Perfect competition in goods, labor markets, and asset markets
- Flexible commodity prices, wages and asset prices
- No capital accumulation (for tractability)
- No fiscal sector (for tractability)
- Closed economy (for tractability)
- Only asset is a nominal bond (for simplicity)

Outline of analysis

- The problem of households and firms
- Equilibrium: monetary policy neutrality
- Monetary policy and the determination of nominal variables
Households

Representative household solves

\[ \max E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, N_t \right) \]  \hfill (1)

subject to

\[ P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \]  \hfill (2)

for \( t = 0, 1, 2, \ldots \) plus a solvency constraint (to be discussed further in class).
Optimality conditions

\[- \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \]  \hspace{1cm} (3)

\[Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \]  \hspace{1cm} (4)
Derivation and Interpretation

- Households adjust labor to the point where the real utility reward to an additional unit of $N$, which is $(W/P)U_c$, is equated to the real cost, which is $U_n$

  Utility trade-off: $U_{c,t}dC_t + U_{n,t}dN_t = 0$

  Budget trade-off: $P_t dC_t = W_t dN_t$
Households adjust current saving to the point where the utility cost of having a little bit less consumption today, \( U_{c,t} \), is equated to the expected utility from consuming a little bit more tomorrow

\[
\beta E_t \{ U_{c,t+1} \frac{P_t}{P_{t+1}} \frac{1}{Q_t} \}
\]

Starting with utility consequences, if one consumes one unit less then one loses \( U_{c,t} dC_t \) and tomorrow one gains \( \beta E_t (U_{c,t+1} dC_{t+1}) \)

The budget constraint implies \( \frac{1}{P_{t+1}} dC_{t+1} = -\frac{P_t}{Q_t} dC_t \). Note the bond is a pure discount bond, with a face value of one. Its price today is \( Q_t \) and the quantity purchased today is \( \frac{P_t}{Q_t} \).
**Convenient specification of utility:**

\[
U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}
\]

**implied optimality conditions:**

\[
\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi
\]

(5)

\[
Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}
\]

(6)

**Interpretation of parameters:**

1/\(\varphi\) : labor supply elasticity; 1/\(\sigma\) intertemporal subsitution elasticity
Log-linear versions

Exact: \( w_t - p_t = \sigma c_t + \varphi n_t \)  \hspace{1cm}(7)

Approximate: \( c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \rho) \)  \hspace{1cm}(8)

where \( \pi_{t+1} \) is the inflation rate \( \log(P_{t+1}/P_t) \) and where \( i_t \equiv -\log Q_t \) and \( \rho \equiv -\log \beta \) (add interpretation). Note \( i_t \) is the continuously compounded nominal interest rate since the price of bond at maturity is one.
Low frequency implications

Looking across countries with different inflation rates as we did last time.

Perfect foresight steady state (with growth at rate $\gamma$):

$$i = \pi + \rho + \sigma \gamma$$

which is the one-for-one "Fisher" relation between $i$ and $\pi$.

Another implication is that faster growing economies should have a higher real rate

$$r \equiv i - \pi = \rho + \sigma \gamma$$
A time series implication (Hall)

\[
c_{t+1} = c_t + \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) \\
+ [c_{t+1} - E_t \{ c_{t+1} \}]
\]

Note that the [ ] term is an expectation error. As Hall stresses, under rational expectations, this error should be uncorrelated with information used for forecasting. One of our labs will explore versions of the Hall test using EVIEWS.
Ad-hoc loglinear money demand

\[ m_t - p_t = y_t - \eta i_t \]

with \( \eta \) controlling effect of interest rate (expected inflation) on real balances.

Reminiscent of "Cagan" money demand function discussed last time.

Note: some (perhaps most) economists would criticize this approach (discuss)
Firms

Representative firm with technology

\[ Y_t = A_t N_t^{1-\alpha} \quad (9) \]

(might stand in for CRTS production function with capital fixed over duration of analysis).

Profit maximization:

\[ \max P_t Y_t - W_t N_t \]

subject to (9), taking the price and wage as given (perfect competition)

(note that there is, however, no capital in this model: labor is the only factor of production)
Optimality condition:

\[
\frac{W_t}{P_t} = (1 - \alpha)A_t N_t^{-\alpha}
\]

In log-linear terms

\[
w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha)
\]  \hspace{1cm} (10)

Sometimes described as "implicit labor demand" condition. Using this language, (7) would be viewed as labor supply condition.
Equilibrium

Goods market clearing

\[ y_t = c_t \]  (11)

Aggregate production relation:

\[ y_t = a_t + (1 - \alpha)n_t \]
Labor market clearing

\[ \sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha) \]

Asset market clearing:

\[ B_t = 0 \]

\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \]

If we added a real bonds with a face value of one good, the equilibrium condition \( B_t^r = 0 \), since all the bonds are internal to the economy, implies

\[ Q_t^r = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\} \]

(12)

where \( Q_t^r \) is the real bond price at \( t \).
Implied equilibrium values for real variables

\[ n_t = \psi_{na}a_t + \vartheta_n \]
\[ y_t = \psi_{ya}a_t + \vartheta_y \]
\[ r_t = i_t - E_t\{\pi_{t+1}\} = \rho + \sigma E_t\{\Delta y_{t+1}\} = \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \]
\[ \omega_t = w_t - p_t = y_t - n_t + \log(1 - \alpha) = \psi_{\omega a}a_t + \log(1 - \alpha) \]

where \[ \psi_{na} \equiv \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)} \quad ; \quad \vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)} \quad ; \quad \psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} \]
\[ \vartheta_y \equiv (1 - \alpha)\vartheta_n \quad ; \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} \]

\[ \Rightarrow \text{ neutrality: real variables determined independently of monetary policy} \]
\[ \Rightarrow \text{ money, inflation, and the nominal interest rate do not enter no in these equations} \]
\[ \Rightarrow \text{ specification of monetary policy needed to determine nominal variables} \]
\[ \Rightarrow \text{ optimal monetary policy: to be studied, but not yet clear.} \]
Monetary Policy and Price Level Determination

Example I: An Exogenous Path for the Nominal Interest Rate

exogenous stationary process \{i_t\} with mean \( \rho \)

\[ E_t\{\pi_{t+1}\} = i_t - r_t \]

where \{r_t\} is determined above.

Any path for inflation that satisfies

\[ \pi_{t+1} = i_t - r_t + \xi_{t+1} \]

where \( E_t\{\xi_{t+1}\} = 0 \) for all \( t \) is consistent with equilibrium: inflation can be subject to random shocks that are "not fundamental".
Since $\pi_{t+1} = p_{t+1} - p_t$, any path for the price level which satisfies

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

where $E_t\{\xi_{t+1}\} = 0$ for all $t$ is consistent with equilibrium.

In fact, even if there are no random shocks, the initial price level $p_0$ cannot be determined: only the change in the price level (inflation) is pinned down by the equilibrium conditions.

Implied path for the money supply:

$$m_t = p_t + y_t - \eta i_t$$

and hence it inherits the indeterminacy of $p_t$. 
Example II: A Simple Interest Rate Rule and the Taylor principle

\[i_t = \rho + \phi_\pi \pi_t\]

Combined with the definition of the real rate (and short-hand \(\hat{r}_t = r_t - \rho\)):

\[\phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \hat{r}_t\]

If \(\phi_\pi > 1\), unique stationary solution:

\[\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\hat{r}_{t+k}\}\]

If \(\phi_\pi < 1\), any process \(\pi_t\) satisfying \(\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_{t+1}\), where \(E_t\{\xi_{t+1}\} = 0\) for all \(t\) is consistent with a stationary equilibrium.

_price level indeterminacy_
Example III: An Exogenous Path for the Money Supply \( \{m_t\} \)

Combining money demand and Fisherian equations:

\[
p_t = \left( \frac{\eta}{1 + \eta} \right) E_t \{p_{t+1}\} + \left( \frac{1}{1 + \eta} \right) m_t + u_t
\]

where \( u_t \equiv (1 + \eta)^{-1}(\eta r_t - y_t) \) is pinned down by the real equilibrium. Assuming \( \eta > 0 \) and solving forward we obtain:

\[
p_t = \frac{1}{1 + \eta} \sum_{k=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{m_{t+k}\} + u'_t
\]

where \( u'_t \equiv \sum_{k=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{u_{t+k}\} \).
In terms of expected future money growth rates

\[ p_t = m_t + \sum_{k=1}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \left\{ \Delta m_{t+k} \right\} + u_t \]

(13)
Implied nominal interest rate:

\[ i_t = \eta^{-1}(y_t - (m_t - p_t)) = \eta^{-1} \sum_{k=1}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{ \Delta m_{t+k} \} + v_t \]

where \( v_t \equiv \eta^{-1}(u_t + y_t) \) is unaffected by policy (due to neutrality).

Nominal interest rate depends on expected future money growth because inflation depends on current and expected future money growth.
Example: Serially correlated money growth

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \]

Assume no real shocks \((y_t = 0)\).

Price response:

\[ p_t = m_t + \frac{\eta \rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t \]

\[ \implies \text{large price response if } \rho_m > 0 \text{ (Cagan)} \]

Nominal interest rate response:

\[ i_t = \frac{\rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t \]

\[ \implies \text{no liquidity effect (if } \rho_m \geq 0) \]
A Model with Money in the Utility Function

Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t}, N_t \right) \]

Budget constraint

\[ P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t \]

Letting \( A_t \equiv B_{t-1} + M_{t-1} \):

\[ P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t \leq A_t + W_t N_t + D_t \]

\( 1 - Q_t = 1 - \exp\{-i_t\} \approx i_t \); opportunity cost of holding money
Optimality Conditions

\[ -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \]

\[ Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \]

\[ \frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\} \]

where marginal utilities evaluated at \((C_t, \frac{M_t}{P_t}, N_t)\)

- utility separable in real balances \(\implies\) neutrality
- utility non-separable in real balances (e.g. \(U_{cm} > 0\)) \(\implies\) non-neutrality due to effect of expected inflation on real money and, hence, on labor supply and consumption
How important is the implied non-neutrality?

Utility specification:

$$U \left( C_t, \frac{M_t}{P_t}, N_t \right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

$$X(C_t, M_t/P_t) \equiv \left[ (1 - \vartheta) C_t^{1-\nu} + \vartheta \left( \frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \text{ for } \nu \neq 1$$

$$\equiv C_t^{1-\vartheta} \left( \frac{M_t}{P_t} \right)^{\vartheta} \text{ for } \nu = 1$$


Simulation in Walsh

Policy Rule: \( \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \)

Calibration: \( \nu = 2.56 \quad ; \quad \sigma = 2 \quad \implies \quad U_{cm} > 0 \)

Effects of exogenous monetary policy shock (Figs.)
Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

Social Planner’s problem

\[
\max U \left( C_t, \frac{M_t}{P_t}, N_t \right)
\]

subject to

\[ C_t = A_t N_t^{1-\alpha} \]

Optimality conditions:

\begin{align*}
- \frac{U_{n,t}}{U_{c,t}} & = (1 - \alpha) A_t N_t^{-\alpha} \\
U_{m,t} & = 0
\end{align*}

(14)

(15)

Optimal policy (Friedman rule): \[ i_t = 0 \text{ for all } t \]
**Intuition:** the real cost of producing money here is 0 (not in production function) and efficiency require that the cost of holding money is equal to its production cost.

**Implied average inflation:** \[ \pi = -\rho < 0 \]

**Implementation problem:** Find the values of real balances \((d_t^*)\) and the real interest rate \((r_t^*)\) which are consistent with the above equations when \(i_t = 0\) for all dates. Since \(i_t = 0\), implied equilibrium inflation:

\[ \pi_t = -r_{t-1}^* \]

Deflation is optimal.
Implementation via an interest rate rule

\[ i_t = \phi (r^*_t + \pi_t) \]

for some \( \phi > 1 \). Combined with the definition of the real rate:

\[ E_t \{i_{t+1}\} = \phi i_t \]

whose only stationary solution is \( i_t = 0 \) for all \( t \) and for any \( \phi \).
Implementation via money growth rule

\[ m_t = d_t + P_t \]
\[ m_t - m_{t-1} = d_t^* - d_{t-1}^* + \pi_t \]
\[ = d_t^* - d_{t-1}^* - r_t^* \]

Negative money growth occurs unless the real demand for money is rapidly increasing.

Under both policies, there can be issues about the determination of the initial price level, but these are eliminated if \( M_0 \) is specified or some other aspect of policy sets \( P_0 \).