Basic model of bank runs

1. Simple stylized model illustrates key ideas

• An **optimal financial contract** (bank deposit contract) is derived from underlying assumptions on preferences, technology and **information**
• The financial contract is desirable because it provides **liquidity** in a specific sense
• The financial contract is one that can be implemented by a **competitive banking system**
• The behavior of individuals under the contract is studied as a Nash **game**: there are multiple equilibria (two) under the contract.
• One of these equilibria can be interpreted as a **bank-run equilibrium**: it is worse than the optimal outcome
• **Deposit insurance** can eliminate the bank run equilibrium
2A: Model’s technology

- The economy has three periods
- An investment can be made by individuals or organizations at date 0. At date 1, the owner can decide to terminate it or continue it. The payouts are as shown below

<table>
<thead>
<tr>
<th>Date</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminate</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Continue</td>
<td>-1</td>
<td>0</td>
<td>R&gt;1</td>
</tr>
</tbody>
</table>

- DD description: The productive technology yields $R > 1$ units of output in period 2 for each unit of input in period 0. If production is interrupted in period 1, the salvage value is just the initial investment... the choice between $(0, R)$ and $(1, 0)$ is made in period 1. ...Constant returns to scale imply that a fraction can be done in each option.)
Digression on continuum economies

• When we talk about “competition” we frequently discuss how an individual is “small” relative to the economy as a whole: technically, this requires some notion of infinity. One is that agents are “named” on the line segment [0,1].

• This is convenient when there is a shock that hits individuals, but does so in a manner that is fully idiosyncratic. We can then equate:
  – The probability (t) of guy “x” being hit with a shock
  – The fraction (t) of population being hit by a shock
2B: Individuals with uncertain, idiosyncratic consumption needs

- The DD model has individuals either preferring to consume at date 1 or at date 2 randomly, as a result of the realization of a preference (consumption urgency) shock.

  \[ u(c_1) \text{ with probability } t \text{ (impatient: needs funds at date 1) } \]
  \[ \rho u(c_1 + c_2) \text{ with probability } 1-t \text{ (patient: can wait until date 2) } \]

- General idea: motivate demand for liquidity
- Specific implementation: convenient for results
Impatient individuals

• Assumed to just want early consumption \( (c_1) \)
• If an individual is impatient for sure, then he’d just invest short-term if he is operating on his own (autarchy).
• His consumption would be 1 if he is operating on his own (autarchy)
• His consumption would be \( c_1 = 1 \) (his initial wealth); his utility would be \( u(1) \)
Patient individuals

• Assumed to prefer to defer consumption if return is high enough
• These guys like more $c_1 + c_2$ so if they can get more than one unit of later consumption in exchange for earlier consumption they’ll do so.
• If an individual is patient for sure, then he’d just invest long-term since $R>1$ if he is operating on his own (autarchy)
• His consumption would be $c_2 = R \times 1$ (return times his initial wealth); his utility would be $u(R)$
Uncertainty about types

• The type of individuals is realized at date 1 in the above time line. It is a risk to the individual.

\[ c_d^\tau : \text{consumption of type } \tau (1=\text{impatient}, 2=\text{patient}) \text{ at date } d (1,2) \]

\[ c_1^1 : \text{example: consumption of impatient type } (\tau=1) \text{ at date } 1 (d=1) \]

• Use expected utility as criterion
Expected utility

• Probability-weighted expected utility

\[ tu(c^1_1) + (1 - t) \rho u(c^2_1 + c^2_2) \]

• Note that impatient guys (type 1) don’t care about second period consumption, so that this does not enter in expected utility
3. Optimal allocation with types uncertain at date 0 but observable at date 1

- Pool all guys in the economy
- Maximize utility subject to economy-wide resource constraint

Objective: \[ tu(c_1^1) + (1-t)\rho u(c_1^2 + c_2^2) \]

Constraint: \[ tc_1^1 + (1-t)c_1^2 + (1-t)\frac{1}{R}c_2^2 = 1 \]

- First period consumption for patient types is inefficient if \( R > 1 \).
Moving consumption across types: along the resource constraint

\[ t(dc_1^1) + (1-t) \frac{1}{R} (dc_2^2) = 0 \]

\[ dc_2^2 = -\frac{t}{1-t} Rdc_1^1 \]
Effect of change on expected utility; Nature of optimal allocation with full information

\[ dEU = tu'(c_1^1)dc_1^1 + (1-t)\rho u'(c_2^2)dc_2^2 \]

\[ = tu'(c_1^1)dc_1^1 + (1-t)\rho u'(c_2^2)[1 - \frac{t}{(1-t)}]Rdc_1^1 \]

\[ = t[u'(c_1^1) + \rho u'(c_2^2)(-R)]dc_1^1 \]

\[ = 0 \text{ (at optimum)} \]

\[ u'(c_1^1) = \rho Ru'(c_2^2) \]

\[ c_2^2 > c_1^1 \text{ if } \rho R > 1 \text{ as in Diamond-Dybvig} \]
Insurance in the optimal contract

Suppose that \( u(c) = \frac{1}{1-\sigma} c^{1-\sigma} \) then \( u'(c) = c^{-\sigma} \)

\( \sigma = \) coefficient of relative risk aversion

\[ \text{EFF:} \quad (c_1^1)^{-\sigma} = R \rho (c_2^2)^{-\sigma} \implies c_2^2 = c_1^1 (R \rho)^{\frac{1}{\sigma}} = \gamma c_1^1 \]

\[ \text{RC:} \quad tc_1^1 + (1-t)c_2^2 \frac{1}{R} = 1 \implies c_1^1 = \frac{1}{t + (1-t) \frac{\gamma}{R}} \]

\( c_1^1 > 1 \iff \gamma < R \quad \text{[not too much substitution]} \)

\( c_2^2 > c_1^1 \iff R \rho > 1 \quad \text{[sufficiently productive long-term investment]} \)
4. optimal contract with private information

• Suppose that it is not possible to observe type.
• Then, contracts may be constrained by the requirement that individuals must voluntarily report type.
• Who might have this incentive here?
• Impatient guys would not: by reporting themselves as patient, they would receive second period consumption which they do not value
• Patient guys might....
Incentive compatibility

• A truthful report of type is incentive compatible if the individual is better off when he reports correctly than not.

• For a patient type, this requires that $u(\text{consumption of patient}) > u(\text{consumption of impatient})$.

• Under the $R\rho > 1$ condition, this is satisfied (growth in consumption).

• So the full information allocation is also one that can be implemented under private information (this is most commonly not the case).
5. Implementing optimum as a bank contract

• Everyone deposits 1 unit endowment w/ bank
• Period 1 (early) withdrawers get \( r_1 > 1 \).
• Period 2 (late) withdrawers get \( r_1 r_2 < R \)
• Bank actions
  
  its investment must yield \( tr_1 \) for bank at date 1,
  so terminate fraction \( tr_1 \) at date 1
  its investment must yield \((1-t)r_1 r_2\) for bank at date 2
  so complete fraction \((1-t)r_1\) at date 2 getting \((1-tr_1)R\)
Implementing optimum as a bank contract cont’d

• Link between optimal quantities and returns

\[ r_1 = c_1^1 \]
\[ r_2 r_1 = c_2^2 \]

• Can think of individual banks offering interest rate packages to individuals: people will flow to the one with the highest expected utility
6. Bank runs

- We have seen that one equilibrium outcome of the deposit contract is equivalent to the socially optimal allocation
- Can there be other equilibria?
- Diamond-Dybvig is an example of modeling financial outcomes as Nash games
Scenario

• “In our model, the demand deposit contract gives each agent withdrawing in period 1 a fixed claim of \( r_1 \) per unit deposited in period 0. Withdrawal tenders are served sequentially in random order until the bank runs out of assets. This approach allows us to capture the flavor of continuous time (in which depositors deposit and withdraw at different random times) in a discrete model. Note that the demand deposit contract satisfies a sequential service constraint, which specifies that a bank’s payoff to any agent can depend only on the agent’s place in line and not on future information about agents later in line.”
Consumption of a depositor at place j in line, given that a fraction place $f_j$ of prior depositors has withdrawn and that a fraction $f$ of depositors will withdraw

• Consumption from withdrawing

$$r_1 \text{ if } r_1 f_j < 1 \quad [\text{funds are still available}]$$

$$0 \quad \text{if } r_1 f_j > 1 \quad [\text{bank has gone bust}]$$

• Consumption from not withdrawing

$$\max(0, R(1 - r_1 f) / (1 - f))$$

• Note from DD: the bank is mutually owned and liquidated in period 2, so that agents not withdrawing in period 1 get a pro rata share of the bank’s assets in period 2.
A bank run equilibrium

• A bank run equilibrium has all agents panicking and trying to withdraw their deposits at date 1: if this is anticipated, all agents will prefer to withdraw at date 1. This is because the face value of deposits is larger than the liquidation value of the bank’s assets.

• In terms of the prior discussion, for $f$ sufficiently larger than $t$ (including $f=1$), the consumption from withdrawing exceeds the consumption from not withdrawing. Hence, all patient agents will withdraw unless the bank has gone bust by the time that it reaches their place in line.
Welfare ranking

• Banking without runs is better than autarchy
• Autarchy is
• Autarchy is better than a bank run: one gets 1 for sure in autarchy, but only in expected value in a bank run (risk aversion)
7. deposit insurance

• Can prevent bank runs completely
  – Government says “each will get his money, at his option, either now or later”
  – Patient depositors no longer believe that their return can be less than $r_1$, so they never have an incentive to run

• Hence, deposit insurance improves welfare by eliminating a bad equilibrium.

• In the DD model, it will never be called into action so long as it is on the books.