Macroeconomics Qualifying Examination
Boston University
August 17, 2012

Be careful, the weighting of the questions is different from past qualifying examination in macroeconomics. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 30 points each. Part B consists of two multipart questions: each multipart question is worth 50 points. The exam is designed to take about three hours to complete and the total points on the exam are 220 (that will be scaled down to 100).

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Subtotal 120

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Subtotal 100
Short Questions (Part A): 4 questions

1. Consider a complete markets exchange economy with the probability of history $s^t$ given by $\pi_i(s^t)$ and agent $i$ receiving endowment $y_i^t(s^t)$ after history $s^t$. Suppose $s^0$ is known so $\pi(s^0) = 1$. The agents maximize

$$U^i = \sum_t \sum_{s^t} \beta^t u(c_i^t(s^t)) \pi_i(s^t)$$

where $u(c) = \frac{1}{\gamma_i} \exp \{-\gamma_i c\}$. A full set of Arrow-Debreu securities are traded at date 0. Let $\bar{y}(s^t) = \sum_i y_i^t(s^t)$ be the aggregate endowment after history $s^t$.

(a) State the optimization problem of agent $i$. 

(b) Show that $c^i(s^t) = a^i\bar{y}(s^t) + b^i$ for all $t$ and all $s^t$, where $a^i$ and $b^i$ are constants specific to individual $i$, but constant across $t$ and $s^t$. 
2. Consider an overlapping generations economy with one good where each generation is represented by a single price-taking individual who lives two periods and has the life-time utility

\[ U(c, \ell) = \log(c) + \log(1 - \ell) , \]

where \( c \) and \( \ell \) are the consumption and the labor in the second period. Each individual is endowed with one fixed unit of labor in the first period.

The production technology is represented by the function \( Y = K^\alpha L^{1-\alpha} \). Capital depreciates completely from one period to the next. Assume \( \alpha = 1/2 \). Production factors are paid their marginal productivity.

(a) Write the conditions that determine in the equilibrium, in a steady state.
(b) Show that there is a unique steady state. (You may use a diagram). Show that in the steady state, $\ell < 1/2$. You are not asked to solve for the steady state.
(c) A government introduces a tax on labor for the second period in the life-time. (That is roughly equivalent to an increase of the earning test for the payment of retirement benefits). The proceeds of the tax are consumed by the government. Analyze qualitatively the impact of this policy on the level of labor. (You may use the graph that you may have presented in the previous question).
3. There is an economy with a representative infinitely lived family who has a utility function

\[ U = \int_0^\infty e^{-\rho t} e^{nt} u(c_t, \ell_t, K_t) dt, \]  
with \( u(c, \ell, K) = \log(c) + \log(1 - \ell) + \gamma \log(K), \)

and where \( c_t \) and \( \ell_t \) are the levels of consumption and labor per capita, and \( \rho \) is the time preference rate. Note that individuals have some utility from wealth. We assume \( \gamma = 1 \). The production technology is represented by the function

\[ Y = K_t^\alpha L_t^\beta, \]  
with \( \alpha + \beta = 1 \),

and where \( K_t \) and \( L_t \) are the levels of total capital and labor. Capital does not depreciate.

(a) Determine the steady state of this economy. Determine explicitly \( \ell \) and the rate of return on capital as a function of \( \alpha \) and \( \rho \). Compare \( r \) and \( \rho \) and comment.
(b) A social planner thinks that this private taste for capital accumulation is ridiculous and that consumers should behave as if $\gamma = 0$. He calls you, his economic advisor. What do you suggest? You will limit your advice to the steady state.
4. An unemployed worker maximizes $E \sum_{t=0}^{\infty} \beta^t y_t$, where $y_t$ equals the wage if the worker is employed, and zero if she is unemployed. Each period the worker is unemployed she draws a first period wage $w_1 = \theta + \epsilon$, where $\theta$ and $\epsilon$ are independent random variables with $\theta \sim N(\mu, \sigma_\theta^2)$, $\epsilon \sim N(0, \sigma_\epsilon^2)$. The worker then chooses whether to accept this job (and work at wage $w_1$ that period), or reject it (and enter the following period unemployed). After the first period on the job, the worker’s wage will be $w_\tau = \theta$ for $\tau = 2, 3, \ldots$ where $\tau$ denotes tenure and $\theta$ was drawn the first period of the job. Thus, $\epsilon$ is “noise” in the first period wage. Based on observing $\theta$, the worker then chooses whether to continue in the job, or enter the following period unemployed. Successive draws of $\theta$ and $\epsilon$ are independent, but to draw a new $\theta$, the worker must be unemployed and also draw a new $\epsilon$.

Hint: You may assume that the worker cannot quit after $\theta$ has been revealed and she has once accepted to keep this job with wage $\theta$. (She would not choose to do so anyway.) You should allow the worker to quit when $\theta$ is first revealed, however.

(a) Denote the conditional distribution of $\theta|w_1$ as $G(\theta|w_1)$, and the unconditional distribution of $w_1$ as $F(w_1)$. Write down the worker’s Bellman equations.
(b) Describe the worker’s optimal strategy.
(c) Find the distribution of $\theta$ conditional on $\theta + \epsilon$. 
Long Questions (Part B): 2 questions

1. Consider an economy described by the following log-linear dynamic IS and AS curves:

\[
\begin{align*}
IS & : \ x_t = -(i_t - E_t\pi_{t+1}) + E_t\{x_{t+1}\} + g_t \\
AS & : \ \pi_t = \lambda x_t + E_t\{\pi_{t+1}\} + u_t
\end{align*}
\]

where \(x_t\) denotes the output gap, \(i_t\) is the nominal interest rate, and \(\pi_t\) is the inflation rate. Further assume that \(u_t\) and \(g_t\) satisfy:

\[
\begin{align*}
u_t &= \rho_u u_{t-1} + \varepsilon^u_t \\
g_t &= \rho_g g_{t-1} + \varepsilon^g_t
\end{align*}
\]

with \(\varepsilon^u_t, \varepsilon^g_t\) iid and \(0 \leq |\rho_u| \leq 1, 0 \leq |\rho_g| \leq 1\) Suppose the monetary authority chooses a path for nominal interest rates to minimize:

\[
\frac{1}{2}E_t\sum_{i=0}^{\infty} \beta^i \left( \alpha (x_{t+i})^2 + \pi^2_{t+i} \right)
\]

(a) Assume that the monetary authority cannot credibly commit to a policy. Derive the first order conditions for optimal monetary policy. Explain what these conditions mean.
(b) Given your answer in part a) derive expressions for the volatility of inflation and the output gap as functions of the volatility of the underlying shocks $u_t$ and $g_t$. How does inflation and output-gap volatility vary with $\alpha$? Under what conditions is it optimal to conduct strict inflation targeting (i.e. $\pi_t = 0$ so that there is zero inflation volatility)?
(c) Now consider the situation where the monetary authority can commit to a policy rule. Assume that the monetary authority chooses a rule of the form

\[ x_t = -\omega u_t \]

What is the optimal choice of \( \omega \)? Are there circumstances in which inflation and output gap volatility is the same under discretion and commitment to this form of a rule? Explain.
(d) Show that solution under commitment to a rule of the form $x_t = -\omega u_t$ is equivalent to discretionary monetary policy that minimizes the loss function

$$\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left( \alpha_c (x_{t+i})^2 + \pi_{t+i}^2 \right)$$

where $\alpha_c < \alpha$. Are there circumstances in which inflation and output gap volatility is the same under discretion and commitment to this form of a rule? Explain.
(e) Now consider an economy with partial price indexation. Let $\gamma$ denote the degree of indexation. The AS curve is now:

$$\pi_t - \gamma \pi_{t-1} = \lambda x_t + E_t\{\pi_{t+1} - \gamma \pi_t\} + u_t$$

and the appropriate loss function is

$$-\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left( \alpha (x_{t+i})^2 + (\pi_{t+i} - \gamma \pi_{t+i-1})^2 \right)$$

where $0 \leq \gamma \leq 1$. Again derive the optimal policy under discretion.
(f) Does indexation change the dynamics of inflation and the output gap in response to shocks to $u_t$? Does it change the optimal choice of $\omega$ derived in part c? Explain.
2. Consider an endowment economy with $2N$ agents with preferences $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u$ is strictly increasing, strictly concave, and satisfies Inada conditions. Half of the agents (call them “even”) have an endowment stream $\{y_{e,t}\}_{t=0}^{\infty} = \{1, 0, 1, 0 \ldots \}$, and the rest (call them “odd”) have an endowment stream $\{y_{o,t}\}_{t=0}^{\infty} = \{0, 1, 0, 1 \ldots \}$. Endowments are perishable, there is no storage technology.

(a) Write down a planner’s problem for this economy, and characterize Pareto-optimal allocations. Use the same Pareto weight $\theta^e$ for all even agents, and $\theta^o$ for all odd agents.
(b) Suppose we endow each odd agent with $M$ units of fiat money at the beginning of period 0, and only allow trades of goods for money. Write down the maximization problem the agents face in this environment. Considering equilibria where money circulates, with positive value, and the price level is constant, show that we cannot attain an efficient allocation in this environment.
(c) Show that in the environment of part (b), there exists an equilibrium where money circulates, with positive value, and the price level is constant. Characterize the allocation and price level in such an equilibrium.