Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 20 points each. Part B consists of two multipart questions: each multipart question is worth 50 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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Short Questions (Part A): 4 questions

1. Consider an economy populated by two agents $i = A, B$, with exogenous endowments $\{y_t(s^t)\}_{t=0}^{\infty}$, where $s^t$ is the history of stochastic events up to and including date $t$. For simplicity, assume the aggregate endowment is constant and normalized to one, implying that $y^A_t(s^t) = 1 - y^B_t(s^t) \forall t, s^t$. Also, assume that any point in time, there are only two possible states of nature. In particular, assume the state of nature at date $t$ is drawn i.i.d. as

$$y^A_t(s^t) = \begin{cases} 
\bar{y}^L & \text{with prob. } \bar{\pi} \\
\bar{y}^H & \text{with prob. } 1 - \bar{\pi},
\end{cases}$$

where $0 < \bar{y}^L < \bar{y}^H < 1$. There is no storage or production technology in this economy. Finally, the two agents have preferences for allocations given by the following utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \log(c^t_i(s^t)).$$

(a) Write down the agent’s problem assuming that, in the first period ($t = 0$), given $y^A_0$, securities for any maturity and contingent on any state of nature can be traded in the market. After $t = 0$, previous contracts are implemented but no further trading can occur. Derive the agent’s optimality conditions.
(b) Provide a definition of competitive equilibrium in this economy.
(c) Solve for the equilibrium consumption allocation.
2. Consider a household that chooses consumption, $C_t$, real money balances $M_t/P_t$, bond holdings $B_t/P_t$, and labor supply $N_t$, to maximize

$$\sum_{t=0}^{\infty} \beta^t \left( \ln \left( C_t + v \left( \frac{M_t}{P_t} \right) \right) - \theta N_t \right)$$

subject to

$$\frac{M_t}{P_t} + \frac{1}{1 + i_t} \frac{B_t}{P_t} + C_t = \frac{W_t}{P_t} N_t + \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + T_t$$

where $P_t$ is the price level, $W_t$ is the nominal wage, $i_t$ is the nominal interest rate so that $1/(1+i_t)$ is the discount price of a bond that delivers one unit of currency in $t+1$ and $T_t$ denotes monetary transfers. Assume that $v(x) > 0$ is an increasing, concave function with $v'(x^*) = 0$ for some $x^* < \infty$. There is no capital and output in this economy satisfies: $Y_t = N_t$. Money grows at a constant rate: $(1 + \mu) = \frac{M_{t+1}}{M_t}$.

(a) Write the Bellman equation for this problem. Derive the household’s optimality conditions.
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(b) Determine the equilibrium allocation as a function of the money growth rate $\mu$. What is the effect of an increase in the money growth rate on labor supply and real balances? Explain.
(c) Now consider a social planner that chooses consumption, $C_t$, money balances $M_t/P_t$, and labor supply $N_t$, to maximize

$$\sum_{t=0}^{\infty} \beta^t \left( \ln \left( C_t + \nu \left( \frac{M_t}{P_t} \right) \right) - \theta N_t \right)$$

subject to $C_t = N_t$. Show that the solution to the social planning problem is implemented by the decentralized equilibrium allocation as long as the monetary authority sets the money growth rate to satisfy the Friedman rule.
3. Consider an overlapping generations economy with one good where each generation is represented by a single price taking individual with the life-time utility

\[ U(c_1, c_2) = \text{Min}(c_1, ac_2). \]

Each generation is endowed with one unit of labor in its first period of life. The production technology is represented by the function \( Y = K^\alpha L^{1-\alpha} \). Capital depreciates completely from one period to the next. Assume \( \alpha = 1/2 \). Production factors are paid their marginal productivity.

(a) Show that under some condition on the parameter \( a \)–call it condition (A)–there is a steady state with strictly positive capital. Determine the steady state level of capital when condition (A) holds. If the population is growing, what is the impact on the steady state level of the capital stock? (Provide only an intuitive answer. You do not need to do the algebra).
(b) Population is constant. Under condition (A), analyze the stability property of the economy depending on the initial level of capital $K_1$ at the beginning of the first period. You may use a graphical representation of $K_t$ as a function of $K_{t+1}$. (There is no mistake in the order of time subscripts). Use your analysis (and your graph) to describe the dynamics of the economy when condition (A) is not satisfied. Provide an intuitive description on the mechanisms that relies to the microeconomic incentives in the model. (No graphical interpretation).
4. There is an economy with a representative infinitely lived family who has a utility function

\[ U = \int_0^\infty e^{-\rho t}e^{n t}u(c_t, \ell_t)dt, \quad \text{with} \quad u(c, \ell) = \log(c) + \log(1 - \ell), \]

and where \( c_t \) and \( \ell_t \) are the levels of consumption and labor per capita, \( \rho \) is the time preference rate and \( n \) is the constant population growth rate, \( n > 0 \). The production technology is represented by the function

\[ Y = K_t^\alpha L_t^{1-\alpha}, \]

where \( K_t \) and \( L_t \) are the levels of total capital and labor.

(a) Determine the steady state of this economy.
(b) For this question, \( n = 0 \). Assume that there is a permanent linear tax at the rate \( \theta \) on the income of labor. The proceeds of the tax are consumed by the government and that consumption does not enter the utility of private individuals. What is the impact of the tax on the gross factor prices, on consumption and on the labor supply, in the long-run (steady state). Provide an interpretation of the impact on the labor supply.
(c) The utility $u$ is the Log utility as in the first expression. A planner prefers the allocation that maximizes the discounted utilities per capita and has the function

$$U = \int_0^\infty e^{-\mu} u(c_t, \ell_t) dt,$$

with $u(c, \ell) = \log(c) + \log(1 - \ell)$.

Which tax policy you would recommend to this planner? Available instruments are linear taxes on factor prices, consumption and lump-sum transfers. The government does not consume goods. You will analyze the policy only in the long-run limit, in the steady state.
Long Questions (Part B): 2 questions

1. Consider an economy populated by a unit continuum of identical households with preferences for consumption and leisure given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t),$$

where $n_t$ is the fraction of time devoted to labor. In this economy, firms use capital and labor to produce output according to a Cobb-Douglas production function

$$Y_t = K_t^{1-\alpha} N_t^\alpha,$$

where $K_t$ and $N_t$ are the aggregate inputs of capital and labor respectively. The total capital input, $K_t$, is the product of the amount of capital used $k_t$ and its intensity of utilization $z_t$. Using the capital stock more intensely leads it to depreciate faster. Therefore, capital accumulates according to

$$k_{t+1} = I_t + [1 - \delta(z_t)] k_t,$$

where $\delta(\cdot)$ is an increasing, convex function. Finally, the aggregate resource constraint is

$$Y_t = C_t + I_t.$$

(a) State the social planner’s problem.
(b) From the social planner’s problem, derive a condition that pins down the optimal rate of capital utilization, $z_t$. 
(c) Give an economic interpretation to the optimality condition for $z_t$. 

(d) You will now show that the planner’s choice of capital utilization rate can be decentralized. To do so, you should assume that the households own the capital stock and choose how intensely to rent it out at a given rental rate $r_t$. You should also assume that the households rent their labor to the firm at a wage $w_t$. State the household’s maximization problem.
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(e) State the firm’s maximization problem.
(f) Define a competitive equilibrium of this economy.
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(g) Show that the same condition pins down $z_i$ in the decentralized equilibrium as in the planner's problem.
2. **This problem has two parts, I and II**

We examine the effects of mandatory severance payments – transfers from firms to workers upon termination of a match – in the Mortensen-Pissarides search and matching model.

Time is continuous. There is a continuum of measure one homogenous risk neutral infinitely lived workers with discount rate $r > 0$. When unemployed, a worker’s flow income is normalized to zero.

There is a large number of risk-neutral infinitely-lived firms that open vacancies at flow cost $c > 0$ as long as it is profitable to do so. Each filled job produces $p$ units of output, but ends according to a Poisson process with arrival rate $\lambda$. When the job ends, the firm pays a one-time severance payment $f$ to the worker.

Unemployed workers and vacancies match according to a constant returns to scale matching function $m(u,v)$. Denote the job-finding rate of workers and the hiring rate of firms by $\mu(\theta)$ and $q(\theta)$, respectively, where $\theta$ is the vacancy-unemployment ratio. Upon matching, the worker and firm bargain a wage $w$, which gives share $\beta$ of the match surplus to the worker, and share $1 - \beta$ to the firm.

(a) Write Bellman equations for the value of unemployment $U$, the value of employment $W$, the value of a vacant job $V$ and the value of a filled job $J$. 
(b) Define the match surplus as $S = W + J - U - V$, and use it to derive an implicit equation for the equilibrium vacancy-unemployment ratio $\theta^*$. How does $\theta^*$ depend on the severance payment $f$?
Part II: Consider a one-period version of the Mortensen-Pissarides model with capital. All agents, workers and firms, are risk neutral. Workers have a value of leisure \( z \).

Each firm can only hire one worker, and the output of an employed worker is given by \( f(k) \) where \( k \) is capital and \( f' > 0, f'' < 0 \). To open a vacancy the firm must pay the cost of capital \( r_k \). Firms contact workers with probability \( q(\theta) \) and workers contact firms with probability \( \mu(\theta) \). Wages are determined according to Nash-bargaining, where the workers' bargaining power is \( \gamma \). Assume there is no free entry, and the value of \( \theta \) is exogenous.

(A) Assume capital can be sold freely in a perfect market if the firm fails to hire a worker.

(i) Write the value of an employed worker, and the value of a filled job, after matching has taken place. Write the value of an unemployed worker, and the value of a vacant job at the beginning of the period, before matching has taken place. (Note that the wage may depend on \( k \).)
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(ii) Write the Nash product and find the wage.
(iii) Find the equilibrium level of capital per worker, $k_1$. 
(B) Assume that capital cannot be resold freely if the firm fails to hire a worker (the cost of capital $rk$ is sunk).

(i) Write the value of an employed worker, and the value of a filled job, after matching has taken place. Write the value of an unemployed worker, and the value of a vacant job at the beginning of the period, before matching has taken place. (Note that the wage may depend on $k$.)
(ii) Write the Nash product and find the wage.
(iii) Find the equilibrium level of capital per worker, $k_2$. 
(iv) Rank $k_1$ and $k_2$, and explain the difference in capital levels.