Microeconomic Theory Qualifying Exam
June 2013

Instructions. You have a maximum of 4 hours and 15 minutes to complete this exam (suggested time allocation: 15 mins to review the questions and up to 4 hrs to answer them).

Answer all four questions. The questions are equally weighted.

Write on one side of the provided paper only. Start the answer to each question on a new sheet of paper and be sure to write your candidate number, question number, and page number on each sheet.

Be concise in your answers, and think before you write. Good luck!
Question 1

Let $X$ be a finite set and $\mathcal{B}$ the set of all nonempty subsets of $X$. Suppose we have a choice function $c$ defined on this space. That is, $c : \mathcal{B} \to X$ and satisfies $c(B) \in B$ for all $B \in \mathcal{B}$.

Suppose that our model of the agent is that he only “notices” some of his options. Specifically, given a feasible set $B$, the agent only recognizes the options in $\Gamma(B)$, a nonempty subset of $B$. The agent has a complete, transitive preference $\succeq$ over $X$ with no indifference. I.e., if $x \neq y$, then either $x \succ y$ or $y \succ x$. Given feasible set $B$, the agent chooses the best element of $\Gamma(B)$ according to $\succeq$.

(a) Show that any choice function can be rationalized by an appropriate specification of $\Gamma$, and that choices reveal nothing about the underlying preference.

(b) In light of (a), let’s add an assumption on $\Gamma$. Assume now that $\Gamma$ has the property that if $x \in B \setminus \Gamma(B)$, then $\Gamma(B) = \Gamma(B \setminus \{x\})$. That is, if the agent doesn’t notice $x$ when $B$ is the feasible set, then if we remove $x$ from $B$, what the agent notices doesn’t change.

Can the following choices be generated by this model?

$$
c(\{x, y\}) = x, \quad c(\{x, z\}) = x, \quad c(\{y, z\}) = y, \quad c(\{x, y, z\}) = z.
$$

(Provide a proof in either case.) Are these choices consistent with preference maximization by an agent who always notices all feasible alternatives? Why or why not?

(c) Let $X = \{w, x, y, z\}$ and consider any choice function satisfying $c(\{x, y, z\}) = x$, $c(\{w, x, y\}) = y$, $c(\{x, z\}) = z$, and $c(\{w, y\}) = w$. Show that there is no $\Gamma$ satisfying the property stated in (b) and no preference $\succeq$ satisfying the assumptions above, such that the above procedure generates the indicated choices.
Question 2

Consider the following complete information bilateral bargaining game. Two players are bargaining over how to divide a perfectly divisible surplus. At \( t = 0 \), the surplus has size equal to 1. At \( t = 1 \), the size of the surplus is equal to \( X > 1 \) with probability \( p \in (0,1) \), and it is equal to 1 with probability \( 1 - p \). From time \( t = 2 \) onwards, the size of the surplus remains constant and equal to its size at \( t = 1 \).

Assume that both players are risk-neutral expected utility maximizers, and have a common discount factor \( \delta < 1 \). Therefore, if a player receives a (possibly random) quantity \( y \) of the surplus at time \( t \), her payoff is \( \delta^t \cdot E[y] \).

The game proceeds as follows. At \( t = 0 \), each player is selected to be proposer with probability 1/2. The proposer makes an offer over how to divide the current surplus; i.e., she makes an offer \( y \in \mathbb{R}_+^2 \) with \( y_1 + y_2 = 1 \). The other player, the responder, chooses whether to accept or reject the offer. If the responder accepts the offer, each player consumes her share of the surplus and the game ends. Otherwise, between \( t = 0 \) and \( t = 1 \) both players learn the new size of the surplus: with probability \( p \) the size of the surplus is \( X > 1 \), and with probability \( 1 - p \) the size of the surplus is 1. Then, at each time \( t = 1, 2, ... \) each player is selected to be proposer with probability 1/2. The proposer makes an offer over how to divide the current surplus; i.e., she makes an offer \( y \in \mathbb{R}_+^2 \) with \( y_1 + y_2 = z \), where \( z \) denotes the size of the surplus (either \( X \) or 1). The other player, the responder, chooses whether to accept or reject the offer. If the responder accepts the offer, each player consumes her share of the surplus and the game ends. Otherwise, the game moves on to \( t + 1 \). The game continues until an offer is accepted. Note that the size of the surplus remains constant after \( t = 1 \).

1. Argue that in any subgame perfect equilibrium (SPE) players always reach an agreement at time \( t = 1 \) if they had not reached an agreement at \( t = 0 \). Compute the players’ SPE payoffs at \( t = 1 \) depending on whether the size of the surplus is \( X \) or 1.

2. Using your answer to part (1), compute the set of offers that a player at \( t = 0 \) will be willing to accept if she is the responder?

3. Show that there exists \( \tilde{\delta} < 1 \) such that, if \( \delta > \tilde{\delta} \), the proposer will never find it optimal to make an acceptable offer to the responder at \( t = 0 \). Therefore, if \( \delta > \tilde{\delta} \), there is delay at \( t = 0 \). How does \( \tilde{\delta} \) depend on the values of \( X \) and \( p \)?
Consider the following multitask Principal-Agent model. Both players are risk neutral. The agent has limited liability (can never receive negative income) and a zero outside option.

There are two types of effort $A$ and $B$. Let $a \in [0, 1]$ be the level of $A$ effort, and $b \in [0, 1]$ be the level of $B$ effort. Output is verifiable, and equal to $R$ with probability $b$ and equal to 0 with probability $1 - b$.

To provide incentives, the principal chooses a compensation scheme that pays $w$ if output is $R$ and 0 otherwise. Thus his payoff is $b(R - w)$, the expected value of profit.

The agent’s payoff is $bw + \alpha a - \frac{1}{2}(a + b)^2$, where $R > \alpha$. The parameter $\alpha \geq 0$ measures the size of the benefit the agent derives from $A$ effort.

a) Derive the first-best levels of $a$ and $b$, that is, those that should be chosen if $a$ and $b$ are contractible.

b) For the rest of the problem assume that $a$ and $b$ are not contractible. Suppose that $\alpha$ is publicly known to be equal to zero. Given $w$, what levels of $a$ and $b$ will the agent choose? What then is the principal’s optimal choice $w_G$?

c) Now suppose that the value of $\alpha$ is publicly known and is strictly positive (but still less than $R$). Given $w \geq \alpha$, what levels of $a$ and $b$ will the agent choose? What if $w < \alpha$? Solve for the optimal contract (call it $w_B$). How does the optimal contract depend on the size of $\alpha$?

d) Suppose for the rest of the problem that $\alpha$ equals 0 with probability $\mu$, and that with probability $1 - \mu$, $\alpha$ equals $\alpha_+$, where $\frac{3}{4}R \leq \alpha_+ < R$. The principal offers either the contract $w_G$ derived in part (b) or the contract $w_B$ derived in part (c). Show that there is a cutoff probability $\mu^*$ such that $w_G$ is offered if $\mu > \mu^*$, while $w_B$ is offered if $\mu < \mu^*$.

e) Assume now that the agent privately learns $\alpha$ before signing a contract with the principal. Show that the principal cannot gain relative to his decision rule in part (d) by screening, i.e. by offering a menu of contracts $(w_0, w_+)$, where $w_0$ is for the 0-type and $w_+$ is for the $\alpha_+$-type. Give some intuition for the result.
Question 4

For each part below, indicate whether the statement is True or False and justify your answer completely, either by a proof or by a counterexample. If the assertion is True only under additional unstated assumptions, state them. Where there are two assertions (as in (b) and (d)), one could be True and the other False. Therefore, address each part separately.

(a) In an exchange economy with two risk averse consumers, Alice and Bob, if Alice is strictly more risk averse than Bob, then in any interior efficient allocation where Bob is perfectly insured, Alice must also have complete insurance.

(b) (i) Two individuals with the same convex and strongly monotone preferences and the same initial endowments must receive indifferent consumption bundles in any interior Walrasian equilibrium. (ii) Further, if preference is strictly convex, then the equilibrium bundles for the two individuals must be identical.

(c) Your true love loves flowers. Unfortunately, you can’t remember whether the right thing to do is to bring an odd or an even number of flowers — you believe there is 50% chance each of them is the right thing to do. If you do the right thing, you’ll get a kiss. If you do the wrong thing, you’ll get a kick. You are an expected utility maximizer, so you decide to choose an odd number with probability \( p \) and an even number with probability \( 1 - p \).

Your unique optimal strategy is to set \( p = \frac{1}{2} \).

(d) Consider a security market with four securities and four states. Their dividends are given by the matrix \( D \),

\[
D = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
1 & 2 & 1 & 1 \\
0 & 0 & 0 & 1 
\end{bmatrix},
\]

where the entry in row \( s \) and column \( k \) gives the dividend paid by security \( k \) in state \( s \). Security prices are \((q_1, q_2, q_3, q_4) = (4, 9, 5, 3 + \delta)\).

(i) Then the security market is arbitrage-free for every positive \( \delta \).

(ii) Let \( \delta = 2 \) above. A new security is added that pays a dividend equal to 1 in every state. Then there are many prices for this security that are consistent with no arbitrage. (If you agree, describe all such prices. If you disagree, prove that there is at most one such price.)