Student ID:

Macroeconomics Qualifying Examination
Boston University
September 2010

Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 15 points each. Part B consists of two multipart questions: each multipart question is worth 60 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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1. Consider the following economy populated by a large number of identical agents. Each individual divides his or her one unit of time between working($n$), leisure($\ell$) and school($s$). That is
\[ n_t + s_t + \ell_t = 1. \]

The individual’s budget constraint is
\[ c_t + k_{t+1} = wh_t n_t + (1 + r - \delta_h) k_t \]

and human capital accumulates according to
\[ h_{t+1} = (1 - \delta_h) h_t + s + t. \]

Households have preferences for consumption given by
\[ \sum_t \beta^t [\log(c_t) + \alpha \log(\ell_t)]. \]

Assume that wages, $w$, and interest rates, $r$, are constant across time.

(a) State the recursive formulation of the household’s problem.
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(b) Derive the first order necessary conditions (FONCs) for the household's problem.

(c) Express the FONCs in a way that eliminates the value function, its derivatives and any Lagrange multipliers (as is the case with the Euler equation for a standard consumption-saving problem).
2. Consider the Mortensen-Pissarides search and matching model.

(a) Suppose unemployed workers and vacant jobs match according to a Cobb-Douglas matching function \( m(u, v) = u^\alpha v^{1-\alpha} \). Write down expressions for the job-finding rate of workers \( \mu(\theta) \) and hiring rate of firms \( q(\theta) \), where \( \theta \) is the vacancy-unemployment ratio.

(b) Suppose all filled jobs produce \( p \) units of output, and end according to a Poisson process with arrival rate \( \lambda \). Write Bellman equations for the value of unemployment \( U \), the value of employment \( W \), the value of a vacancy \( V \) and the value of a filled job \( J \), in continuous time. Denote unemployment benefits by \( b \), the wage by \( w \), the vacancy-cost by \( c \), and the discount rate by \( r \).
(c) Suppose there is free entry into vacancy-creation by firms, and that wage-bargaining results in half of the match surplus going to the worker and half to the firm. Defining the match surplus as \( S = W + J - U - V \), this means that \( W - U = J - V = \frac{1}{2} S \). Derive an implicit equation determining the equilibrium \( \theta \).
3. There is an economy with an infinitely lived representative household who takes prices as given. The household has the utility function

\[ U = \int_0 e^{-\rho t} \log(C_t) \, dt, \tag{1} \]

where \( C_t \) represents consumption. Production is determined by the production function \( Y = K^{\beta} L^{1-\beta} \), where \( K \) is capital and \( L \) is effective labor that is supplied (that may vary because of technological change). There is no capital depreciation in this problem.

(a) Assume that because of technological change, the quantity of effective labor grows at the rate \( \mu > 0 \). Determine the rate of return of capital in the steady state.
(b) Assume that the economy is in a steady state with $\mu > 0$ constant. At time 0, $\mu$ jumps to a higher level $\mu^* > \mu$. Analyze the dynamics of the economy.

(c) Assume that in the previous question, the jump from $\mu$ to $\mu^*$ takes place at time $T > 0$ but that it is announced at time 0. Analyze the dynamics of the consumption, the capital accumulation and of the rate of return over time.
4. Household choose consumption $C_t$ and labor $N_t$ to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right]$$

subject to

$$K_{t+1} = (1 - \delta)K_t + R_t K_t + W_t N_t - C_t - T_t$$

where $T_t$ denotes taxes that are used to finance government spending. Firms in this economy choose labor and capital inputs to maximize profits $\Pi_t = Y_t - W_t N_t - R_t K_t$ given the production function $Y_t = K_t^\alpha N_t^{1-\alpha}$. Resources satisfy $Y_t = C_t + I_t + G_t$ where $I_t = K_{t+1} - (1 - \delta)K_t$.

(a) Suppose

$$U(C_t, N_t) = \log(C) - \nu(N)$$

where $\nu(N) = \frac{N^{1+\phi}}{1+\phi}$. Assume that government expenditures are financed through lump-sum taxes. Describe the effect of a permanent increase in the government expenditure share $g_t = G_t / Y_t$ on the steady-state values of labor and capital.
(b) Now assume that

\[ U(C, N) = \log(C - v(N)) \]

How does this change your answer to part a? Provide economic intuition.

(c) Now suppose that the government taxes labor in order to pay for government expenditures. Again assume that \( U(C, N) = \log(C) - v(N) \) as in part a). Describe the effect of a permanent increase in the government expenditure share on the steady-state value of labor. How does the response compare to that derived in part a)? Explain.
Long Question 1

1. Consider an economy with three consumers, $i = 1, 2, 3$. There is one good in the economy, which is not storable and arrives in endowment streams owned by the three consumers. The endowments are

$$y_t^1 = 1 + s_t$$
$$y_t^2 = 2 - s_t$$
$$y_t^3 = 1,$$

where $s_t$ is stochastic and independently and identically distributed according to

$$s_t = \begin{cases} 
0 & \text{with probability } \frac{1}{2}, \\
1 & \text{with probability } \frac{1}{2}.
\end{cases}$$

As usual, use the notation $s^t$ to refer to the history $\{s_0, s_1, \ldots, s_t\}$ and let $\pi(s^t)$ be the probability that history $s^t$ is realized. All three consumers have identical preferences given by

$$U(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} = \beta^t \log \left( c^i_t(s^t) \right) \pi(s^t).$$
(a) Consider a social planner's problem. Assume the planner assigns Pareto weight \( \phi^i \) to each agent and the weights satisfy \( \sum_i \phi^i = 1 \). The planner's social welfare function is \( \sum_i \phi^i U(c^i) \). State the planner's problem and solve for the consumption allocation that solves it.
(b) Suppose there is a complete market of Arrow-Debreu securities with trade occurring once and for all at date zero. Assume that \( s_0 \) is not yet known at the time trade occurs. Let \( q_t(s^t) \) be the date zero price of a unit of the good at date \( t \) after history \( s^t \). State the optimization problem of agent \( i \).
(c) Define a competitive equilibrium of the economy with Arrow-Debreu securities.
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(d) What consumption allocation can be supported as a time zero competitive equilibrium with no taxes or transfers? Verify your answer.
(e) Normalize \( q_0(s_0 = 0) \) to one and recall that trade occurs before \( s_0 \) has been realized. Find the prices of all the Arrow-Debreu securities. [It is sufficient to provide expressions for \( q_0(s_0 = 0) \) and \( q_0(s_0 = 1) \) and a recursion that expresses the of history \( s^{t+1} \) in terms of the price of history \( s^t \).]
(f) Now change the endowment process as follows

\[ y_t^1 = 1.5 + s_t \]
\[ y_t^2 = 1.5 - s_t \]
\[ y_t^3 = 1, \]

where \( s_t \) is stochastic and independently and identically distributed according to

\[ s_t = \begin{cases} 
  -0.5 & \text{with probability } \frac{1}{2}, \\
  0.5 & \text{with probability } \frac{1}{2}. 
\end{cases} \]

All other aspects of the problem are as described above. Prove that \( E_0 (c_t^3(s_t^4)) > 1. \)
2. Consider an economy described by the following log-linear dynamic IS and AS curves:

\[
\begin{align*}
IS & : x_t = -r_t + E_t \{ x_{t+1} \} + g_t \\
AS & : \pi_t = \lambda x_t + \beta E_t \{ \pi_{t+1} \} + u_t
\end{align*}
\]

where

\[
r_t = i_t - E_t \{ \pi_{t+1} \}
\]

denotes the exante real interest rate, \( x_t \) denotes the output gap, \( i_t \) is the nominal interest rate, and \( \pi_t \) is the inflation rate. Further assume that \( u_t \) and \( g_t \) satisfy:

\[
\begin{align*}
u_t &= \rho_u u_{t-1} + \varepsilon^u_t \\
g_t &= \rho_g g_{t-1} + \varepsilon^g_t
\end{align*}
\]

with \( \varepsilon^u_t, \varepsilon^g_t \) iid and \( 0 \leq |\rho_u| \leq 1, 0 \leq |\rho_g| \leq 1 \) Suppose the monetary authority chooses a path for nominal interest rates to maximize:

\[
-\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i (\alpha (x_{t+i})^2 + \pi_{t+i}^2)
\]

(a) Assume that the monetary authority cannot credibly commit to a policy. Derive the first order conditions for optimal monetary policy. Explain what these conditions mean.
(b) Given your answer in part a) derive expressions for the volatility of inflation and the output gap as functions of the volatility of the underlying shocks $u_t$ and $g_t$. How does inflation and output-gap volatility vary with $\alpha$? Under what conditions is it optimal to conduct strict inflation targeting (i.e. $\pi_t = 0$ so that there is zero inflation volatility)?
(c) Now consider the situation where the monetary authority can commit to a policy rule. Assume that the monetary authority chooses a rule of the form

\[ x_t = -\omega u_t \]

What is the optimal choice of \( \omega \)?
(d) Are inflation and the output gap more or less volatile under the optimal policy rule with commitment derived in part c) compared to the optimal policy under discretion? Explain.
(e) Now consider the fully optimal policy under commitment. Write down the Ramsey problem and derive the optimality conditions.
(f) Recent policy discussions within Central Banks have focused on the potential benefits of price-level targeting as opposed to inflation targeting. To what extent does the optimal policy implied by this model provide any guidance as to whether or not price-level targeting is a desirable alternative to inflation targeting? Explain.