Maca economics Qualifying Examination
Boston University
September 2008

Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 15 points each. Part B consists of two multipart questions: each multipart question is worth 60 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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Subtotal 60

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Subtotal 120
A. Shorter Questions

1. Govt spending in the neoclassical growth model: Consider the neoclassical growth model without uncertainty: there is a representative consumer, with utility $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where $c_t$ is consumption. The consumer supplies inelastically one unit of labor. There is an aggregate production function $y_t = F(k_t, n_t)$ where $n_t$ is labor. Capital is accumulated according to $k_{t+1} = (1-\delta)k_t + i_t$ where $i_t$ is investment. The government levies lump-sum taxes to finance an exogenous stream of expenditures $\{g_t\}$. The resource constraint of the economy is $c_t + i_t + g_t \leq y_t$.

(a) Write the equations characterizing the evolution of the economy given some initial capital $k_0$. 

Student ID:

(b) What is the effect on output, consumption and investment of a temporary decrease in lump-sum taxes that is compensated by higher lump-sum taxes in the future? what if the decrease in lump-sum taxes is compensated by lower government spending in the future? (Note: for the second question, an intuitive answer is enough.)

(c) Discuss the statement: “government deficits do not have any real effect in the neoclassical growth model, because of Ricardian equivalence: public savings decrease, but private savings increase by an equal amount, so that total savings is unchanged.”
2. Welfare Cost of Business Cycles: Let $U(C)$ denote the life-time utility from consumption. Assume that the representative agent has constant relative risk-aversion preferences. Let $\gamma$ denote the coefficient of relative risk-aversion:

$$U(C) = E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}.$$ 

Assume that the representative agent is endowed with a consumption stream $C_t$ such that:

$$C_t = A e^{\mu t} e^{-\frac{1}{2} \sigma^2 \varepsilon_t},$$

where $A > 0$, $\mu > 0$ and $\log \varepsilon_t$ is a normally distributed random variable with mean zero and variance $\sigma^2$. The representative agent would prefer a deterministic consumption path to a stochastic path with the same mean. Assume that we can multiply the risky consumption path by $1 + \lambda$ in all dates and states of the world. What is the level of $\lambda$ that would make the representative agent indifferent between the deterministic and the compensated, risky stream?
Student ID:
3. Effects of income taxes: Suppose that there are many identical agents, with utility

\[ u(c_t, n_t) = \log(c_t) - \frac{X}{1 + \gamma} n_t^{1+\gamma} \]

Suppose further that each agent can produce output \( y \) according to

\[ y_t = a_t n_t \]

where \( a_t \) is a time-varying level of productivity, but that a fraction of income must be given to the government, so that

\[ c_t = (1 - \tau_t)a_t n_t + T_t \]

where \( \tau_t \) is the tax rate and \( T_t \) is the volume of lump sum transfers made by the government to each agent.

(a) If all agents optimize, taking as given a sequence of tax rates but with transfers equal to zero at all dates, then what will be the equilibrium series of consumption and labor supply? How will each respond to taxation? Why?
Student ID:

(b) Now suppose that the government rebates the proceeds of income taxation in a lump sum manner. How will this change the effects of taxes and productivity.
4. Consider a risk neutral investor’s decision to invest in a project with value $V(t)$. The project value is stochastic and is governed by the Geometric Brownian process

$$dV(t) = \alpha V(t)dt + \sigma V(t)dW(t).$$

where $W(t)$ is a Wiener process. To invest in the project and receive $V(t)$, the investor must pay the cost $I$ at the time of investment. The investor discounts future cash flows according to the rate $\rho$.

(a) Let $F(V)$ denote the option value of waiting to invest. Derive a second order differential equation that determines $F(V)$ in the continuation region.
Student ID:

(b) Assume $F(V) = AV^\beta$. Provide an equation that determines $\beta$. What is the effect of an increase in $\sigma$ on the option value and investment decision? Explain.
B. Long Questions

1. Externalities and Multiple Equilibria: Consider an economy in which the government must raise a fixed amount of resources, $g$, with taxation of production ($y$) at rate $\tau$.

\[ g = \tau y \]

Suppose further that production supply takes form

\[ y = \max\{(b - \alpha \tau), 0\} \]

where we assume that $b - \alpha < 0$.

(a) Graph the government’s tax revenue as a function of its tax rate. What is the implication of the condition that $b - \alpha < 0$?
(b) At what tax rate is the revenue maximized? What is the maximum tax revenue?
Student ID:

(c) Suppose that the government adopts the fiscal rule for setting the tax rate,

\[ \tau = \frac{g}{y} \]

with \( g < \bar{g} \). Show that there are two equilibria, one with high taxes and one with low taxes.
(d) In what sense is there an external effect that leads to multiple equilibria? For the purpose this question, you may want to use the utility function

\[ u = c - \frac{1}{2\alpha} (y - (b - \alpha))^2 \]

and the budget constraint

\[ c = (1 - \tau)y \]
2. Optimal Monetary Policy: Suppose the economy is characterized by the following system of equations:

\[
\begin{align*}
IS & : x_t = -(i_t - E_t \pi_{t+1}) + E_t(x_{t+1}) \\
AS & : \pi_t = x_t + \beta E_t(\pi_{t+1}) + u_t
\end{align*}
\]

where \(x_t\) is the output gap, \(i_t\) is the nominal interest rate and \(\pi_t\) is the inflation rate. Further assume that \(u_t\) satisfies:

\[u_t = \rho_u u_{t-1} + \epsilon^u_t\]

where \(0 \leq \rho_u \leq 1\) and \(\epsilon^u_t\) is a mean zero, iid random variable. Suppose that the monetary authority chooses the nominal interest rate to maximize

\[L_o = -\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t (\alpha (x_{t+i} - k)^2 + \pi_{t+i}^2)\]

where \(k > 0\) is the socially desirable level of \(x_t\).

(a) Consider a Markov Perfect Equilibrium and derive the F.O.C. for optimal monetary policy in the absence of commitment.
(b) In the absence of commitment, is the long-run rate of inflation zero? Why or why not? Explain.
Assuming the true loss function to society is indeed $L_\alpha$, suppose the govt. can appoint a central banker who sets monetary policy to minimize the following loss function:

$$L^c = -\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left( \alpha^c (x_{t+i} - k)^2 + \pi_{t+i}^2 \right)$$

where $\alpha^c < \alpha$. How would appointing such a central banker affect the long-run rate of inflation and output gap? Is $\alpha^c = 0$ socially optimal? Explain.
(d) Now suppose that monetary policy is conducted under full commitment. Specify the policy problem and derive the optimality conditions for monetary policy.
(e) To what extent is there an inflationary bias when monetary policy is conducted under full commitment? Explain.