Maconomics Qualifying Examination
Boston University
September 2007

Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 15 points each. Part B consists of two multipart questions: each multipart question is worth 60 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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1. Consider an economy with two agents without uncertainty. The two agents have the same utility function:

\[ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}. \]

Agent 1 has the following endowment: \( e_1 = \{1, 0, 1, 0, \ldots\} \), i.e. \( e_{1,t} = 1 \) if \( t \) is even and \( e_{1,t} = 0 \) if \( t \) is odd (note that the first date is \( t = 0 \)); and agent 2 has the endowment \( e_2 = \{0, 1, 0, 1, \ldots\} \) i.e. \( e_{2,t} = 0 \) if \( t \) is even and \( e_{2,t} = 1 \) if \( t \) is odd.

a. Define and compute a competitive equilibrium for this economy, assuming complete markets.
b. Is this equilibrium Pareto-Optimal? Are there other Pareto-Optimal allocations?

c. Give the price of a consol, i.e. an asset with payoff \( \{1, 1, 1, \ldots \} \).
2. Professor Kotlikoff argued that many different fiscal systems are behaviorally identical. Consider a two period consumption-saving model with a lifetime utility function of the form

\[ u(c_1) + \beta u(c_2) \]

where \( c_t \) is consumption in period \( t \), \( \beta \) is a positive discount factor, and \( u(c) \) is an increasing concave function, differentiable as needed. Suppose that the household’s budget constraints take the form

\[ c_1 = (1 - \tau_1)y_1 + T_1 - s \]
\[ c_2 \leq (1 - \tau_2)y_2 + T_2 + (1 + r)s \]

where \( y_t \) is income at date \( t \), \( \tau_t \) is the income tax rate at date \( t \), \( T_t \) is a lump-sum transfer at date \( t \), \( s \) is saving and \( r \) is the interest rate.

**a.** Show a perturbation in the mix of transfers and taxes that will be behaviorally neutral for the household and fiscally neutral for the government.
b. Define Ricardian equivalence. Explain why this model would display "Ricardian equivalence" between transfers and income taxes. Explain one modification to the model that would break Ricardian equivalence.
3. Let us find the value at time $t$ of a payoff $X_{t+1}$ at time $t+1$. For example, if you buy a stock today at price $P_t$, the payoff next period is the stock price $P_{t+1}$ plus dividend $D_{t+1}$, $X_{t+1} = P_{t+1} + D_{t+1}$. The payoff $X_{t+1}$ is a random variable; at date $t$, an investor does not know exactly how much he will get from his investment at date $t+1$. We consider an endowment economy, where $e$ denotes the endowment level.

a. Write the maximization problem and the first-order condition of an investor characterized by a two-period utility function $u(C_t) + E_t[\beta u(C_{t+1})]$, where $\beta$ is the subjective discount factor. Let $\xi$ be the amount of the asset the investor chooses to buy in the first period. Find the price $P_t$ of the asset as a function of $M_{t+1} = \beta u'(C_{t+1})/u'(C_t)$ and interpret it.
b. Assume that there are $S$ states of nature tomorrow. Let $s$ denote an individual state and $P_c(s)$ the price of a contingent claim. Write the price $P_t$ of the asset as a function of $P_c(s)$. How does $M$ relate to $P_c(s)$?

c. Assume that the investor can buy any contingent claim. He starts in the first period with an initial wealth $W$ to invest. He receives in the next period a random labor income $e$. He cares about terminal consumption. Write his maximization problem and
find his optimal payoff $X$ as a function of $M, e$ and a Lagrange multiplier.
d. How does your answer to the previous question change if the investor values the first period too?


e. Let us assume now power utility (i.e. CRRA, with risk-aversion coefficient $\gamma$) and ignore labor income. Find the optimal payoff $X$ as function of $W$, $M$ and $\gamma$. Does the
return on the optimal investment depend on initial wealth?
4. Consider a household that gains utility from the service flow of durable goods. Assume that the service flow is proportional to the current stock of durables $D_t$. The household then chooses durable consumption expenditures $C_t$ to maximizes

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(D_t), \quad 0 < \beta < 1$$

subject to the accumulation equation

$$D_t = D_{t-1} + C_t$$

and the budget constraint:

$$A_{t+1} = RA_t + W_t - C_t,$$

where $A_t$ denotes financial assets, $R$ is the gross rate of returns on assets, and $W_t$ follows a stationary stochastic process.

a. Write down Bellman’s equation for this problem.
b. Derive the household euler equation. In words, what does this say?

c. Let $\beta R = 1$. Suppose per-period utility is quadratic in the stock of consumer durables

$$U(D_t) = \left( aD_t - \frac{b}{2} (D_t)^2 \right)$$

Are consumption expenditures $C_t$ a random walk? If yes, explain why. If not, what stochastic process does $C_t$ follow?
1. Consider an economy populated by many agents, indexed by $i$, with the same preferences:

$$E \sum_{t=0}^{\infty} \beta^t u(c_{it}),$$

where $u(c) = -\exp(-\gamma c)$ is a CARA utility function. We normalize the population size to 1. Each individual’s income follows an iid process: it is $y_{it} = \bar{y} + \sigma \epsilon_{it}$, where $\epsilon_{it}$ is a random variable which is iid across time and people and is distributed $N(0, 1)$. A law of large number implies that total income in this economy is constant over time and equal to $\bar{y}$.

This problem’s aim is to compare the consumption allocation under different market structures.

a. Assume first that agents have no access to markets or storage, so that each agent’s consumption equals his current income. What is the expected discounted utility of a given consumer as of time 0? What is the marginal propensity to consume out of current income? (i.e. formally, how does the consumption today react to a good shock today (i.e. a high draw of $\epsilon$))
b. Now assume instead that agents live in ‘families’ of two, behaving optimally within the family, and exchange with nobody outside the family. Assume for simplicity that within each household, the two agents have equal weights. What is the consumption of each agent, as a function of the income of the two agents? What is the expected discounted utility of a given consumer as of time 0? What is the marginal propensity to consume out of current income?
c. Now assume instead that markets are complete (and agents live alone). What is the consumption of each agent? What is the expected discounted utility of a given consumer as of time 0? What is the marginal propensity to consume out of current income?
d. Finally assume instead that agents can save or borrow using a risk-free asset with gross return \( R = 1 + r \), and there is no borrowing constraint. Define \( b_t \) = assets at the end of the previous period, then the budget constraint in each period is
\[
b_{t+1} = Rb_t + y_t - c_t.
\] [Hint: write the Bellman equation. Write the first-order condition and envelope condition of the Bellman equation, and write the Euler equation for this problem. Use the guess-and-verify method. A possible guess is \( V(b, y) = -\exp(-\gamma (a_0 b + a_1 y + a_2)) \). You do not have to give the exact value of \( a_2 \).] Explain how consumption is determined. What is the expected discounted utility of a given consumer as of time 0? What is the marginal propensity to consume out of current income?
e. Compare the welfare across the four market structures. Can you rank them? Discuss briefly.

f. Compare the marginal propensity to consume out of current income. Discuss briefly.

g. Explain how the cross-sectional distribution of income and of consumption evolve for
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each of the four cases.

h. Suppose that an agent learns today that his income tomorrow (i.e. his $e$ tomorrow) will be high. How does this affect his consumption today in each of the four cases?
2. The economy is made up of a representative agent that chooses consumption, \( C_t \), and leisure, \( L_t \), to maximize:

\[
U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\log(C_t) + \theta \log(L_t)) \right\}
\]

Firms in this economy rent capital, \( K_t \), and labor, \( N_t \), from consumers to maximize profits. All markets are competitive. Firms have access to the following production technology:

\[
Y_t = (A_tN_t)^{(1-\alpha)} K_t^{\alpha} Q_t, \quad 0 < \alpha < 1
\]

where \( Q_t \) represents per capita aggregate income which the firm views as exogenous when making its own production decisions and \( A_t \) represents a stochastic process for the level of technology. The number of firms equals the number of consumers which may be normalized to one. Capital depreciates fully each period and the total time endowment for consumers is one so that resource constraints are:

\[
C_t + K_{t+1} \leq Y_t
\]

\[
N_t + L_t \leq 1
\]

a. Consider the decentralized economy:

i. Derive the FOC.

ii. Find closed-form solutions for labor \( N_t \) and new capital, \( K_{t+1} \) as functions of the
state variables in the economy.

iii. What are the steady-state values for $N$ and $K$? How do these vary with $\varepsilon$? Explain.

b. For the decentralized economy, what conditions if any do you need to impose on
parameter values to ensure stability of the system?
c. Now consider the social planner problem:
   i. Derive the FOC.

ii. Find closed-form solutions for labor $N_t$ and new capital, $K_{t+1}$ as functions of the state variables in the economy.
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iii. What are the steady-state values for $N$ and $K$? How do these differ from your answers for the decentralized economy? Under what conditions is the steady-state $K$ higher in the social-planner problem than in the decentralized solution?
d. Assume \( \log(A_t) \) is iid. For the decentralized economy, describe the dynamics for \( \log(Y_t) \) as a function of lagged values of \( \log(Y_t) \) and \( \log(A_t) \). To what extent does this model magnify business cycle shocks? To what extent does it propagate business cycle shocks? How does your answer depend on the parameter \( \epsilon \)?
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e. Does the decentralized economy provide more or less magnification than the social planner's solution? Explain.

f. How would the degree of magnification and propagation in this model change if there is less than full depreciation? Explain.