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**Macroeconomics Qualifying Examination**  
Boston University  
June 2011

**Instructions.** There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 20 points each. Part B consists of two multipart questions: each multipart question is worth 50 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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Short Questions (Part A): 4 questions

1. Suppose the economy is characterized by the following system of equations:

\[ IS : \quad x_t = -(i_t - E_t \pi_{t+1}) + E_t(x_{t+1}) \]
\[ AS : \quad \pi_t = x_t + \beta E_t(\pi_{t+1}) + u_t \]

where \( x_t \) is the output gap, \( i_t \) is the nominal interest rate and \( \pi_t \) is the inflation rate. Further assume that \( u_t \) satisfies:

\[ u_t = \rho_u u_{t-1} + e^u_t \]

where \( 0 \leq \rho_u \leq 1 \) and \( e^u_t \) is a mean zero, iid random variable. Suppose that the monetary authority chooses the nominal interest rate to maximize

\[ L_o = -\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t (\alpha (x_{t+t} - k)^2 + \pi_{t+t}^2) \]

where \( k > 0 \) is the socially desirable level of \( x_t \).

(a) Derive the F.O.C. for optimal monetary policy in the absence of commitment. Explain the economic intuition behind this optimality condition.
(b) What is the long-run rate of inflation in this model? Does it deviate from the target rate of zero inflation implied by the loss function? If so, what factors contribute to the deviation?
2. There is a one good economy with continuous time and a representative agent who is endowed with a fixed flow of labor that is normalized to one and the utility function

\[ U = \int_0^\infty e^{-\rho t} (\log(c_t) + \log(1 - \xi_t)) \, dt. \]

Production is determined by the function \( Y_t = K_t^\alpha L_t^{1-\alpha} \), with \( K_t \) and \( L_t \) as the levels of capital and labor at time \( t \).

(a) The economy is in a steady state. Determine the rate of return on capital and the level of the labor supply as functions of the parameters of the model.
(b) The economy is in a steady state when the government announces suddenly (unexpected from the agent) that a linear tax on labor will be implemented from time 0 on and forever, at the rate of 50 percent. The proceeds of the tax are consumed by the government and that consumption does not enter in the utility of the agent. Determine the impact of the tax in the long run on the rate of return on capital and on the labor supply.
3. Consider the Mortensen-Pissarides search and matching model. Denote the job-finding rate of workers by \( \mu(\theta) \) and the hiring rate of firms by \( q(\theta) \), where \( \theta \) is the vacancy-unemployment ratio. Suppose newly filled jobs produce \( p_1 \) units of output, and end according to a Poisson process with arrival rate \( \delta_1 \), while mature jobs produce produce more \( p_2 (p_1) \), and are less likely to end \( \delta_2 (< \delta_1) \). A newly filled job matures at Poisson arrival rate \( \lambda \).

(a) Write Bellman equations for the value of unemployment (\( U \)), the value of employment in new and mature jobs (\( W_1, W_2 \)), the value of a vacancy (\( V \)) and the value of a new and mature filled job (\( J_1, J_2 \)), in continuous time. Denote unemployment benefits by \( b \), wages by \( (w_1, w_2) \), the vacancy-cost by \( c \), and the discount rate by \( r \).
(b) Suppose there is free entry into vacancy-creation by firms, and that wage-bargaining results in half of the match surplus going to the worker and half to the firm. Defining the match surplus as $S_i = W_i + J_i - U - V$, this means that $W_i - U = J_i - V = \frac{1}{2}S_i$ for $i = 1, 2$. Write Bellman equations for the surpluses $S_1, S_2$. Provide an equation implicitly determining the equilibrium $\theta$. 
4. There is an overlapping generation model with no durable good and money. Individuals live two periods. Each generation is represented is a continuum of mass one of individuals who operates in perfectly competitive markets. The representative individual has the utility function

\[ U(c_1, c_2) = (1 - a)\log(c_1) + a\log(c_2). \]

where \( c_1 \) and \( c_2 \) are the consumptions in the two period. The individual is endowed with one unit of labor in each period. There is no storable good in this economy. The production of the good is equal to the quantity of labor in the economy. All markets have perfect competition.

(a) The quantity of money is equal to \( M = 1 \). Determine a condition on \( a \) such that there is an equilibrium with a constant and positive price of money in the economy. Interpret this condition. What is the price level \( P \) (the price of the good in money) in this case? (Partial credit will be given for a correct writing of the budget constraint). If \( a = 3/4 \), what is the price level?
(b) Assume that \( a = 3/4 \). The government announces in period 0 that in period 1 and in period 1 only, it will purchase consumption goods on the market, by an amount \( \bar{c} \), and distribute them for free to old people and finance that purchase by printing money. Assume that \( \bar{c} \) is small. Determine (using economic theory) the qualitative impact of this policy on the welfare of the young people and the old people in period 1 and on the welfare of the old people in period 0. Demonstrate without algebra and using your answer in question 1, that after the announcement, \( 2 < P_0 < P_1 \).
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Long Questions (Part B): 2 questions

1. A representative household chooses consumption goods, labor supply and real balances \((C_t, N_t, M_t/P_t)\) to maximize

\[
E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, M_t/P_t, N_t) \right]
\]

subject to

\[
P_tC_t + M_t + TR_t = W_t N_t + M_{t-1} + \Pi_t
\]

where \(C_t = \int_0^\infty C_t^{(e-1)/\varepsilon} dt\), \(P_t\) is the price index, \(TR_t\) are lump-sum transfers and \(\Pi_t\) represents nominal profits from the production sector, which households take as given. Assume the per-period utility function satisfies:

\[
U(C_t, M_t/P_t, N_t) = \log(C_t) + \log\left(\frac{M_t}{P_t}\right) - \theta N_t
\]

There is a continuum of firms on \([0,1]\). The ith firm faces a downward sloping demand curve of the form:

\[
Y_{it} = (P_{it}/P_t)^{-e} C_t
\]

Each firm has access to the following production technology:

\[
Y_{it} = A_t N_{it}
\]

and \(A_t\) follows the random walk process:

\[
A_t = A_{t-1} \exp(v_t), \; v_t \sim N(0, \sigma_v^2).
\]

Assume that firm i chooses its price \(P_{it}\) to maximize profits:

\[
\max P_{it} Y_{it} - W_t N_{it}
\]

where \(W_t\) is the nominal wage. Assume that the nominal wage \(W_t\) is set one period in advance to satisfy

\[
W_t E_{t-1}\{\lambda_t^L/P_t\} = E_{t-1}\{\lambda_t^N\}
\]

where \(\lambda_t^L = \frac{\partial U(C_t, M_t/P_t, N_t)}{\partial C_t}\) and \(\lambda_t^N = \frac{\partial U(C_t, M_t/P_t, N_t)}{\partial N_t}\) represent the marginal utility of consumption and the marginal disutility of leisure respectively. Intuitively, households choose the nominal wage one period in advance to satisfy the ex ante labor-leisure first-order condition. Ex post, household labor supply is set to meet the quantity of labor demanded by firms.
(a) The labor-leisure first-order condition is determined by (***) above. Show that the households optimization over real balances \((M/P)\) implies:

\[
\frac{1}{C_t} = \frac{P_t}{M_t} + E_t \left[ \frac{\beta}{C_{t+1}} \frac{P_t}{P_{t+1}} \right]
\]

Show that profit maximization by firms implies:

\[
P_{it} = \mu \frac{W_t N_{it}}{Y_{it}}.
\]

for some constant \(\mu > 1\). Interpret these conditions.
(b) Assume that the growth rate of money is constant. Derive a relationship between real balances, output and the growth rate of money. How do real balances respond to changes in the growth rate of money? Explain.
(c) Again assume that the growth rate of money is constant. Is the amount of output supplied in this economy at the social efficient level? How does the money growth rate influence the long-run efficiency of this economy? Explain.
(d) Now assume that money growth is a random walk:

\[ M_t = M_{t-1} \exp u_t, \quad u_t \sim N(0, \sigma_u^2). \]

Characterize the dynamic effects of a shock to the money growth rate on the log-deviations from steady-state of output, labor and inflation. Explain the intuition behind these results.
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(c) Suppose the monetary authority chooses the money growth rate to stabilize labor in this economy. How should money growth respond to technology shocks? How would your answer differ if the model allowed for one period output price rigidity rather than one period wage rigidity? Explain.
2. Consider an economy that consists of $N$ individuals. At each date $t$, these individuals are endowed with different labor productivities. Let $s_t$ be the stochastic event at date $t$ and $s^t$ be the history of events that have occurred up to date $t$. Then the labor productivity of individual $i$ at date $t$ after history $s^t$ is $c^t_i(s^t)$. The individuals can decide how much labor to supply. Let $n_i^t(s^t)$ be the labor supply of individual $i$ after history $s^t$. An individual that has productivity $e$ and supplies $n$ units of labor is said to supply $e \times n$ effective units of labor. An individual cannot supply more than one unit of (raw) labor: $0 \leq n \leq 1$.

The economy has a production process that is linear in labor and in particular, one effective unit of labor generates $A(s^t)$ units of output that is used for consumption. Output is perishable and must be consumed within the period in which it is produced.

The consumers have identical preferences for consumption and labor supply given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c^t_i(s^t), n^t_i(s^t)) \pi(s^t|s^0),$$

where $\pi(s^t|s^0)$ is the probability that history $s^t$ is realized.

At date zero, the individuals trade a complete set of contracts for consumption and labor at all future dates and all histories. Let $q_0^t(s^t)$ be the price of a unit of consumption at date $t$ after history $s^t$ in terms of date zero consumption.

Assume that all of the individuals are identical at date zero when they agree on the contracts.
(a) Given that markets are competitive, how much of the consumption good at date $t$ will an individual with productivity $e^t$ receive in exchange for a unit of their labor?
(b) Suppose that individuals place no value on leisure and in particular have the period utility function \( u(c, n) = \log(c) \). Characterize the equilibrium allocation of consumption and labor supply levels.
(c) Now suppose that individuals do value leisure and have the period utility function
\[ u(c, n) = \frac{1}{1-\sigma} \left( c - \frac{\psi n^{1+\sigma}}{1+\sigma} \right)^{1-\sigma} \]. Again, characterize the equilibrium allocation of consumption and leisure.
(d) Using your answer to the previous part, solve for aggregate output as a function of $A_t$ and the labor productivity endowments.