Student ID:

Macroeconomics Qualifying Examination
Boston University
June 2010

Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 15 points each. Part B consists of two multipart questions: each multipart question is worth 60 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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1. Consider the social planner's problem for a representative-agent economy with preferences given by
\[ \sum_{t=0}^{\infty} \beta^t \log(c_t) \]
and technology given by
\[ y_t = k_t^\alpha L^{1-\alpha} \]
\[ k_{t+1} = y_t - c_t. \]

The labor input, \( L \) is fixed and constant across periods.

(a) Derive the Euler equation for the planner's choice of \( k_{t+1} \).
(b) What is the steady state level of the capital stock?

(c) What is the policy rule for the choice of $k_{t+1}$ as a function of $k_t$?
2. An unemployed worker receives a random number $n$ job offers each period, where $n$ is independently distributed over time. The probability that the worker receives $n$ offers per period is $\pi_n \geq 0$ for $n = 1, 2, \ldots N$, where $\sum_{n=1}^{N} \pi_n = 1$. Each offer is drawn independently from the distribution $F(w)$. During unemployment the worker receives unemployment benefit $b$, and jobs last forever. The worker maximizes the expected present value of income $\sum_{t=0}^{\infty} \beta^t y_t$, where $y_t = b$ if the worker is unemployed, and $y_t = w$ if he is employed at wage $w$. Let $V(w)$ be the value function for a worker with wage offer $w$ in hand.

(a) Write the Bellman equation for the worker.

(b) Characterize his optimal strategy.
3. There is an overlapping generation economy where people live two periods, earn one unit of good in the first period and have a utility function $U(C_1, C_2) = \min(C_1, C_2)$. There is no capital. The only asset is money that has a price $P_t$ in period $t$. Population is constant and normalized to one. The quantity of money is fixed and normalized to one.

(a) Analyze the dynamics of the price level.

(b) Is there more than one equilibrium?
4. Consider an economy described by the following log-linear dynamic IS and AS curves:

\[
IS : \quad x_t = -(r_t - r^n_t) + E_t\{x_{t+1}\}
\]

\[
AS : \quad \pi_t = \lambda x_t + \beta E_t\{\pi_{t+1}\} + u_t
\]

where

\[
r_t = i_t - E_t\{\pi_{t+1}\}
\]

denotes the exante real interest rate, \(x_t\) denotes the output gap, \(i_t\) is the nominal interest rate, and \(\pi_t\) is the inflation rate. Further assume that the natural rate shock \(r^n_t\) and the cost-push shock \(u_t\) satisfy:

\[
r^n_t = \rho_r r^n_{t-1} + \varepsilon^r_t
\]

\[
u_t = \rho_u u_{t-1} + \varepsilon^u_t
\]

with \(\varepsilon^r_t, \varepsilon^u_t\) iid and \(0 \leq |\rho_r|, |\rho_u| \leq 1\). Suppose the monetary authority chooses a path for the nominal interest rate to maximize:

\[
-\frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i (\alpha (x_t)^2 + \pi_t^2)
\]

subject to the IS and AS curves. Assume that the monetary authority cannot credibly commit to a policy and hence takes future expectations as given.

(a) Derive the first order conditions for optimal monetary policy under discretion. Explain what these conditions mean.
(b) How does the nominal interest rate respond to movements in $r_t$? Provide intuition for your answer in terms of monetary policy tradeoffs.
Long Question 1

1. **Part I:** This problem examines the effects of mandatory severance payments — transfers from firms to workers upon termination of a match — in the Mortensen-Pissarides search and matching model.

Time is continuous. There is a continuum of measure one homogenous risk neutral infinitely lived workers with discount rate \( r > 0 \). When unemployed, a worker's flow income is normalized to zero.

There is a large number of risk-neutral infinitely-lived firms that open vacancies at flow cost \( c > 0 \) as long as it is profitable to do so. Each filled job produces \( p \) units of output, but ends according to a Poisson process with arrival rate \( \lambda \). When the job ends, the firm pays a one-time severance payment \( f \) to the worker.

Unemployed workers and vacancies match according to a constant returns to scale matching function \( m(u, v) \). Denote the job-finding rate of workers and the hiring rate of firms by \( \mu(\theta) \) and \( q(\theta) \), respectively, where \( \theta \) is the vacancy-unemployment ratio. Upon matching, the worker and firm bargain a wage \( w \), which gives share \( \beta \) of the match surplus to the worker, and share \( 1 - \beta \) to the firm.

(a) Write Bellman equations for the value of unemployment \( U \), the value of employment \( W \), the value of a vacant job \( V \) and the value of a filled job \( J \).
(b) Define the match surplus as $S = W + J - U - V$, and use it to derive an implicit equation for the equilibrium vacancy-unemployment ratio $\theta^*$. How does $\theta^*$ depend on the severance payment $f$?
(c) Find an expression for the equilibrium wage $w^*$. How does $w^*$ depend on the severance payment $f$? Explain.
**Part II**: This problem examines the effects of non-labor income on job search behavior, in the McCall search model.

Each period an unemployed worker draws an offer to work forever at wage $w$. Wages are iid draws from the distribution $F(w)$, with support $[0, B]$. The worker also has a second source of income, denoted $\epsilon_t$, which represents his non-labor income. The realization of $\epsilon_t$ is iid over time, drawn from distribution $G(\epsilon)$ with support $[0, \infty)$. The draws of $w$ and $\epsilon$ are independent. The worker maximizes the expected present value of income $\sum_{t=0}^{\infty} \beta^t y_t$, where $y_t = w + \phi \epsilon_t$ if the worker is employed at wage $w$, and $y_t = b + \epsilon_t$, if the worker is unemployed. Here $0 < \phi < 1$, as the employed have less time to maximize their non-labor income. Write down the worker’s Bellman equation and show that his reservation wage rises in non-labor income. *Hint: Use $V^e(\epsilon, w)$ to denote the value of an employed worker with wage $w$ and current non-labor income $\epsilon$. Use $V(\epsilon, w)$ to denote the value of an unemployed worker with wage offer $w$ in hand and current non-labor income $\epsilon$.****
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1. **Long Question 2:** There is an overlapping generation economy. Each generation is represented by an individual who lives two periods, supplies one unit of labor in his first period and consumes only in his second period. The production of the unique good uses capital $K$, labor $L$ and land $M$ and the production function is given by $Y = K^\alpha L^\beta M^{1-\alpha-\beta}$ with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. The rate of depreciation of capital is zero. Land is in fixed supply and is normalized to 1 throughout the problem.

   (a) Population is constant. Determine the rate of return $r$ and the price of land $p$ in the equilibrium steady state. (Recall that there is no uncertainty in this model.)
(b) There is a permanent tax on capital income at the rate \( \theta \). The government consumes the proceeds of the tax. What is the impact of the tax on the level of production in the economy (increase, decrease or no impact), in the steady state? Provide the intuition for the result.
(c) Assume that the population is growing at the rate $n$. Can there be balanced growth in this economy?
(d) Population is constant and but the productivity of labor in generation $t$ is equal to $A_t$ with $A_t = (1 + \mu)^t$. If there is a balanced growth in this economy (in the variable resources), determine the growth rate $g$, and compared it with $\mu$. What is the evolution of the price of land? (Do not attempt to determine the sufficient condition for the steady state nor analyze the stability of the dynamic path).