Macroeconomics Qualifying Examination
Boston University
June 2007

Instructions. There are two parts to this exam. Part A consists of a series of four shorter questions, which are worth a total of 15 points each. Part B consists of two multipart questions: each multipart question is worth 60 points. The exam is designed to take about three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

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Short 1: Suppose that there is an individual who has an intertemporal objective of the form

\[ \int_0^T u(c_t) e^{-rt} dt \]

with \( u(c) = \frac{1}{1-\sigma} c^{1-\sigma} \) and with a flow constraint

\[ \frac{d}{dt} a_t = ra_t + (y_t - c_t) \]

and a terminal constraint \( a_T \geq 0 \).

(a) What factors govern the growth rate of consumption?
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(b) What is the level of consumption at date 0?

(c) What is a discrete time version of this problem?
Short 2: Technology and capital accumulation: The economy is made up of a representative agent that chooses consumption $C_t$ to maximize:

$$U = E_o \left\{ \sum_{t=0}^{\infty} \beta^t C_t \right\}$$

Firms in this economy rent capital, $K_t$, from consumers to maximize profits. All markets are competitive. Firms have access to the following production technology:

$$Y_t = A_t K_t^\alpha, \quad 0 < \alpha < 1$$

where $A_t$ represents a stochastic process for the level of technology. Capital depreciates by the amount $\delta$ each period so that the resource constraint is:

$$C_t + K_{t+1} \leq Y_t + (1 - \delta)K_t$$

Finally, assume $\log(A_t)$ is a random walk with drift:

$$\log(A_t) = -\frac{1}{2} \sigma_e^2 + \log(A_{t-1}) + \varepsilon_t$$

with $\varepsilon_t \sim N(0, \sigma_e^2)$ which implies $E_t(A_{t+1}) = A_t$.

a) Write down the household and firms’ optimization problems. Derive the optimality conditions.
b) Find a closed form solution for capital, $K_{t+1}$ as a function of the state variables in the economy.

c) Starting from steady-state, consider a one time permanent shock to $A_t$ in this model. How long does output take to converge to its new steady-state in response to such a shock? How does your result depend on the form of the utility function?
Short 3: Consider an economy with a representative agent with utility function:

$$E \sum_{t=0}^{\infty} \frac{\beta^t C_{t}^{1-\gamma}}{1-\gamma}.$$ 

Suppose consumption growth is iid (independent and identically distributed) and normal:

$$\Delta \log C_t = \mu_c + \sigma_c \epsilon_t,$$

with $\epsilon_t$ iid and distributed $N(0, 1)$.

a) Consider an asset ('stock') with dividends that evolve according to:

$$\Delta \log D_t = \mu_d + \sigma_d \epsilon_t,$$

i.e. the shock to dividend growth $\epsilon_t$ is the same as the shock to consumption growth, but with a different standard deviation $\sigma_d \neq \sigma_c$. Compute the price-dividend ratio $P_t/D_t$ of this asset.
b) Suppose we suddenly learn than from now on, the growth rate of log dividends $\mu_d$ is higher than we thought. What is the effect on the price-dividend ratio? Explain your result.

c) Suppose we suddenly learn than from now on, consumption growth volatility $\sigma_c$ is lower than we thought. What is the effect on the price-dividend ratio? Explain your result.
Short 4: The representative agent has preferences over consumption \( C_t \):

\[
U = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \frac{C_t^{1-\sigma}}{1 - \sigma}
\]

Let \( Y_t \) be the ‘full income’, i.e. the endowment of the agent if he spends all his time ‘harvesting’. Let \( u_t \) denote the time spent going to the bank or shopping (the other only alternative time occupation). The agent is endowed with one unit of time per period. His revenue is proportional to the time spent ‘harvesting’, (i.e. not shopping). Money is given by lump-sum transfer. Let \( M_t \) denote the total amount of money in the economy. Assume that the money growth rate is constant \( \left( \frac{M_{t+1}}{M_t} = 1 + \mu \right) \). The agent can buy for \( q \) dollar a 1 dollar bond (he pays \( q \) now, and get 1 next period).

The agent begins the period with \( N_t \) units of money. The market for goods opens first. At the end of the day, the agent has access to one-period nominal bonds \( B_{t+1} \).

Assume that the CIA constraint takes the form:

\[
P_t C_t \leq KN_t u_t,
\]

where \( K \) is a constant. Let us denote \( b_t = \frac{B_t}{M_t}, \ w_t = C_t / Y_t, \ x_t = N_t / M_t \) and \( m_t = M_t / (P_t Y_t) \).

a) Write the budget constraint at date \( t \).
b) Assume that the 'full income' grows at the constant rate $\gamma$. Let $W(N, M, Y, B)$ denote the value function. Write the expression for value function that corresponds to the steady-state maximization problem.

c) Guess and check that this value function has a solution of the form $W(N, M, Y, B) = Y^{1-\sigma}u(x, b)$. 

d) Using the first-order condition and envelope condition, what is the price of a one-period nominal bond?
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**Long 1**
Suppose that there is a government which can borrow and lend with the rest of the world. It must raise taxes on labor income to fund an exogenous stream of goods expenditure. The government's budget constraint takes the form

$$\sum_{t=0}^{T} \rho^t [g_t - \tau_t n_t] \geq 0$$

where $g$ is real government purchases of goods and service, $\tau_t$ is the labor income tax rate at date $t$, and $n_t$ is the amount of work at date $t$. The real wage is normalized to one and the real interest rate is implicit in the market discount factor, $0 < \rho < 1$.

a) Is it feasible for this government to make the tax rate constant over time. If it is constant, then what must the level be?
b) Suppose that the representative individual has preferences described by

$$\sum_{t=0}^{T} \beta^t [\log(c_t) - \chi \frac{1}{1 + \gamma_t} n_t^{1+\gamma_t}]$$

What does this specification suggest about the response of labor supply to taxation, if the budget constraint is

$$\sum_{t=0}^{T} \rho^t [(1 - \tau_t) n_t - c_t]$$
c) What does this specification of utility and preferences suggest about the response of the time path of consumption to taxation?

d) What would you conjecture the time path of optimal labor taxation would be in this economy? Why?
e) Set up the optimal Ramsey tax problem for this economy

f) Determine the optimal pattern of taxation and compare it to that described in part (d)
Long 2

Let us entertain a variant of the segmented markets model of Alvarez, Atkeson and Kehoe (2001) to study the optimal choice of a currency regime.

Consider a small open economy perfectly integrated with world goods markets. There is a unit measure of households who consume an internationally-traded good. The world currency price of the consumption good is fixed at one. The households’ intertemporal utility function is \( W_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \) where \( \beta \) is the households’ time discount factor, \( C_s \) is consumption in period \( s \), while \( E_t \) denotes the expectation conditional on information available at time \( t \). The households face a cash-in-advance constraint which binds in equilibrium.

Households are prohibited from consuming their own endowment. We assume that a household consists of a seller-shopper pair. While the seller sells the household’s own endowment, the shopper goes out with money to purchase consumption goods from other households. We assume that households are heterogenous. In particular, only a fraction \( \lambda_t \) of the population, called traders, have access to the asset markets, where \( 0 < \lambda_t < 1 \). The rest, \( 1 - \lambda_t \), called non-traders, can only hold domestic money as an asset. All money injections occur in asset markets.

There are two potential sources of uncertainty in the economy. First, each household receives a random endowment \( y_t \) of the consumption good in each period. We assume that \( y_t \) is an independently and identically distributed random variable with mean \( \bar{y} \) and variance \( \sigma_y \). We assume that the shopper can access a proportion \( v_t \) of the household’s current period (\( t \)) sales receipts, in addition to the cash carried over from the last period (\( M_t \)), to purchase consumption. We assume that \( v_t \) is an independently and identically distributed random variable with mean \( \bar{v} \) and variance \( \sigma_v \).

The timing runs as follows. First, both the endowment and velocity shocks are realized at the beginning of every period. Second, the household splits. Sellers of both households stay at home and sell their endowment for local currency. Shoppers of the non-trading households are excluded from the asset market and, hence, go directly to the goods market with their overnight cash to buy consumption goods. Shoppers of trading households first carry the cash held overnight to the asset market where they trade in bonds and receive any money injections for the period. They then proceed to the goods market with whatever money balances are left after their portfolio rebalancing. After all trades for the day are completed and markets close, the shopper and the seller are reunited at home.
a) Let \( S_t \) denote the price of domestic good (and the exchange rate) and \( M_t^{NT} \) the beginning of period \( t \) nominal money balances of the non-traders. Write the CIA constraint and the consumption \( C_t^{NT} \) of the non-traders (as a function of \( M_t^{NT}, S_t, \nu_t \) and \( y_t \)).

b) Let \( B \) denote aggregate one-period nominal domestic government bonds, \( i \) the interest rate on these bonds. Traders have also access to foreign bonds (denoted \( f \) and denominated in terms of the foreign consumption good). Let \( r \) be the exogenous and constant world real interest rate, and \( T \) aggregate (nominal) lump-sum transfers (i.e., negative taxes) from the government. Let \( \hat{M}_t^T \) denotes the money balances with which the trader leaves the asset market and \( M_t^T \) denotes the money balances with which the trader entered the asset market. We know that:

\[
\hat{M}_t^T = M_t^T + (1 + i_{t-1}) \frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} + S_t (1 + r) f_t - S_t f_{t+1} + \frac{T_t}{\lambda}
\]

Write the CIA constraint for the traders and the first-order conditions of the traders’ maximization problem.
c) The government in this economy holds foreign bonds (reserves, denoted $h_t$) which earn the world rate of interest $r$. The government can sell nominal domestic bonds, issue domestic money, and make lump sum transfers to the traders. Let us write the budget constraint of the government as:

$$S_t h_{t+1} - (1 + r)S_t h_t + (1 + i_{t-1})B_t - B_{t+1} + T_t = M_{t-1} - M_t.$$ 

Write the equilibrium condition in the money market, the quantity theory equation, and the flow constraint for the economy as a whole (i.e., the current account). Let $k \equiv h + \lambda f$ denotes per-capita foreign bonds for the economy as a whole.
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d) Derive the equilibrium consumption of non-traders.

e) Derive the equilibrium consumption of traders $C_t^T$ (as a function of $k_{t+1}$, $k_t$, $y_t$, $M_{t+1}$, $M_t$, $S_t$, $r$ and $\lambda$).
f) Let us now consider the special case of quadratic utility under different exchange rate regimes:

\[ W_t^{i,j} = E \sum_{t}^{\infty} \beta^t [c_t^{i,j} - \zeta (c_t^{i,j})^2] \text{ for } i=\text{T,NT and } j=\text{flex,peg} \]

\[ W_t^{j} = \lambda W_t^{j} + (1 - \lambda) W_t^{j} \text{ for } j=\text{flex,peg}. \]

Assume that under flexible exchange rates, the monetary authority sets a constant path of the money supply (\( M_t = \overline{M} \)). The two kinds of agents start with the same initial money balances (\( M_0^T = M_0^{NT} = \overline{M} \)). Under fixed exchange rates, the monetary authority sets a constant path of the exchange rate equal to \( S = \overline{M} / [1 - (\overline{r} - \overline{y})] \). Under flexible exchange rates, the monetary authority sets a constant path of the money supply: \( M_t = \overline{M} \).

What is the consumption of traders and non-traders under flexible or fixed exchange rates?
g) Show that when velocity shocks are the only source of uncertainty in the economy (i.e. $\sigma_y = 0$, $y_t = \bar{y}$), the flexible exchange rate regime welfare-dominates the fixed exchange rate regime for both agents and hence, is the optimal exchange rate regime for the economy. Show that when all agents in the economy are traders, i.e. $\lambda = 1$, the fixed and flexible exchange rate regimes are welfare equivalent.

Hint: Note that the quadratic utility specification implies that the expected value of periodic utility can be written as $E(c - \zeta c^2) = E(c) - \zeta E(c^2) = E(c) - \zeta Var(c)$ where $Var(c)$ denotes the variance of consumption.