Boston University
Department of Economics
Qualifying Examination in Macroeconomics

June 2006

Abstract
There are two sections of this examination.

Part A: Shorter questions (6 questions, 10 points each)
These questions are designed to test general knowledge of macroeconomics as learned in the first-year courses. You should provide a short and clear answer to each: there is no reason to exceed the provided space (1 page per question). Plan to spend 10 minutes or so answering each question.

Part B: Longer questions (2 questions, 60 points each)
These questions are designed to test detailed knowledge of problems and methods covered in the first-year courses. There are three questions and you must choose two of three to answer.

These questions frequently have a cumulative structure, with the results of early parts helping you answer the later parts. However, dynamic optimization theory (and common sense) suggests that it is a good idea to read through the whole question before starting to write answers to the question.

Unless specified otherwise, all parts of all problems are weighted equally. There is a separate page for each part of each longer problem, so that you are not space constrained. Do not feel that you must fill each page. No credit will be given for correct but extraneous information and some credit may be taken away for incorrect and extraneous information.

Please write only on the fronts of pages, as these exams will be copied for distribution to the graders.

Please write legibly using a dark pen.

Please write your Exam Code at the top of each page!
A. Shorter questions

1. Optimal consumption over time (10 points): Consider an economy with a representative agent who has the following utility function:

\[-E \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t + \theta c_{t-1}}{2} - c^* \right)^2\]

We assume that \(\frac{c_t + \theta c_{t-1}}{2} < c^*\) always. The representative agent can borrow or save at a risk-free asset \(R\) with \(\beta R = 1\). The budget constraint is

\[\forall t \geq 0 : b_{t+1} = R b_t + y_t - c_t,\]

where \(b_t = \text{assets at the end of } t - 1, \text{ and } b_0 \text{ is given.}\)

(a) Interpret briefly the utility function for \(\theta > 0\) and for \(\theta < 0\).

(b) Derive the first-order conditions.

(c) What is the testable implication of this model? How could you test it?
2. New Keynesian Macroeconomics (10 points): Suppose the economy is described by the following New Keynesian model:

\[
\begin{align*}
  x_t &= E_t(x_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1})) \\
  \pi_t &= \beta E_t(\pi_{t+1}) + \kappa x_t \\
  i_t &= \phi_\pi \pi_t + \phi_x x_t
\end{align*}
\]

(a) If \( \phi_x = 0 \), explain intuitively why \( \phi_\pi > 1 \) is needed to ensure that the equilibrium will be unique.

(b) Assume that both \( \phi_x \) and \( \phi_\pi \) are nonnegative. Explain intuitively why some values of \( \phi_\pi < 1 \) are still consistent with stability when \( \phi_x > 0 \).

(c) Write the system of equations above in the form \( E_t z_{t+1} = M z_t \) where \( z_t = [x_t, \pi_t]' \).

(d) How would you derive a condition involving \( \beta \), \( \phi_\pi \), \( \phi_x \) and \( \kappa \) that ensures a unique stationary equilibrium? Note that you are not asked to derive this condition.
3. Asset pricing (10 points). Consider an economy with a representative agent with expected utility, CRRA preferences:

\[ U = E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}. \]

Assume that log consumption growth is normal and iid, i.e.

\[ \Delta \log c_t = \mu + \sigma \varepsilon_t, \]

with \( \varepsilon_t \) iid \( N(0,1) \). Recall the log-normal formula: if \( X \) is \( N(\mu, \sigma^2) \), then \( Ee^X = e^{\mu + \frac{\sigma^2}{2}} \).

(a) Compute the price of a bond that pays one unit of good for sure in one year (= in one period). How is this price related to the risk-free rate? [Hint: one way to do it is as follows. let \( p_t \) be the bond price; consider maximizing the utility at time \( t \) by buying a quantity \( q \) of bonds:

\[ \max_q u(c_t - q p_t) + \beta E_t(c_{t+1} + q) + \ldots \]

use the first-order condition and that we know that in equilibrium \( q = 0 \) is optimal.]

(b) Compute the price of a bond that pays one unit of good for sure in two years (two periods). Deduce the interest-rate for this two-year bond. (Express the interest rate in interest rate per year).
(c) A stylized fact is that the yield curve is generally upward sloping, i.e. the per year interest rate on a two-period bond is larger than the one on a one-year. Does the model replicate this fact for some parameter value?

(d) A leading theory of the yield curve is the expectation hypothesis, i.e. the interest rate on a two-period bond is the weighted average of expected future one-period interest rates:

\[ r_{t \rightarrow t+2} = \frac{1}{2} (r_{t \rightarrow t+1} + E_t r_{t+1 \rightarrow t+2}), \]

where \( r_{t \rightarrow k} \) is the return per year on a bond bought at year \( t \) that delivers a unit of good in year \( k \). Does this relation hold in general in this model? Discuss briefly.
4. *Money in the utility function* (10 points)

Assume that utility is:

\[ U = \sum_{t=0}^{\infty} \beta^t u(c_t, \frac{M_t}{P_t}), \]

where \( c_t \) denotes real consumption, \( M_t \) money holdings and \( P_t \) the price level. Assume that money holdings chosen in period \( t \) do not yield utility until period \( t + 1 \). This is the key variation from the standard money-in-the-utility framework.

The budget constraint is:

\[ \omega_t = c_t + \frac{M_{t+1}}{P_t} + b_t + k_t, \]

where \( \omega_t \) denotes the household's real wealth, \( b_t \) the number of bonds and \( k_t \) the level of capital. The household's real wealth \( \omega_t \) is given by:

\[ \omega_t = f(k_{t-1}) + (1 - \delta)k_{t-1} + (1 + r_{t-1})b_{t-1} + m_t, \]

where \( f \) is the production function, \( \delta \) the depreciation rate of capital and \( r_t \) the real return on bonds.

(a) Write the value function. Hint: Assume that the value function \( V \) depends on \( \omega_t \) and \( m_t \) and plug the expression for \( \omega_{t+1} \) in the Bellman equation.
(b) Derive the first-order conditions for the household’s choices. Apply the envelope theorem.

(c) Show that

\[
\frac{u_m(c_{t+1}, m_{t+1})}{u_c(c_{t+1}, m_{t+1})} = i_t.
\]

and discuss how this condition relates to "the demand for money."
5. *Endogenous Growth* (10 points) Consider an economy in which there is a representative household who has preferences for consumption over time of the form

$$U = \int_0^\infty u(c_t)e^{-\rho t}dt$$

(1)

with $\rho > 0$ and $u(c) = \frac{1}{1-\sigma}c^{1-\sigma}$. Suppose that the technology is of the form

$$\frac{d}{dt}k_t = ak_t - c_t$$

(2)

with $a > 0$. Suppose further that the government levies income taxes on individuals, so that the individual’s budget constraint takes the form

$$\frac{d}{dt}k_t = (1 - \tau)ak_t - c_t$$

(3)

Suppose finally that tax revenues are used amount to pay for government purchases, so that there is a fiscal rule of the form

$$g_t = \tau ak_t$$

(4)

with $\tau$ exogenous.

(a) Determine the equilibrium growth rate in this economy and discuss how it depends on the rate of taxation.

(b) Suppose that the tax rate is increased. Does consumption rise or fall? Why?
6. *Investment with adjustment costs* (10 points): Consider a firm that chooses investment to maximize the expected present discounted value of future profit streams subject to quadratic costs of adjustment (assume per period profits are of the form $\Pi(K, A) - C(I)$ where $A$ is a profit shock and $C(I) = \frac{2}{\delta}I^2$ denotes adjustment costs). Define Tobin’s $Q$ as $Q = V(K, A)/K$ where $V(K, A)$ denotes firm value.

(a) Derive the investment first-order condition.

(b) Is the investment rate $(I/K)$ a function of Tobin’s $Q$?
B. Longer questions

1. *Immigration and Macroeconomic Activity* (60 points). There has been a great deal of discussion in recent months about the effect of immigration on the U.S. economy and society.

Consider a closed economy with a neoclassical production function, exogenous technical progress and a fixed saving rate and a fixed population (the Solow model), as described by the following equations

\[
\frac{d}{dt}K_t = sY_t - \delta K_t \\
Y_t = A_t K_t^\alpha N^{1-\alpha} \\
\frac{d}{dt}A_t = \gamma A_t
\]

where K is the capital stock; Y is output; A is productivity; and N is the population. The parameters are the saving rate (s), the depreciation rate (\( \delta \)) and the growth rate of technical progress (\( \gamma \)).

(a) What is the steady-state growth path of this economy?
(b) Suppose that the economy starts with a level of the capital stock below this path. What is the nature of the transitional dynamics of output, capital, the real wage and the real interest rate?
(c) Suppose that there were two closed economies that never-the-less share a border. If the only difference is that productivity is higher in the home country than the foreign country and each country is on its steady state path, what incentives would there be for workers to move to the home country from the foreign country? How large would these incentives be?
(d) Now suppose that workers move, so that the population of the home country is now $\theta N$ with $\theta > 1$ but that its capital stock is initially unchanged. What will be the immediate effects on output, the real wage and the real interest rate?
(e) Opponents of immigration sometimes argue that real wages will be permanently depressed by immigration. Is this view correct in the Solow model?
(f) Alternatively, suppose that the production function is \( Y_t = A_t K_t^\alpha N^\nu L^{1-\alpha-\nu} \), where \( L \) is the quantity of land. Supposing that the home country is initially on a steady-state path and that there is a movement of workers as above (the population of the home country is now \( \theta N \) with \( \theta > 1 \) but its capital stock is initially unchanged). What will be the immediate effects on output, the real wage and the real interest rate?
(g) Opponents of immigration sometimes argue that real wages will be permanently depressed by immigration. Is this view correct in this modification of the Solow model?
2. **Monetary Economics** (60 points).

Let $c_{1,t}$ and $c_{2,t}$ be respectively the household’s purchases of cash goods and credit goods in period $t$ and $\theta_t$ a random shock to the relative desirability of cash goods and credit goods (a proxy for a money demand shock). The household consists of a shopper and a seller. The seller receives a random endowment of $y_t$ unit of goods in period $t$. The shopper starts period $t$ with $m_t$ units of money and $b_t$ units of nominal bonds and proceeds to the financial market. There he receives nominal transfers of $T_t$ and engages in financial transactions which leave him with $z_t$ units of money which he takes to the cash goods market. The representative household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1,t}, c_{2,t}, \theta_t) - \phi(z_t/m_t),$$

subject to the following constraints:

$$z_t/p_t \equiv (m_t + b_t + T_t)/p_t - b_{t+1}/[(1 + R_t)p_t] \geq c_{1,t},$$

$$(m_t + b_t + T_t)/p_t - b_{t+1}/[(1 + R_t)p_t] - c_{1,t} + y_t - c_{2,t} - m_{t+1}/p_t \geq 0,$$

where $p_t$ denotes the price level and $R_t$ nominal interest rate. The function $\phi$ in the household preferences reflects a cost (in terms of disutility) of undertaking financial market transactions in order to change money balances. Assume that $\phi(1) = \phi'(1) = 0$ and that $\phi$ is convex. Thus the household could avoid these costs if the shopper chose to proceed directly to the cash-goods market with its starting money balances $m_t$. However, attempting to change the amount of cash to be taken to the cash-goods market requires financial market transactions which impose some costs.

(a) Denote $\beta^t \lambda_t$ and $\beta^t \mu_t$ the nonnegative multipliers associated respectively with constraints (1) and (2). Write the first-order conditions.
(b) Write the government budget constraint and the resource constraint. What is $z_t$ in equilibrium?
(c) Let \( x_t \) be the gross money growth rate in period \( t + 1 \). Write \( p_t/p_{t+1} \) as a function of \( c_{1,t}, c_{1,t+1} \) and \( x_{t+1} \).
(d) Use results in the three previous questions to obtain an expression in terms of $c_{1,t}, c_{1,t+1}, u_{2,t}, u_{1,t+1}, x_{t+1}$ and $\phi'$ that does not use the Lagrangian multipliers.
(e) Does the Fisher relation hold in the steady-state? Does it hold along the dynamic path?
(f) Now, consider optimal monetary policy (policy which maximizes the welfare in this economy). Write the maximization problem of the policy maker.
(g) Is the Friedman rule optimal?
(h) Is the solution to the maximization problem of the policy maker time-consistent?
3. Real business cycles with preference and productivity shocks (60 points). An economy consists of a representative household that makes choices over consumption $C_t$ and labor $N_t$ to maximize

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - (\theta_t N_t)^\sigma \right] \right]$$

subject to

$$K_{t+1} = R_t K_t + W_t N_t - C_t + \Pi_t$$

where $R_t$ is the rate of return on capital, $W_t$ is the real wage and $\Pi_t$ represent profits (dividends) paid out by firms. We thus assume that households own the capital stock which they rent to firms in this economy. Firms in this economy choose capital and labor inputs to maximize

$$\Pi_t = Y_t - R_t K_t - W_t N_t$$

subject to

$$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha}.$$ 

The aggregate resource constraint is

$$K_{t+1} + C_t = Y_t.$$ 

The exogenous stochastic processes evolve according to:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon^A_t$$

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \epsilon^\theta_t$$

$$-1 \leq \rho_A, \rho_\theta \leq 1$$

where $\epsilon^A_t, \epsilon^\theta_t$ are mean-zero iid random variables known at time $t$.

(a) Derive the equilibrium conditions for this economy.
(b) Provide expressions for the stochastic process for the log of output and the log of labor productivity as functions of current and past shocks (in terms of log-deviations from steady-state).
(c) How would you interpret a shock to $\theta_i$? Assuming $\text{var}(\epsilon_i^\theta) > 0$, based on your results in part b, can this model help explain empirical regularities not captured by the RBC model driven solely by technology shocks? Why or why not?
(d) Suppose $\rho_A = \rho_\theta = 1$. Compare the response of labor productivity to a 1% increase in $\varepsilon_t^A$ versus a 1% decrease in $\varepsilon_t^\theta$. In either case, how does labor productivity respond in the short-run versus the long-run. Does either shock have a permanent effect? Explain in words why such responses occur.
(e) Without computing the full solution, discuss how allowing for less than full depreciation \((0 < \delta < 1)\) modifies your answers regarding the effect of a shock to \(A_t\) on the economy. Provide some intuition for your results.