Instructions. There are five parts to this exam. They are all weighted equally. Some may seem longer than others, but that is only because their sub questions are smaller.

The exam is designed to take about three hours to be completed. The average time per question takes 36 minutes.

Budget your time carefully. Try to avoid leaving an answer blank. That may be costly.

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1. Consider an economy with a continuum (measure 1) of two-period-lived households. Household $i$ has utility function

$$\mathbb{E}_1 \left\{ \frac{c_{i,1}^{1-\rho}}{1-\rho} + \beta \frac{c_{i,2}^{1-\rho}}{1-\rho} \right\}$$

where $c_{i,t}$ is the consumption of household $i$ in period $t$. Households enter period 1 without any assets. In the first period, they trade a risk-free bond at interest rate $1+r$. This bond is in zero net supply. There is a borrowing constraint such that households cannot borrow at all. That is, $b_i \geq 0$ where $b_i$ is the savings of household $i$ in period 1.

In each period, all households supply one unit of labor and receive labor income $w_t s_{i,t}$ where $s_{i,t}$ is the effective skill of household $i$ in period $t$.

A representative firm produces a consumption good using the production function

$$y_t = AN_t$$

where $y_t$ is the production of the good, $N_t = \int s_{i,t}di$ is the aggregate effective labor input and $A$ is a productivity level.

In the first period, households have either low or high productivity ($s_L$ or $s_H$) with a fraction $q$ having low productivity and $1-q$ having high productivity. The households know their productivities when they make their choices for the first period. In the second period, all households have the same productivity, $s_2$. Assume $s_L < s_2 < s_H$. 

(a) What is the distribution of wealth at the start of the second period?
(b) What is the equilibrium allocation of consumption in the two periods?
(c) What is the equilibrium real interest rate?
(d) Suppose that the government makes a small transfer to the low-productivity group in the first period and finances this with debt. The debt will be repaid using a lump-sum tax in the second period paid by those who had high productivity in the first period. How do the distribution of wealth, the consumption allocation and real interest rate change? You may provide an economic answer or a mathematical answer. An economic answer can be as good (or even better) provided that it is clear and complete.
2. Consider an economy that consists of a unit continuum of individuals. At each date $t$, these individuals are endowed with different labor productivities. Let $s_t$ be the stochastic event at date $t$ and $s^t$ be the history of events that have occurred up to date $t$. Then the labor productivity of individual $i$ at date $t$ after history $s^t$ is $e^i_t(s^t)$. The individuals can decide how much labor to supply. Let $n^i_t(s^t)$ be the labor supply of individual $i$ after history $s^t$. An individual that has productivity $e$ and supplies $n$ units of labor is said to supply $e \times n$ effective units of labor.

The economy has a production process that is linear in effective labor

$$Y_t = A_t(s^t) \int n^i_t(s^t)e^i_t(s^t)di,$$

where $Y_t$ is output that is used for consumption. Output is perishable and must be consumed within the period in which it is produced. The “wage” of an individual is $A_t(s^t)e^i_t(s^t)$.

The consumers have identical preferences for consumption and labor supply given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left[ \log(c^i_t(s^t)) - \frac{n^i_t(s^t)^{1+\psi}}{1+\psi} \right] \pi(s^t|s^0),$$

where $\pi(s^t|s^0)$ is the probability that history $s^t$ is realized and $0 < 1/\psi < \infty$ is the Frisch elasticity of labor supply.

At date zero, the individuals trade a complete set of contracts for consumption and labor at all future dates and all histories. Let $q^0_t(s^t)$ be the price of a unit of consumption at date $t$ after history $s^t$ in terms of date zero consumption.

Assume that all of the individuals are identical at date zero when they agree on the contracts.
(a) Characterize the labor supply and consumption of each household.
(b) The wage of an individual can change with $A_t$ or $e^i_t$. Does $n^i_t$ respond to these two wage shocks in the same way? Why or why not? Discuss which features of the environment are important for your results.
3. Fiscal Policy in a Neoclassical Growth Model: Consider the following version of the neoclassical growth model where $\tau_t$ represents lump sum taxes paid by the representative agent to the government. There is no population growth, so the number of agents in the economy remains constant over time.

$$
U = \sum_{t=0}^{\infty} \beta^t \frac{(c_t - \psi n_t^\theta)^{1-\sigma} - 1}{1 - \sigma}
$$

$$
y_t = c_t + i_t + \tau_t
$$

$$
y_t = A k_t^{1-\alpha} n_t^\alpha
$$

$$
k_{t+1} = i_t + (1-\delta) k_t
$$

where $0 < \beta < 1$, $\theta > 1$, $\psi > 0$, $0 < \alpha < 1$, and $\sigma > 0$.

The government runs a balanced budget,

$$
\tau_t = g_t,
$$

where $g_t$ represents the level of government spending.
(a) Assume that $g_t = g$ is constant. Compute the steady state levels of output, capital, and consumption.
(b) Suppose that the economy is in the steady state and learns that there will be an increase in $g$ in the future. What is the immediate impact today of this news on the supply of labor and the level of output?
(c) Now suppose that life-time utility takes the form:
\[
U = \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) + \theta \log(1 - n_t) \right]
\]
Assume, in addition that \( \delta = 1 \) and that taxes and government spending are proportional to output:
\[
\tau_t = g_t = \gamma y_t.
\]
Show that the solution to the model takes the form
\[
\begin{align*}
c_t &= \mu y_t \\
n_t &= \eta
\end{align*}
\]
where \( \mu \) and \( \eta \) are constants. Compute the value of \( \mu \). What effect does \( \gamma \) have on labor supply \( \eta \), the savings rate and the growth rate of capital \( \frac{k_{t+1}}{k_t} \)?
(d) Would the result that you just found in part (d) obtain if \( \delta < 1 \)? If so, why? If not, what alternative result would be more standard?
4. Suppose the economy is characterized by the following system of equations:

\[
\begin{align*}
IS & : \quad x_t = -(i_t - E_t \pi_{t+1}) + E_t(x_{t+1}) \\
AS & : \quad \pi_t = \lambda x_t + \beta E_t(\pi_{t+1}) + u_t
\end{align*}
\]

where \( x_t \) is the output gap, \( i_t \) is the nominal interest rate and \( \pi_t \) is the inflation rate. Further assume that \( u_t \) satisfies:

\[
u_t = \rho u_{t-1} + e_t^u
\]

where \( 0 \leq \rho < 1 \) and \( e_t^u \) is a mean zero, iid random variable. Suppose that the monetary authority chooses the nominal interest rate to maximize

\[
L_o = -\frac{1}{2} E_t \sum_{t=0}^\infty \beta^t (\alpha(x_{t+i} - k)^2 + \pi_{t+i}^2)
\]

where \( k > 0 \) is the socially desirable level of \( x_t \). For simplicity assume that \( \beta = 1 \).
(a) Derive the F.O.C. for optimal monetary policy in the absence of commitment. Explain the economic intuition behind this optimality condition.
(b) What is the long-run rate of inflation in this model? Does it deviate from the target rate of zero inflation implied by the loss function? If so, what factors contribute to the deviation?
(c) Characterize the effect of a shock to $u_t$ on inflation and the output gap. Explain the economic intuition behind these results.
(d) Assuming the true loss function to society is indeed $L_o$, suppose the govt. can appoint a central banker who sets monetary policy to minimize the following loss function:

$$L^c = -\frac{1}{2}E_t \sum_{t=0}^{\infty} \beta^t (\alpha^c (x_{t+i} - k)^2 + \pi_{t+i}^2)$$

where $\alpha^c < \alpha$. Suppose $k = 0$. Would appointing such a central banker be socially beneficial? Is this still true if $k > 0$? Explain.
5. There is an economy with overlapping generations. Population is stationary and normalized to 1 per generation. Each generation lives two periods, supplies a fixed quantity of labor in the first period that is normalized to 1 and consumes in both periods. Each individual has a utility function \( u(c_1, c_2) = (1 - a) \log(c_1) + a \log(c_2) \). The unique good is produced by a technology that is represented by the function

\[
Y = K^\alpha L^\beta T^{1-\alpha-\beta}, \quad \alpha + \beta < 1,
\]

where \( K \) is the capital (that does not depreciate), \( L \) is labor and \( T \) is the quantity of land and is in fixed supply, \( T = 1 \). Factors are paid their marginal productivity and the economy is perfectly competitive.
(a) Determine the steady state rate of return of capital, $r$. You will call the price of land $p$. Provide an intuitive explanation for the qualitative impact of $a$ and $\beta$ on $r$.

\[
\begin{align*}
\begin{cases}
  a\beta Y &= K + p \\
  r &= \frac{\alpha Y}{K} = (1 - \alpha - \beta) \frac{Y}{p}.
\end{cases}
\end{align*}
\]

\[
p = \frac{1 - \alpha - \beta}{\alpha} K, \quad a\beta \frac{Y}{K} = \frac{1 - \beta}{\alpha}.
\]

\[
r = \frac{1 - \beta}{a\beta}.
\]
(b) Assume that a tax is put on the income of land at the rate \( \theta \). The proceeds of the tax are consumed by the government. Analyze the impact of the tax on the price of land in the steady state.
(c) Assume now that there is no tax and that the economy is not in a steady state. Write the dynamic equations. Explain clearly how you would analyze the stability of the steady state, without solving the algebra.

\[
\begin{align*}
& a\beta K_t^\alpha = K_{t+1} + p_t \\
& 1 + \alpha K_{t+1}^{\alpha-1} = \frac{1}{p_t} (p_{t+1} + \frac{1 - \alpha - \beta}{K_{t+1}^{\alpha+\beta}}).
\end{align*}
\]