The Basic New Keynesian Model

by

Jordi Galí

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Motivation and Outline

Evidence on Money, Output, and Prices:

- Short Run Effects of Monetary Policy Shocks
  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

- Micro Evidence on Price-setting Behavior: significant price and wage rigidities

Failure of Classical Monetary Models

A Baseline Model with Nominal Rigidities

- monopolistic competition
- sticky prices (staggered price setting)
- competitive labor markets, closed economy, no capital accumulation
Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U (C_t, N_t)$$

where

$$C_t \equiv \left[ \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i)C_t(i) \, di + Q_tB_t \leq B_{t-1} + W_tN_t - T_t$$

for $t = 0, 1, 2, \ldots$ plus solvency constraint.
Optimality conditions

1. Optimal allocation of expenditures

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]

implying

\[ \int_0^1 P_t(i)C_t(i) \, di = P_t C_t \]

where

\[ P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \]

2. Other optimality conditions

\[ \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \]

\[ Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \]
Specification of utility:

\[ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} \]

implied log-linear optimality conditions (aggregate variables)

\[ w_t - p_t = \sigma c_t + \varphi n_t \]
\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \]

where \( i_t \equiv -\log Q_t \) is the *nominal interest rate* and \( \rho \equiv -\log \beta \) is the *discount rate.*

Ad-hoc money demand

\[ m_t - p_t = y_t - \eta i_t \]
Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology
  \[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]
- Probability of resetting price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$
Aggregate Price Dynamics

\[ P_t = \left[ \theta(P_{t-1})^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]

Dividing by \( P_{t-1} \):

\[ \Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \]

Log-linearization around zero inflation steady state

\[ \pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (1) \]

or, equivalently

\[ p_t = \theta p_{t-1} + (1 - \theta)p_t^* \]
Optimal Price Setting

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{Q_{t,t+k} \left( P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \}
\]

subject to

\[
Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k}
\]

for \( k = 0, 1, 2, \ldots \) where

\[
Q_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)
\]

Optimality condition:

\[
\sum_{k=0}^{\infty} \theta^k E_t \{Q_{t,t+k} Y_{t+k|t} \left( P_t^* - M \Psi_{t+k|t} \right) \} = 0
\]

where \( \psi_{t+k|t} \equiv \psi'_{t+k}(Y_{t+k|t}) \) and \( M \equiv \frac{\epsilon}{\epsilon - 1} \)
Equivalently,

\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( \frac{P^*_t}{P_{t-1}} - \mathcal{M}MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0
\]

where \( MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k} \) and \( \Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1} \)

**Perfect Foresight, Zero Inflation Steady State:**

\[
\frac{P^*_t}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\mathcal{M}}
\]
Log-linearization around zero inflation steady state:

\[ p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \hat{m}c_{t+k|t} + p_{t+k} - p_{t-1} \} \]

where \( \hat{m}c_{t+k|t} \equiv mc_{t+k|t} - mc \).

Equivalently,

\[ p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + p_{t+k} \} \]

where \( \mu \equiv \log \frac{\epsilon}{\epsilon - 1} \).

Flexible prices (\( \theta = 0 \)):

\[ p_t^* = \mu + mc_t + p_t \]

\[ \implies mc_t = -\mu \text{ (symmetric equilibrium)} \]
**Particular Case:** $\alpha = 0$ (constant returns)

$$\implies MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta \theta E_t\{p_{t+1}^*\} + (1 - \beta \theta)\widehat{mc}_t + (1 - \beta \theta)p_t$$  \hspace{1cm} (2)

Combining (1) and (2):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta}$$
Generalization to $\alpha \in (0, 1)$ (decreasing returns)

Define

$$mc_t \equiv (w_t - p_t) - m \rho n_t$$

$$\equiv (w_t - p_t) - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log(1 - \alpha)$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1 - \alpha} (a_{t+k} - \alpha y_{t+k|t}) - \log(1 - \alpha)$,

$$mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k})$$

$$= mc_{t+k} - \frac{\alpha \epsilon}{1 - \alpha} (p^*_t - p_{t+k})$$ (3)

Implied inflation dynamics

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \hat{mc}_t$$ (4)

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon}$$
Equilibrium

Goods markets clearing

\[ Y_t(i) = C_t(i) \]

for all \( i \in [0, 1] \) and all \( t \).

Letting \( Y_t \equiv \left( \int_0^1 Y_t(i)^{\frac{1-\epsilon}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \),

\[ Y_t = C_t \]

for all \( t \). Combined with the consumer’s Euler equation:

\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \] (5)
**Labor market clearing**

\[
N_t = \int_0^1 N_t(i) \, di
\]

\[
= \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di
\]

\[
= \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di
\]

Taking logs,

\[
(1 - \alpha)n_t = y_t - a_t + d_t
\]

where \( d_t \equiv (1 - \alpha) \log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}} \, di \) (second order).

Up to a first order approximation:

\[
y_t = a_t + (1 - \alpha) n_t
\]
Marginal Cost and Output

\[ mc_t = (w_t - p_t) - m pn_t \]
\[ = (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1 - \alpha) \]
\[ = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \] (6)

Under flexible prices

\[ mc = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y^n_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \] (7)

\[ \implies y^n_t = -\delta_y + \psi_{ya} a_t \]

where \( \delta_y \equiv \frac{(\mu - \log(1 - \alpha))(1 - \alpha)}{\sigma + \varphi + \alpha(1 - \sigma)} > 0 \) and \( \psi_{ya} \equiv \frac{1 + \varphi}{\sigma + \varphi + \alpha(1 - \sigma)} \).

\[ \implies \hat{mc}_t = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y^n_t) \] (8)

where \( y_t - y^n_t \equiv \tilde{y}_t \) is the output gap
New Keynesian Phillips Curve

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]  \hspace{1cm} (9)

where \( \kappa \equiv \lambda \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \).

Dynamic IS equation

\[ \tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma} \left( i_t - E_t \{ \pi_{t+1} \} - r^n_t \right) \]  \hspace{1cm} (10)

where \( r^n_t \) is the natural rate of interest, given by

\[ r^n_t \equiv \rho + \sigma \ E_t \{ \Delta y^n_{t+1} \} \]
\[ = \rho + \sigma \psi_{ya} E_t \{ \Delta a_{t+1} \} \]

Missing block: description of monetary policy (determination of \( i_t \)).
Equilibrium under a Simple Interest Rate Rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \quad (11) \]

where \( v_t \) is exogenous (possibly stochastic) with zero mean.

Equilibrium Dynamics: combining (9), (10), and (11)

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = \mathbf{A}_T
\begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix} + \mathbf{B}_T (\tilde{r}_t - v_t) \quad (12)
\]

where

\[
\mathbf{A}_T \equiv \Omega \begin{bmatrix}
\sigma & 1 - \beta \phi_\pi \\
\sigma \kappa & \kappa + \beta (\sigma + \phi_y)
\end{bmatrix} ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix}
1 \\
\kappa
\end{bmatrix}
\]

and \( \Omega \equiv \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \)
Uniqueness \iff A_T has both eigenvalues within the unit circle

Given $\phi_\pi \geq 0$ and $\phi_y \geq 0$, (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

is necessary and sufficient.
Effects of a Monetary Policy Shock
Set \( \tilde{r}_t^n = 0 \) (no real shocks).
Let \( v_t \) follow an AR(1) process
\[
v_t = \rho_v v_{t-1} + \varepsilon_t^v
\]

Calibration:
\[
\rho_v = 0.5, \phi_\pi = 1.5, \phi_y = 0.5/4, \beta = 0.99, \sigma = \varphi = 1, \theta = 2/3, \eta = 4.
\]

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).
Exercise: analytical solution
Effects of a Technology Shock

Set \( v_t = 0 \) (no monetary shocks).

Technology process:

\[
a_t = \rho_a a_{t-1} + \varepsilon_t^a.
\]

Implied natural rate:

\[
\tilde{r}_t^n = -\sigma \psi_y a(1 - \rho_a) a_t
\]

Dynamic effects of a technology shock (\( \rho_a = 0.9 \)) (Figure 2)

Exercise: AR(1) process for \( \Delta a_t \)
Equilibrium under an Exogenous Money Growth Process

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \epsilon_t^m \]  \hspace{1cm} (13)

Money market clearing

\[ \hat{l}_t = \hat{y}_t - \eta \hat{i}_t \]  \hspace{1cm} (14)

\[ \hat{l}_t = \hat{y}_t + \hat{y}_t^n - \eta \hat{i}_t \]  \hspace{1cm} (15)

where \( l_t \equiv m_t - p_t \) denotes (log) real money balances.

Substituting (14) into (10):

\[ (1 + \sigma \eta) \hat{y}_t = \sigma \eta E_t \{ \hat{y}_{t+1} \} + \hat{l}_t + \eta E_t \{ \pi_{t+1} \} + \eta \hat{r}_t^m - \hat{y}_t^n \]  \hspace{1cm} (16)

Furthermore, we have

\[ \hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t \]  \hspace{1cm} (17)
Equilibrium dynamics

\[
\mathbf{A}_{M,0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_{t-1} \end{bmatrix} = \mathbf{A}_{M,1} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{l}_{t-1} \end{bmatrix} + \mathbf{B}_M \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta m_t \end{bmatrix}
\]

(18)

where

\[
\mathbf{A}_{M,0} \equiv \begin{bmatrix} 1 + \sigma \eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} ; \quad \mathbf{A}_{M,1} \equiv \begin{bmatrix} \sigma \eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad \mathbf{B}_M \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\]

Uniqueness \(\iff\) \(\mathbf{A}_M \equiv \mathbf{A}_{M,0}^{-1}\mathbf{A}_{M,1}\) has two eigenvalues inside and one outside the unit circle.
Effects of a Monetary Policy Shock
Set \( \hat{r}_t^n = y_t^n = 0 \) (no real shocks).

Money growth process

\[
\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m
\]

where \( \rho_m \in [0, 1) \)

Figure 3 (based on \( \rho_m = 0.5 \))

Effects of a Technology Shock
Set \( \Delta m_t = 0 \) (no monetary shocks).

Technology process:

\[
a_t = \rho_a a_{t-1} + \varepsilon_t^a
\]

Figure 4 (based on \( \rho_a = 0.9 \)).

Empirical Evidence
Technical Appendix

Optimal Allocation of Consumption Expenditures

Maximization of $C_t$ for any given expenditure level $\int_0^1 P_t(i) C_t(i) \, di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[ \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda \left( \int_0^1 P_t(i) C_t(i) \, di - Z_t \right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}} C_t^\frac{1}{\epsilon} = \lambda P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods $(i,j)$ we have:

$$C_t(i) = C_t(j) \left( \frac{P_t(i)}{P_t(j)} \right)^{-\epsilon}$$

which can be plugged into the expression for consumption expenditures to yield

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Z_t P_t$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of $C_t$, yielding

$$\int_0^1 P_t(i) C_t(i) \, di = P_t C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Log-Linearized Euler Equation
We can rewrite the Euler equation as
\[ 1 = E_t\{\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho)\} \] (19)

In a perfect foresight steady state with constant inflation \( \pi \) and constant growth \( \gamma \) we must have:
\[ i = \rho + \sigma \gamma + \pi \]

with the steady state real rate being given by
\[ r \equiv i - \pi = \rho + \sigma \gamma \]

A first order Taylor expansion of \( \exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho) \) around that steady state yields:
\[ \exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho) \simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \]
\[ = 1 + i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho \]

which can be used in (19) to obtain, after some rearrangement of terms, the log-linearized Euler equation
\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \]

**Aggregate Price Level Dynamics**

Let \( S(t) \subset [0, 1] \) denote the set of firms which do not re-optimize their posted price in period \( t \). The aggregate price level evolves according to

\[ P_t = \left[ \int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]
\[ = \left[ \theta(P_{t-1})^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]
where the second equality follows from the fact that the distribution of prices among firms not adjusting in period $t$
corresponds to the distribution of effective prices in period $t - 1$, with total mass reduced to $\theta$.
Equivalently, dividing both sides by $P_{t-1}$:

\[ \Pi_{t}^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{20} \]

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1}$.
Log-linearization around a zero inflation ($\Pi = 1$) steady state implies:

\[ \pi_t = (1 - \theta)(p_t^* - p_{t-1}) \tag{21} \]

**Price Dispersion**

From the definition of the price index:

\[ 1 = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\epsilon} \, di \]

\[ = \int_0^1 \exp\{(1 - \epsilon)(p_t(i) - p_t)\} \, di \]

\[ \simeq 1 + (1 - \epsilon) \int_0^1 (p_t(i) - p_t) \, di + \frac{(1 - \epsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 \, di \]

thus implying the second order approximation

\[ p_t \simeq E_t\{p_t(i)\} + \frac{(1 - \epsilon)}{2} \int_0^1 (p_t(i) - p_t)^2 \, di \]
where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i) \, di$ is the cross-sectional mean of (log) prices.
In addition,

$$\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di \approx \int_0^1 \exp \left\{ -\frac{\epsilon}{1-\alpha} (p_t(i) - p_t) \right\} \, di$$

$$\approx 1 - \frac{\epsilon}{1-\alpha} \int_0^1 (p_t(i) - p_t) \, di + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 \, di$$

$$\approx 1 + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \int_0^1 (p_t(i) - p_t)^2 \, di$$

$$\approx 1 + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \text{var}_i\{p_t(i)\} > 1$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\epsilon \alpha}$, and where the last equality follows from the observation that, up to second order,

$$\int_0^1 (p_t(i) - p_t)^2 \, di \approx \int_0^1 (p_t(i) - E_i\{p_t(i)\})^2 \, di$$

$$\equiv \text{var}_i\{p_t(i)\}$$

Finally, using the definition of $d_t$ we obtain

$$d_t \equiv (1 - \alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di \approx \frac{1}{2} \frac{\epsilon}{\Theta} \text{var}_i\{p_t(i)\}$$