Boston University


May 23, 2011

Instructions:

1. The exam has three parts:
   (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   You are required to answer all three parts of the exam.

2. A total of three hours is allowed for the exam.
   It is recommended, but not mandatory, that you budget one hour for each part of the exam.

3. Do not write your name or ID number on the exam. It has already been coded.
   Write your answers for each part in the space provided. Do not write on the back of the page. If you run out of space, ask for additional paper and attach it to the exam.
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   Use of calculator is permitted.

4. This is PART (I) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
PART I: MICROECONOMICS (2 QUESTIONS)

I.1 The demand for widgets is

\[ Q = 100 - P \]

where \( P \) is the price of widgets. There are two firms in the industry: Acme Widget Corp and Best Widget Corp. They each have the following cost function for producing widgets:

\[ C_i = 10 q_i + \frac{1}{2} q_i^2 \]

where \( q_i \) is the level of output of firm \( i \).

(a) Suppose the firms behave as price-taking perfect competitors, choosing quantities. What will be the short-run equilibrium price and quantity produced of widgets? How much will each firm produce?

(b) Suppose the firms agree to collude and maximize their joint profits. Now what will be the short-run equilibrium price and quantity produced of widgets? How much will each firm produce?

(c) Who gains and who loses as a result of this collusion? Calculate the respective gains and losses.

(d) How much better off or worse off is society as a whole as a result of this collusion? Draw a graph that illustrates as clearly as possible the net change in social welfare.
1.2 Daniel Archer is a family farmer. He has set aside enough money to look after his family for the next six months and finds he has $250,000 available to invest. He has three options:

(i) Put the money in a 6-month bank deposit that will pay interest of 5% at the end of six months.
(ii) Buy regular seed for $200,000 and put the remainder of the money in a 6-month bank deposit like the one in (i). He would then plant the seed and harvest a crop in 6 months’ time. The crop would be valued at $500,000 if the rains are good, but only $50,000 if there is a drought. The probability of a good rainy season is 0.7 and the probability of a drought is 0.3.
(iii) Buy drought-resistant seed for $250,000 and plant that instead. The harvest would again be in 6 months, but the yields with the drought-resistant seed will be worth $400,000 if the rains are good and $350,000 if there is a drought.

Archer wants to maximize his expected utility of wealth, which is

\[ U(W) = W^{1/2} \]

where \( W \) is the family’s wealth in 6 months’ time.

(a) Which of the three options will Archer choose? Explain fully.

(b) Suppose Archer could buy crop insurance if he plants regular seed. The premium would be due at the time of the harvest and the insurance would pay him $450,000 if there had been a drought. What is the actuarially fair premium for this insurance? What is Archer’s maximum willingness to pay for it? Explain fully.
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4. This is PART (II) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
PART II: MACROECONOMICS (2 QUESTIONS)

II.1 Solow Model. Let $A(t)$, $L(t)$, $K(t)$, and $Y(t)$ denote the level of technology, labor, capital, and output produced at time $t$. Assume that technology and labor evolve according to:

$$\frac{dA}{dt} = gA(t)$$
$$\frac{dL}{dt} = nL(t)$$

where $g$ and $n$ are the exponential rates of growth. Assume that the production function of the economy is Cobb-Douglas, with $0 < \alpha < 1$:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

Consider a model where households save a constant fraction of income $sY(t)$, where $s$ denotes the savings rate, and consume all other output, so consumption satisfies $C(t) = (1-s)Y(t)$, and capital accumulation satisfies $\frac{dK}{dt} = sY(t) - \delta K(t)$, where $\delta$ denotes the instantaneous rate of depreciation.

a) Derive the equation that describes the evolution of capital per effective unit of labor over time: $k(t) = K(t)/[A(t)L(t)]$. Solve for the balanced growth path level of $k(t)$, or $k^*$, as well as $y^*$ and $c^*$ output and consumption per effective unit of labor on the balanced growth path.

b) Plot actual investment versus break-even investment as a function of $k(t)$, labeling each line appropriately. Show the effect of an increase in $g$ on the balanced growth path level of $k(t)$, call this $k_{high, g}^*$. Does the balanced growth path level of $k(t)$ rise or fall in response to the increase in the growth rate of technology? Draw a graph that shows the time path of $k(t)$ after the increase in $g$.

c) Now suppose that the savings rate of the economy, $s$, is always equal to $g$, so that the increase in $g$ is accompanied by an increase in $s$. Draw another graph that shows the effect of both increases on the actual investment and break-even investment lines. Can it be determined whether the new balanced growth path level of $k(t)$, or $k_{high, g=s}^*$, is higher or lower than $k^*$?

d) Now consider the effect of these changes on the balanced growth path level of $c(t)$. Does $c(t)$ rise or fall in response to the increase in $g$? (Or in other words, is $c_{high, g}$ larger or smaller than $c^*$?) Also, is $c_{high, g=s}$ higher or lower than $c_{high, g}$? Explain.
II.2 **Real Business Cycle model.** Let \( A(t) \), \( N(t) \), and \( Y(t) \) denote the levels of technology, labor, and output produced at time \( t \). Assume that the production function is

\[
Y(t) = A(t)N(t)^{1-\sigma},
\]

and that both goods and labor markets are competitive.

The representative agent's problem is to maximize:

\[
\sum_{t=0}^{\infty} \beta^t u(C_t, N_t),
\]

where \( u(C_t, N_t) = \log \left( C_t - \frac{N_t^{1+\phi}}{1+\phi} \right) \), subject to the budget constraint:

\[
S_{t+1} = (1+r_t)S_t + w_tN_t + \pi_t - C_t,
\]

where \( \pi_t \) is firm's profits, and \( r_t \) is the real interest rate, and \( S_t \) is real savings.

Equilibrium in the goods market requires that \( C_t = Y_t \) i.e. \( S_t = 0 \).

a) Write down the first-order conditions for the optimal choice of labor by the household and the firm.

b) Write down the first-order condition for the optimal choice of savings by the household.

c) Using the equilibrium condition, find the equilibrium employment, output, consumption and the real wage, as function of productivity \( A(t) \).

d) Using (c), explain the effect of an increase in productivity on employment, output, consumption and the real wage.

e) From now on, assume that \( \log A(t) \) is i.i.d. and normally distributed with mean 0 and variance \( \sigma^2 \). Give an expression for the variance of log employment, \( Var(\log N(t)) \), and for the variance of log output, \( Var(\log Y(t)) \). Which is more volatile?

f) Is labor productivity \( Y(t) / N(t) \) procyclical or countercyclical? Explain.
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4. This is PART (III) of the exam and you should answer TWO QUESTIONS, as follows:
   (Econometrics) Answer only 1 question*, III-B1 or III-B2.
   *Do not submit answers to more than one question. You will not receive any credit if the examiners cannot determine which question you intend to answer.

Good Luck!
PART III-A: STATISTICS (1 QUESTION: ANSWER III-A1)

III-A1

(A) True or False. To show that a statement is false, you can give a counterexample. Please avoid long explanations (but, no explanations no points).

i. The sample mean is an unbiased estimator of the population mean. (Assume that the population mean exists.)

ii. The sample standard deviation (square root of the sample variance) is an unbiased estimator of the population standard deviation (square root of the population variance). (Assume that the population variance exists and is strictly positive.)

iii. If \( \hat{\theta} \) is an inconsistent estimator for the parameter \( \theta \), then \( \hat{\theta} \) is also a biased estimator for \( \theta \).

iv. If we construct a 95% confidence interval for the mean many times for different samples from the same population, then the proportion of confidence intervals containing the sample mean converges to 95%.

v. In hypothesis testing, the sum of the probability of type I error and the probability of type II error is one.

(B) Let \( Z_1, \ldots, Z_4 \) be a random sample of size 4 from a random variable \( Z \sim N(0,1) \).

i. Find \( c \) such that the statistic

\[
\frac{cZ_1}{\sqrt{Z_2^2 + Z_3^2 + Z_4^2}}
\]

follows a t-student distribution.

ii. What are the degrees of freedom of this t-student distribution?
III-B1

The following population regression function is estimated by ordinary least squares on a large sample:

\[ LY_i = \beta_0 + \beta_1 \text{GND}_i + \beta_2 \text{XD1}_i + \beta_3 \text{XD3}_i + \beta_4 Z_i + \beta_5 GZ_i + \epsilon_i \]

where

- \( LY_i \) = the natural logarithm of \( Y \) for person \( i \);
- \( \text{GND}_i = 1 \) if person \( i \) is male, 0 if female;
- \( X_i \) = a continuous variable with values from 20 through 95;
- \( \text{XD1}_i = 1 \) if \( X_i < 30 \), 0 otherwise;
- \( \text{XD3}_i = 1 \) if \( X_i > 50 \), 0 otherwise;
- \( Z_i \) = a continuous variable with values from 0 through 18;
- \( GZ_i = \text{GND}_i \times Z_i \).

a. Avoiding the use of the word logarithm, state in words what the estimate of each of the parameters, \( \beta_0, \beta_1, \beta_2, \beta_3, \beta_4 \) and \( \beta_5 \), would mean.

b. Suggest TWO alternative ways of testing whether the value of \( LY \) is greater when \( X > 50 \) than when \( X < 30 \), given gender and \( Z \). You should describe both the formula for the test statistics that your propose as well as the degrees of freedom for your tests.

c. You suspect that \( \epsilon_i \) may have a heteroskedastic distribution. How might you test for this? Be as precise as possible in describing your test.

(III-B2 on next page.)
Consider the simultaneous equation system with three endogenous variables and two exogenous variables:

\[ Y_1 = a_1 Y_2 + a_2 Y_3 + a_3 X_1 + u_1 \]  
\[ Y_2 = b_1 Y_1 + b_2 X_2 + u_2 \]  
\[ Y_3 = c_1 Y_1 + c_2 X_1 + u_3 \]  

a) Explain precisely why it is inappropriate to estimate the second equation via OLS.
b) Not all equations are ‘identified’ in this system. What does it mean to say that an equation is ‘identified’?
c) Which equations are identified and which ones aren’t?
d) Suppose you have access to software that can only run OLS. Describe how exactly you will estimate any equation that is identified via 2SLS.
e) Will your estimates be unbiased? Consistent? Efficient?