Boston University

Comprehensive Examination
for M.A.E.P. and M.A. in Economics

May 22, 2008

Instructions:

1. The exam has three parts: (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   You are required to answer all three parts of the exam.

2. A total of three hours is allowed for the exam.
   It is recommended, but not mandatory, that you budget one hour for each part of the exam.

3. Do not write your name or ID number on the exam. It has already been coded.
   Write your answers for each part in the space provided. Do not write on the back of the page. If you run out of space, ask for additional paper and attach it to the exam.
   If the sheets of the exam get separated, be sure to re-attach them before submitting your exam.
   You will receive a blue book for scratch work. It will be collected at the end of the exam. Do not write your answers in the scratch book. It will be ignored in the grading.
   Write legibly: you will not get credit if the examiners cannot understand what you write.
   Use of calculator is permitted.

4. This is PART (I) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
PART I: MICROECONOMICS (2 QUESTIONS)

I.1 [50 points] Barack consumes only two goods, x and y, and his utility function is

\[ U(x, y) = xy + y. \]

Let \( p_x \) and \( p_y \) represent the prices of \( x \) and \( y \) respectively and let \( I \) represent Barack's income.

(a) Find Barack’s ordinary demand functions for \( x \) and \( y \).
(b) Find Barack’s compensated demand functions for \( x \) and \( y \).
(c) Suppose that Barack’s income is $100, the price of \( y \) is $10 per unit and the price of \( x \) rises from $20 to $25 per unit. Find the minimum amount of money Barack would have to be given after the price increase in order to make him just as well off as he was before the increase (the Compensating Variation of this price change).
I.2 [50 points] The market demand for widgets is given by:

$$Q_d = 2000 - 100P$$

where $P$ and $Q_d$ are the price and the market demand respectively.

The per-firm cost function for producing widgets is

$$C(q) = 0.5q^2 - 10q + 200$$

where $q$ is the firm-level output.

(a) In the short run, there are 100 price-taking firms operating in the industry. What will be the short run equilibrium price and quantity of widgets? How much will each firm produce, and how much profit will each firm make?

(b) What will be the long run equilibrium price and quantity of widgets? How much will each firm produce, and how many firms will operate in the industry?

(c) Suppose that the production of each widget releases 0.1 unit of carbon and that this carbon emission results in environmental damage according to the function

$$E(x) = 0.5 x^2$$

where $x$ is the total amount of carbon emitted and $E(x)$ is the damage cost of that emission. What is the socially optimal number of widgets that should be produced in the long run under these circumstances? How many firms should there be in the industry?

(d) How much is the deadweight loss in your solution to (b) under the circumstances of part (c)? What tax on widgets would lead to achievement of the social optimum (Pigouvian tax)?
Boston University

Comprehensive Examination for M.A.E.P. and M.A. in Economics

May 22, 2008

Instructions:

1. The exam has three parts:
   (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   You are required to answer all three parts of the exam.

2. A total of three hours is allowed for the exam.
   It is recommended, but not mandatory, that you budget one hour for each part of the exam.

3. Do not write your name or ID number on the exam. It has already been coded.
   Write your answers for each part in the space provided. Do not write on the back of the page. If you run out of space, ask for additional paper and attach it to the exam.
   If the sheets of the exam get separated, be sure to re-attach them before submitting your exam.
   You will receive a blue book for scratch work. It will be collected at the end of the exam. Do not write your answers in the scratch book. It will be ignored in the grading.
   Write legibly; you will not get credit if the examiners cannot understand what you write.
   Use of calculator is permitted.

4. This is PART (II) of the exam and it has TWO QUESTIONS.
   Answer both questions.

GOOD LUCK!
1. Suppose households make consumption-savings decisions to maximize

\[ \sum_{t=0}^{\infty} \beta^t E_0(U(C_t)) \]

subject to

\[ A_{t+1} = (1 + r)A_t + W_t - C_t \]

where \( A_t \) denotes financial assets (savings), \( W_t \) denotes the wage. Assume that \( W_t = W_{t-1} + \varepsilon_t \) and \( \varepsilon_t \) is a mean-zero serially uncorrelated shock to income. Let \( \beta = \frac{1}{1+r} \) and

\[ U(C) = aC - \frac{b}{2}C^2 \]

(a) Show that the optimal consumption savings decision implies:

\[ E_t(C_{t+1}) = C_t \]

Explain this condition.
(b) Derive the relationship between consumption and wealth where wealth is defined as financial wealth plus the expected present discounted value of wages.
(c) What effect does a $1 increase in the current wage have on current consumption? Explain.
(d) What is the expected value of the regression coefficient $\gamma$ in the regression:

$$\Delta C_{t+1} = \gamma \Delta W_t + \nu_{t+1}$$

Is it zero? Why or why not? Explain the economic intuition behind your answer.
(e) What effect does an increase in future income uncertainty have on current consumption and current savings in this model? (i.e. suppose \(\operatorname{var}(\varepsilon_{t+h}) = \mathbb{E}(\varepsilon_{t+h}^2)\) the variance of future income shocks increases). Explain.
2. Consider the following general equilibrium model. Households choose labor supply $N_t$, bond holdings $B_t$ and money balances $M_t$ to solve

$$\max E_t \left( \sum_{s=0}^{\infty} \beta^s \left[ \log(C_{t+s}) + \gamma \log \left( \frac{M_{t+s}}{P_{t+s}} \right) - \frac{N_{t+s}^{1+\phi}}{1+\phi} \right] \right)$$

subject to the budget constraint in nominal terms:

$$B_{t+1} + M_t = (1 + i_t) B_t + M_{t-1} + W_t N_t + \Pi_t - P_t C_t + T_t^M$$

where $B_t$ represents nominal bonds, $i_t$ denotes the nominal interest rate, $W_t$ denotes the nominal wage, and $P_t C_t$ denotes nominal expenditures on consumption. Here $P_t$ is the price index. We assume that households own the firms in the economy and receive the profits from those firms each period. The term $\Pi_t$ denotes nominal profits (dividends) that the households receive from firms while $T_t^M$ denotes monetary transfers. There are many firms in this economy who produce according to a constant return to scale production function:

$$Y_t = N_t$$

Assume that the monetary authority sets the growth rate of money to satisfy:

$$\ln M_t - \ln M_{t-1} = u_t^M.$$

where $u_t^M$ is a mean-zero, iid normally-distributed random variable.
(a) Show that households choose their bond holdings \( B_t \), their labor supply \( N_t \) and their real balances \( M_t/P_t \) to satisfy:

\[
\frac{1}{P_tC_t} = (1 + i_{t+1})\beta E_t \frac{1}{P_{t+1}C_{t+1}}
\]

\[
\frac{W_t}{P_tC_t} = N_t^\phi
\]

\[
\frac{1}{P_tC_t} = \beta E_t \frac{1}{P_{t+1}C_{t+1}} + \gamma \frac{1}{M_t}
\]

Interpret each of these conditions.
(b) Show that in equilibrium these conditions imply:

\[ M_t = \Phi P_t Y_t \]

for some constant \( \Phi \). Provide an algebraic expression for \( \Phi \).
(c) Assume that firms set prices one period in advance as a constant markup over expected nominal marginal cost so that in equilibrium

\[ \ln P_t = \ln \mu + E_{t-1} \ln W_t \]

where \( \mu > 1 \) is the gross markup. Show that expected output is constant in this model, i.e.

\[ E_{t-1} \ln Y_t = y \]

for some constant \( y \).
(d) Using the assumptions of part c) describe the effect a 1% unanticipated increase in money growth on output and inflation in both the short run (time t) and in the long run (time t+1).
(e) Is money neutral in this model? In the short-run? In the long-run? Explain.
Boston University

Comprehensive Examination
for M.A.E.P. and M.A. in Economics

May 22, 2008

Instructions:

1. The exam has three parts:
   (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   You are required to answer all three parts of the exam.

2. A total of three hours is allowed for the exam.
   It is recommended, but not mandatory, that you budget one hour for each part of the exam.

3. Do not write your name or ID number on the exam. It has already been coded.
   Write your answers for each part in the space provided. Do not write on the back of the page. If you run out of space, ask for additional paper and attach it to the exam.
   If the sheets of the exam get separated, be sure to re-attach them before submitting your exam.
   You will receive a blue book for scratch work. It will be collected at the end of the exam. Do not write your answers in the scratch book. It will be ignored in the grading.
   Write legibly: you will not get credit if the examiners cannot understand what you write.
   Use of calculator is permitted and applicable statistical tables are provided.

4. This is PART (III) of the exam and you should answer TWO QUESTIONS, as follows:
   III-A (Statistics); and either III-B1 or III-B2 (Econometrics) but not both.
   Do not submit answers to both III-B1 and III-B2. You will not receive credit if the examiners cannot determine which question you intend to answer.

Good Luck!
PART III-A: STATISTICS (1 QUESTION)

III-A. [50 points]  Let \( X \sim N(\mu, \sigma^2) \) and \( Y \mid X \sim N(X, \tau^2) \).

a. What is the joint pdf of \( X \) and \( Y \)?

b. What is the pdf of \( Y \)? (Hint: Making use of the fact that the mgf of a normal variate with mean \( \mu \) and variance \( \sigma^2 \) is given by \( \exp(\mu t + \frac{1}{2} t^2 \sigma^2) \), may be less messy than performing elaborate integrations.)

c. Hence, or otherwise find \( E(Y) \) and \( \text{Var}(Y) \).

d. What is \( \text{Cov}(X, Y) \)?

e. Suppose you have a sample on \( n \) observations on \( X, Y \) pairs such as \( (X_1, Y_1), \ldots, (X_n, Y_n) \). Provide two different unbiased estimators of \( \mu \). Argue that one is more efficient than the other.

f. Provide a consistent estimator for \( \tau^2 \).

Note: the pdf of \( X \) is
\[
\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right). 
\]
PART III-B: ECONOMETRICS (1 QUESTION)

Answer EITHER Question III-B1 OR Question III-B2. [50 points]
(Do NOT attempt both. If you had done so, please indicate clearly which answer you wish to be read and delete the other(s) unambiguously. You will not receive credit if the examiners cannot determine which question you intend to answer.)

III-B1. Suppose we have a linear model with \( Y | X \sim N(X \beta, \sigma^2 \Omega) \) where \( \Omega \) is some symmetric, positive definite matrix; i.e. the disturbances are 'non-spherical'. Assume also that the model does have a constant term.

a. Write down the expression of the generalized least squares estimator (GLS) and explain why it is BLUE.

b. What is the feasible generalized estimator (FGLS)? Explain with the simple example of a model where the error term is AR(1).

c. Show that if \( P\Omega = \Omega P \) where \( P = X(X'X)^{-1}X' \) (the projection matrix), then the OLS and the GLS estimators coincide.

d. The above is a simple condition that can be used to check the efficiency of the OLS estimator in the non-spherical error case (in fact it is a necessary and sufficient condition). Suppose \( \Omega_{ij} = 1 \) if \( i = j \) and \( \Omega_{ij} = \rho \) if \( i \neq j \). Can you think of a situation where you will encounter this model? Now, using the above result show that the OLS and GLS estimators coincide.

III-B2. (See next page.)
II-B2. Using data on individual adults ages 16 through 70, you estimate an equation by ordinary least squares and obtain the following results (showing t-statistics for a zero null hypothesis in parentheses).

\[
\ln w = 1.83 + 0.081 \text{sch} - 0.478 \text{ag1} + 0.146 \text{ag3} + 0.246 \text{ag4} - 0.136 \text{gen} - 0.017 \text{sch*gen}
\]

\[
(9.92) \quad (2.16) \quad (10.69) \quad (3.35) \quad (7.10) \quad (2.48) \quad (1.70)
\]

Number of observations = 67, Adjusted R-squared = 0.27

where \( \ln w \) = the natural logarithm of wage

\( \text{sch} \) = years of schooling

\( \text{ag1} \) = one if person age 16-20, zero otherwise

\( \text{ag3} \) = one if person age 31-40, zero otherwise

\( \text{ag4} \) = one if person age 41-70, zero otherwise

\( \text{gen} \) = one if female, zero otherwise

The variance-covariance matrix for the estimated coefficients is:

<table>
<thead>
<tr>
<th></th>
<th>intercept</th>
<th>sch</th>
<th>ag1</th>
<th>ag3</th>
<th>ag4</th>
<th>gen</th>
<th>gen*sch</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>0.0340</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sch</td>
<td>-0.0047</td>
<td>0.0014</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ag1</td>
<td>0.0067</td>
<td>0.0890</td>
<td>0.0020</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ag3</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.0080</td>
<td>0.0019</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ag4</td>
<td>-0.0012</td>
<td>-0.0012</td>
<td>-0.0008</td>
<td>0.0003</td>
<td>0.0012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gen</td>
<td>-0.0001</td>
<td>-0.0156</td>
<td>0.0267</td>
<td>0.0076</td>
<td>-0.0001</td>
<td>0.0030</td>
<td></td>
</tr>
<tr>
<td>gen*sch</td>
<td>0.0345</td>
<td>0.0289</td>
<td>0.0890</td>
<td>0.0014</td>
<td>-0.0113</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

a. Critical values of all test statistics must be clearly reported in this section

i. Conduct a 5 percent significance level test of whether the effect on the logarithm of wage of an additional year of schooling is smaller among women than among men.

ii. At a level of 10 years of schooling, how much lower is the logarithm of wage for women than for men, according to these estimates?

iii. Conduct a 5 percent significance level test for whether the logarithm of wage is lower for the age group 16-20 than for the age group 21-30.

iv. Conduct a 5 percent significance level test for whether the logarithm of wage is greater for the age group 41-70 than for the age group 31-40.

v. State, in percentage terms, (approximately) how much greater is the estimated wage for the age group 41-70 than for the age group 31-40.

b. Instead of the specification in Part (a), consider estimating the following specification on the same data set:

\[
\ln w = b_0 + b_1 \text{sch} + b_2 \text{age}
\]

where \( \text{age} \) = age in years

\( b_0, b_1, \) and \( b_2 \) are parameters

and other terms are as before

However, you suspect all of the parameters of this relationship differ between men and women. Explain briefly how you would proceed to test whether the parameters indeed differ between men and women. State the formula for your proposed test statistic, indicating clearly the degrees of freedom for this test statistic.