Comprehensive Exam January 2012
Micro I

1. Consider an exchange economy with two individuals, A and B, and two goods, X and Y. A and B each have an initial endowment of 10 units of good X and 5 units of good Y, and their preferences are described by the utility functions:

\[
\begin{align*}
u_A(x, y) &= x + y \\
u_B(x, y) &= \min\{x, y\}
\end{align*}
\]

Answer the following questions algebraically and illustrate your answers in an appropriate graph.

(a) What is the set of Pareto efficient allocations in this economy? Describe the set of allocations that constitute an Pareto improvement over the endowment allocation. Derive the contract curve.

(b) Find a Walrasian equilibrium. What is the price vector and allocation in equilibrium?

Solution: a) \(A^{PE} = \{(x, y) : y = x\}, A^{PI} = \{(x, y) : x + y \geq 15 \text{ and } \min\{x, y\} \geq 5\}, A^{CC} = A^{PE} \cap A^{PI}; b) p_x/p_y = 1, (x_A, y_A) = (12.5, 2.5), (x_B, y_B) = (7.5, 7.5).\)

2. Suppose that you are the single producer and seller of bicycles in your area. You know that half of your customers are avid bicyclists and have a high valuation for quality, whereas the others value quality less. Normalize the total number of consumers to one. Let \(H\) and \(L\) denote a high-taste and a low-taste consumer, respectively. More specifically, a consumer of type \(i \in \{H, L\}\) has the following preferences over bicycles:

\[U_i(q, p) = s_i q - p\]

Here, \(q\) denotes bicycle quality, \(p\) is the unit price for a bicycle, and \(s_i\) is a taste parameter for \(i \in \{H, L\}\), with \(s_H = 3\) and \(s_L = 2\). A consumer who does not buy a bicycle receives utility zero. The cost of producing a bicycle of quality \(q\) is given by:

\[c(q) = q^2.\]

You, the monopolist, want to maximize your expected profit by choosing \(q\) and \(p\). If you sell a bicycle of quality \(q\) at price \(p\) you receive profits:

\[\pi(p, q) = p - c(q) = p - q^2.\]

(a) Suppose first that you can perfectly observe whether a given consumer has a high valuation or a low valuation for bicycles. What level(s) of quality should you produce, and what price should you charge in order to maximize your expected profits? Calculate your expected profit.

(b) Now suppose that you cannot observe the consumers' valuation of bicycle quality. You do know, however, that any given consumer is just as likely to be high type as low type. Assuming that you want to sell to both types of bicyclists, what is the optimal quality and price choice in this case? What is your expected profit?
(c) Comment on any differences you might have found in your answer to (a) and (b).

Solution: a) \((q_H, p_H) = (3/2, 9/2), \ (q_L, p_L) = (1, 2), \pi = 13n/8;\)

b) \((q_H, p_H) = (3/2, 7/2), \ (q_L, p_L) = (1/2, 1), \pi = n;\)

c) Rent seeking/efficiency trade off: type \(H\) bicycle has optimal quality in both scenarios, type \(L\) bicycle has suboptimal quality in (b), profits lower in (b) due to cost of separating \(H\) and \(L\).
1. Solow model with perfect substitutes production function

(a) evolution of capital per effective unit of labor

Evolution of capital per effective unit of labor: \( k_t = \frac{Y(t)}{A(t)L(t)} \).

\[
\dot{k}(t) = \frac{K}{A(t)L(t)} - \frac{K(t)}{(A(t)L(t))^2} [A(t)L(t) + L(t)A(t)]
\]

\[= \frac{K}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)} \frac{L(t)}{L(t)} - \frac{K(t)}{A(t)L(t)} \frac{A(t)}{A(t)} \]

Therefore,

\[
k(t) = \frac{sY(t) - \delta K(t)}{A(t)L(t)} - nk(t) - gk(t)
\]

\[= s f(k(t)) - (n + g + \delta)k(t)
\]

\[= sak(t) + s - (n + g + \delta)k(t)
\]

(b) Intensive form of the production function of this model

\[y(t) = \frac{Y(t)}{A(t)L(t)} = \alpha k(t) + 1
\]

From \( k(t) = 0 \), we get

\[ sak(t) + s = (n + g + \delta)k(t)
\]

In order to have a balanced growth path,

\[(n + g + \delta) > sa
\]

(c) Balanced growth path

From (b), we get

\[k^* = \frac{s}{n + g + \delta - sa}
\]

Then,

\[y^* = \frac{\alpha s}{n + g + \delta - sa} + 1
\]

\[c^* = (1-s)(\frac{\alpha s}{n + g + \delta - sa} + 1)
\]

(d) In order to have \( \frac{dc^*}{ds} > 0 \),

\[\frac{dc^*}{ds} = (n + g + \delta)[\frac{\alpha - n - g - \delta}{(n + g + \delta - sa)^2}]
\]

Therefore, \( \frac{dc^*}{ds} > 0 \) implies that \( \alpha > n + g + \delta \).
2. Money Demand

a. Household’s optimality condition for bond holding

\[
\frac{1}{P_tC_t} = E_t\left[\frac{1 + i_{t+1}}{1 + \rho} \frac{1}{P_{t+1}C_{t+1}}\right]
\]

b. Household’s optimality condition for money holding

\[
\frac{1}{P_tC_t} = \gamma \frac{1}{P_t} \frac{1}{M_t/P_t} + E_t\left[\frac{1}{1 + \rho} \frac{1}{P_{t+1}C_{t+1}}\right]
\]

c. From a, we get,

\[
\frac{1}{1 + i_{t+1}} = E_t\left[\frac{1}{1 + \rho} \frac{P_tC_t}{P_{t+1}C_{t+1}}\right]
\]

From b, we get

\[
\gamma \frac{C_t}{M_t/P_t} = 1 - E_t\left[\frac{1}{1 + \rho} \frac{P_tC_t}{P_{t+1}C_{t+1}}\right]
\]

Therefore, we get

\[
\frac{C_t}{M_t/P_t} = 1 - \frac{1}{1 + i_{t+1}}
\]

\[
\frac{M_t}{P_t} = \gamma \left(\frac{1 + i_{t+1}}{i_{t+1}}\right) C_t
\]

As interest approaches to zero, money demand approaches to infinity. The intuition is that when the interest is very low, the opportunity cost of holding money is almost zero. Therefore, there is no incentive to hold bond or save.

d. From household optimizing problem, we got the following optimal condition

\[
\frac{1}{1 + i_{t+1}} = E_t\left[\frac{1}{1 + \rho} \frac{P_tC_t}{P_{t+1}C_{t+1}}\right]
\]

From the condition given in the problem, \(P_tC_t = \varphi M_t\), we get

\[
\frac{1}{1 + i_{t+1}} = \frac{1}{1 + \rho} E_t\left[\frac{\varphi M_t}{\varphi M_{t+1}}\right] = \frac{1}{1 + \rho} E_t\left[\frac{M_t}{M_{t+1}}\right] = \frac{1}{1 + \rho} \frac{1}{1 + g^N}
\]

Therefore,

\[
i_{t+1} = (1 + \rho)(1 + g^N) - 1
\]

This is constant
Statistics - MA Comp Jan 2012 Solution

a) i) \[ \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \]

\[ E(\bar{X}) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{n \mu}{n} = \mu \]

So \( \bar{X} \) is an unbiased estimator of \( \mu \)

ii) \[ \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i = \frac{1}{10} \times 160 = 16 \quad \text{Sample mean} \]

\[ S^2 = \frac{1}{n-1} \left( \sum_{i=1}^{10} X_i^2 - n \bar{X}^2 \right) \]
\[ = \frac{1}{9} \left( 3460 - 2560 \right) \]
\[ = 100 \quad \text{Sample variance} \]

(iii) 95% Confidence Interval

\[ P_r \left( A \leq \mu \leq B \right) = 0.95 \]

\[ P_r \left( \frac{16 - B}{\frac{S}{\sqrt{n}}} \leq \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \leq \frac{16 - A}{\frac{S}{\sqrt{n}}} \right) = 0.95 \]

\[ P_r \left( \frac{16 - B}{\frac{S}{\sqrt{n}}} \leq T_{0.975} \leq \frac{16 - A}{\frac{S}{\sqrt{n}}} \right) = 0.95 \]

\[ \frac{16 - B}{\frac{S}{\sqrt{n}}} = -2.262 \]

\[ B = 16 + 2.262 \times 10 \approx 23.153 \]

\[ \frac{16 - A}{\frac{S}{\sqrt{n}}} = 2.262 \]

\[ A = 16 - 2.262 \times 10 \approx 8.847 \]
b) \( \hat{\rho}_{\text{mom}} \)

\[ E(x) = \frac{1}{\mu} \quad \text{so} \quad \bar{x} = \frac{1}{\hat{\rho}_{\text{mom}}} \]

\[ \hat{\rho}_{\text{mom}} = \frac{1}{\bar{x}} \]
i. \[ b_0 = b \]
\[ b_1 = b * c \]
\[ b_2 = b * c^2 \]

ii. \[ b/(1-c) \]

iii. \[ \ln(0.5)/\ln(c) \]

iv. 

v. No difference.

vi. Durbin's \( h = [1 - DW/2] * \sqrt{T/(1-T*V_c)} \)

   where \( DW \) = Durbin-Watson statistic, \( T \) = sample size, \( V_c \) = variance of estimate of \( c \).

   Compare \( h \) to Normal distribution for critical value.

   {Suggest partial credit for mentioning Durbin's \( h \) but not knowing formula}.

   Other correct answers possible, such as Breusch-Godfrey test.

vii. \( y_{t+1} \) is correlated with \( e_t \) because both are correlated with \( e_{t-1} \).