Boston University

Comprehensive Examination

January 25, 2013

Instructions:

1. • The exam has three parts: (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   • You are required to answer all three parts of the exam.

2. • A total of three hours is allowed for the exam.
   • It is recommended, but not mandatory, that you budget one hour for each part of the exam.

3. • Do not write your name or ID number on the exam. It has already been coded.
   • Write your code on the upper right-hand corner of every page.
   • Write your answers for each part in the space provided. Do not write on the back of the page. If you run out of space, ask for additional paper and attach it to the exam.
   • If the sheets of the exam get separated, be sure to re-attach them before submitting your exam.
   • You will receive a blue book for scratch work. It will be collected at the end of the exam. Do not write your answers in the scratch book. It will be ignored in the grading.
   • Write legibly: you will not get credit if the examiners cannot understand what you write.
   • Use of calculator is permitted.

4. • This is PART (I) of the exam and it has TWO QUESTIONS.
   • Answer both questions.

Good Luck!
PART I: MICROECONOMICS (2 QUESTIONS)

I.1 [60 points] Tim’s preferences over goods 1 and 2, the only goods he consumes, are represented by the utility function

\[ U(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\}, \]

where \( x_1 \) denotes the quantity of good 1 and \( x_2 \) denotes the quantity of good 2.

(i) Draw the indifference curve for utility level 24.
(ii) Calculate the MRS at \((x_1, x_2) = (10, 7)\). If Tim chooses to consumes the bundle \((10, 7)\) when the price of good 1 is equal to $4, how much is his income?
(iii) Calculate the optimal consumption bundle \((x_1^*, x_2^*)\) when Tim has an income of 24, \(p_1 = 1\), and \(p_2 = 1\).
(iv) If \( p_2 \) increases to 6, while his income and \( p_1 \) remain the same as in part (iii), what will be the new optimal consumption bundle \((x_1^{**}, x_2^{**})\)? Calculate how much, if any, of the change from \( x_2^* \) to \( x_2^{**} \) is due to the income effect and how much to the substitution effect.
(v) Now, assume that Tim’s income is provided by his father who adjusts Tim’s income so that, no matter what the prices are, Tim always ends up with a utility level of 24 when he consumes the optimal consumption bundle. If \( p_2 = 4 \), calculate Tim’s demand for good 1 as a function of \( p_1 \).
(This type of demand is also known as the Hicksian demand. Let \( x_1^h(p_1, p_2, U_0) \) denote the Hicksian demand for good 1, for a utility level of \( U_0 \) when prices are \( p_1 \) and \( p_2 \). This questions asks you to calculate \( x_1^h(p_1, 4, 24) \).)
1.2 [40 points] A firm has production function given by \( f(K,L) = (2K + 3L)^{1/2} \).

(i) Draw the isoquant for output level 6. Calculate the marginal rate of technical substitution at (4,2).

(ii) What kind of returns to scale does this production function exhibit?

For the rest of the question, assume that the unit cost of capital is $3 and the unit cost of labor is $6.

(iii) If the level of capital is fixed at \( K_0 \) in the short-run, calculate the short-run cost function \( C_s(q) \).

(iv) Calculate the long-run cost function.
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4. This is PART (II) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
PART II: MACROECONOMICS (2 QUESTIONS)

II.1 [40 points]

Consider the discrete-time Solow model. Production is given by

\[ Y_t = K_t^\alpha (A_t L_t)^{1-\alpha} \]

where \( K_t \) is capital, \( A_t \) is technology, and \( L_t \) is labor. Technology and labor grow according to

\[ A_{t+1} = (1+g) A_t \quad \text{and} \quad L_{t+1} = (1+n) L_t \]

Capital is accumulated through

\[ K_{t+1} = I_t + (1-\delta) K_t \]

Households save a constant rate of output \( s \). However, the government taxes these savings at rate \( \tau \), so the effective savings rate is \( s(1-\tau) \). The government then uses the proceeds from these taxes to increase consumption, hence

\[ I_t = s(1-\tau) Y_t \]
\[ C_t = (1-s) Y_t + s\tau Y_t \]

(1) Find a formula for the evolution of capital per efficient worker, \( k_t = K_t / (A_t L_t) \), and steady-state capital per efficient worker \( k^* \).

(2) Explain how an increase in \( \tau \) affects the steady-state level of capital per efficient worker, \( k^* \), and output per efficient worker, \( y^* \). Give both the mathematical argument (you can use a graph for this) and the economic intuition.

(3) What is the effect of \( \tau \) on steady-state consumption per efficient worker \( c^* = C_t / (A_t L_t) \)? Give both the mathematical argument (you can use a graph for this) and the economic intuition.

(4) What is the value of \( \tau \) that maximizes the level and growth rate of consumption per capita \( C_t / L_t \) in the long-run? (Assume that \( s > \alpha \).

(5) How would your answers to (1), (2) and (3) change if the government were throwing money away instead (i.e. \( C_t = (1-s) Y_t \), e.g. by using taxes to finance something that is useless, or at least do not affect consumption or investment)?
II.2 [60 points]

Consider the real business cycle model without capital. Production is given by

\[ Y_t = A_t N_t^{1-\alpha}, \]

where \( A_t \) is technology, and \( N_t \) is labor. The household maximizes utility:

\[ \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{N_t^{1+\phi}}{1+\phi} \right] \]

subject to the budget constraint,

\[ S_{t+1} = (1 + \tau_t) S_t + w_t N_t + \pi_t (1 - \tau_t) - C_t, \quad \text{NOTE: CORRECTION} \]

where \( \pi_t \) are profits, and \( \tau_t \) is a tax on profits.

The firms maximize profits given by \( \pi_t = Y_t - W_t N_t = A_t N_t^{1-\alpha} - W_t N_t \).

The government uses the tax proceeds to finance some spending, and the budget is balanced each period:

\[ G_t = \tau_t \pi_t. \]

The resource constraint is:

\[ C_t + G_t = Y_t, \]

so that \( S_t = 0 \) in equilibrium (there is no capital). Both labor and goods markets are assumed to be competitive.

Denote by \( g_t = G_t / Y_t \) the ratio of government spending to output. Each period, the government decides on \( g_t \), and adjusts the taxes \( \tau_t \) accordingly to balance the budget.

(1) Write the first-order condition characterizing the optimal demand for labor by the firm, and write the first-order condition characterizing the optimal choice of labor by the household.

(2) Compute the equilibrium employment, consumption, output, and the real wage, as a function of \( A_t \), \( g_t \), and the model parameters.

(3) How does an increase in \( A_t \) (holding \( g_t \) constant) affect employment, consumption, output, and the real wage? Provide some intuition for your result.

(4) How does an increase in \( g_t \) (holding \( A_t \) constant) affect employment, consumption, output, and the real wage? Provide some intuition for your result.

(5) In the data, the correlation of labor productivity \( Y_t / N_t \) and of output \( Y_t \) is not very strong. Explain what the model predicts if most shocks are shocks to \( A_t \), and what the model predicts if most shocks are shocks to \( g_t \). (You can answer this without formal derivation.) What is your conclusion?
11.2 [60 points]

Consider the real business cycle model without capital. Production is given by

$$Y_t = A_t N_t^{1-a},$$

where $A_t$ is technology, and $N_t$ is labor. The household maximizes utility:

$$\sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to the budget constraint,

$$S_{t+1} = (1 + r_t)S_t + w_i N_t + \pi_t (1 - \tau_t) - C_t + H_t,$$

where $\pi_t$ are profits, and $\tau_t$ is a tax on profits.

The firms maximize profits given by $\pi_t = Y_t - W_t N_t = A_t N_t^{1-a} - W_t N_t$.

The government uses the tax proceeds to finance some spending, and the budget is balanced each period:

$$G_t = \tau_t \pi_t.$$

The resource constraint is:

$$C_t + G_t = Y_t,$$

so that $S_t = 0$ in equilibrium (there is no capital). Both labor and goods markets are assumed to be competitive.

Denote by $g_t = G_t/Y_t$ the ratio of government spending to output. Each period, the government decides on $g_t$, and adjusts the taxes $\tau_t$ accordingly to balance the budget.

(1) Write the first-order condition characterizing the optimal demand for labor by the firm, and write the first-order condition characterizing the optimal choice of labor by the household.

(2) Compute the equilibrium employment, consumption, output, and the real wage, as a function of $A_t$, $g_t$, and the model parameters.

(3) How does an increase in $A_t$ (holding $g_t$ constant) affect employment, consumption, output, and the real wage? Provide some intuition for your result.

(4) How does an increase in $g_t$ (holding $A_t$ constant) affect employment, consumption, output, and the real wage? Provide some intuition for your result.

(5) In the data, the correlation of labor productivity $Y_t/N_t$ and of output $Y_t$ is not very strong. Explain what the model predicts if most shocks are shocks to $A_t$, and what the model predicts if most shocks are shocks to $g_t$. (You can answer this without formal derivation.) What is your conclusion?
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   Use of calculator is permitted and applicable statistical tables are provided.

4. This is PART (III) of the exam and it has TWO QUESTIONS:
   (Statistics) Answer only 1 question: III-A1 or III-A2.
   (Econometrics) Answer only 1 question: III-B1 or III-B2.
   Do not submit answers to more than one question. You will not receive any credit if the examiners cannot determine which question you intend to answer.

Good Luck!
PART III-A: STATISTICS (CHOOSE ONE QUESTION III-A1 OR III-A2) [50 POINTS]

III-A1  Answer both parts (a) and (b).

a. Suppose that a house that is on the market to be sold has a constant probability of being sold in any given month, \( p \), regardless of how long the house has been on the market. Then the number of months a house is on the market before a sale is a random variable \( X \) with a geometric distribution, that is

\[
f(x) = \begin{cases} 
(1 - p)^{x-1} p & x = 1, 2, 3 \ldots \\
0 & \text{otherwise}
\end{cases}
\]

Suppose you have a random sample with \( n \) observations of data on the number of months houses were on the market before a sale.

i. Write down the likelihood function for the parameter \( p \), given your sample.

ii. Find the Maximum Likelihood estimator, \( \hat{p}_{MLE} \), of \( p \).

b. Suppose that the number of calories in a hamburger from the Happy Hamburger franchise is a normally distributed random variable with unknown mean and variance. A dietician randomly samples 5 hamburgers and determines the calorie content of each one: 390, 440, 420, 380, 410.

i. Find the sample mean, \( \bar{X} \), and sample variance, \( s^2 \), of the calorie content of the hamburgers.

ii. What is known about the distributions of \( \bar{X} \) and \( s^2 \)?

iii. Construct a 99% confidence interval for the true mean calorie content of a hamburger based on this sample.

III-A2  Answer both parts (a) and (b).

a. For random variable \( X \) having a \( \chi^2 \) distribution with probability distribution function

\[
f(x \mid m/2, 1/2) = \frac{1}{2^{m/2} \Gamma(m/2)} x^{m/2 - 1} e^{-x/2}, x > 0
\]

Prove that \( E(X) = m \) and \( \text{Var}(X) = 2m \).

b. Let \( Y \) and \( Z \) be two independent random variables such that \( Y \sim \chi^2(m) \), i.e., a Chi-square distribution with \( m \) degrees of freedom, and \( Z \sim \text{N}(0, 1) \), i.e., a standard normal distribution. Then \( X = Z(Y/m)^{1/2} \sim t(m) \), i.e., a \( t \) distribution with \( m \) degrees of freedom.

Prove that the functional form of \( X \) is

\[
f(x) = \frac{\Gamma\left(\frac{m+1}{2}\right)}{\left(m\pi\right)^{1/2} \Gamma(m/2)} \left(1 + \frac{x^2}{m}\right)^{-\frac{m+1}{2}}
\]
PART III-B: ECONOMETRICS (CHOOSE ONE QUESTION III-B1 OR III-B2) [50 POINTS]

III-B1

You estimate the following model by ordinary least squares on forty observations. Each observation is on one country in the year 2005.

\[ Y_t = b_0 + b_1 X_{1t} + b_2 X_{2t} + b_3 X_{3t} + b_4 X_{4t} + e_t \]

Describe how you would carry out each of the following tests. In each case you must state as precisely as possible the formula for measuring your test statistic, defining the terms in your formulas, and also state the number of degrees of freedom available for your test statistic.

a. A test for whether \( b_2 = b_4 = 0 \).

b. A test for whether all \( b_i \) (i=0 through 4) are equal across two groups: the twenty developing countries in your sample and the twenty industrialized countries.

c. A test for whether all \( b_i \) (i=0 through 4) are equal in the African nations in your sample and in the remaining nations in your sample. There are four African states in your sample.

III-B2

Consider the regression model with four independent variables under the six classical assumptions:

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + u \]

You want to test the null hypothesis \( H_0: 3\beta_1 + 5\beta_3 = 6 \). \{NOTE: CORRECTION\}

a. Let \( \hat{\beta}_1 \) and \( \hat{\beta}_3 \) denote the OLS estimators for \( \beta_1 \) and \( \beta_3 \). Find \( \text{Var}(3\hat{\beta}_1 + 5\hat{\beta}_3) \) in terms of \( \text{Var}(\hat{\beta}_1) \), \( \text{Var}(\hat{\beta}_3) \), and \( \text{Cov}(\hat{\beta}_1, \hat{\beta}_3) \). What is the standard error of \( 3\hat{\beta}_1 + 5\hat{\beta}_3 \) in the same general terms?

b. What is the t-statistic for testing \( H_0 \) using the information in part (a)?

c. Assume \( \hat{\beta}_1 = 0.6 \), \( \hat{\beta}_3 = 0.7 \), \( \text{Var}(\hat{\beta}_1) = 0.15 \), \( \text{Var}(\hat{\beta}_3) = 0.3 \), and \( \text{Cov}(\hat{\beta}_1, \hat{\beta}_3) = 0.12 \). Assume that the sample size \( n=36 \). Do we tend to accept or reject the null hypothesis?
Part III-B: Econometrics (Choose one question III-B1 or III-B2) [50 points]

III-B1

You estimate the following model by ordinary least squares on forty observations. Each observation is on one country in the year 2005.

\[ Y_t = b_0 + b_1 X1_t + b_2 X2_t + b_3 X3_t + b_4 X4_t + e_t \]

Describe how you would carry out each of the following tests. In each case you must state as precisely as possible the formula for measuring your test statistic, defining the terms in your formulas, and also state the number of degrees of freedom available for your test statistic.

a. A test for whether \( b_3 = b_4 = 0 \).

b. A test for whether all \( b_i \) (i= 0 through 4) are equal across two groups: the twenty developing countries in your sample and the twenty industrialized countries.

c. A test for whether all \( b_i \) (i= 0 through 4) are equal in the African nations in your sample and in the remaining nations in your sample. There are four African states in your sample.

III-B2

Consider the regression model with four independent variables under the six classical assumptions:

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + u \]

You want to test the null hypothesis \( H_0: 3\beta_1 + 5\beta_3 = 6 \).

a. Let \( \hat{\beta}_1 \) and \( \hat{\beta}_3 \) denote the OLS estimators for \( \beta_1 \) and \( \beta_3 \). Find \( \text{Var}(3\hat{\beta}_1 + 5\hat{\beta}_3) \) in terms of \( \text{Var}(\hat{\beta}_1), \text{Var}(\hat{\beta}_3) \), and \( \text{Corr}(\hat{\beta}_1, \hat{\beta}_3) \). What is the standard error of \( 3\hat{\beta}_1 + 5\hat{\beta}_3 \) in the same general terms?

b. What is the t-statistic for testing \( H_0 \) using the information in part (a)?

c. Assume \( \hat{\beta}_1 = 0.6 \), \( \hat{\beta}_3 = 0.7 \), \( \text{Var}(\hat{\beta}_1) = 0.15 \), \( \text{Var}(\hat{\beta}_3) = 0.3 \), and \( \text{Cov}(\hat{\beta}_1, \hat{\beta}_3) = 0.12 \). Assume that the sample size \( n=36 \). Do we tend to accept or reject the null hypothesis?