Boston University


January 26, 2015

Instructions:

1. The exam has three parts:
   (I) Micro-economics; (II) Macro-economics; and (III) Statistics & Econometrics.
   You are required to answer all three parts of the exam.

2. A total of three hours is allowed for the exam.
   It is recommended, but not mandatory, that you budget one hour for each part of the exam.

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   Write legibly: you will not get credit if the examiners cannot understand what you write.
   Use of calculator is permitted.

4. This is PART (I) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
PART I: MICROECONOMICS (2 QUESTIONS)

I.1

There are 100 individuals in Randomia. Each of them has a wealth of $10,000 and a utility function given by

\[ U(w) = \sqrt{w} \]

where \( w \) is his or her wealth.

Warren is one of the citizens of Randomia. He has an idea for a project that would cost $6,400 and would then have a 25% chance of returning $36,400 (so that the net return would be $30,000 if the project was successful).

(a) What is the net expected value of this project?
(b) Would Warren adopt the project? Prove your answer.
(c) Suppose Warren decides to share his idea with the rest of Randomia. Each person would pay $64 of the cost and would then be paid $364 if the project was successful. Would the Randomians accept Warren’s proposal? Prove your answer.
(d) Suppose instead that Warren decides to sell shares in his project. There would be 100 shares and each share would entitle the shareholder to a payoff of $364 if the project was successful. What would be the expected value of a share?
(e) If Warren offered shares at a price equal to their expected value (which you found in part d), would Randomians buy these shares? Prove your answer and provide an intuitive explanation for it.
Three identical firms produce a homogenous product at zero cost (zero marginal cost and zero fixed cost). The inverse market demand for the product is given by:

\[ p(q) = 16 - q, \]

where \( p \) is the price, and where \( q \) denotes aggregate output.

a) Suppose that they compete as follows. In the first stage, firm one chooses how much to produce. Firms two and three then observe the output selected by firm one. In the second and last stage, firms two and three simultaneously choose output. Then the three firms receive profits. How much will the three firms produce if their objective is to maximize profits? Assume here that firms two and three act as if the other holds output constant (the Cournot conjecture).

b) Suppose instead that the firms compete in the following manner. In the first stage, firm one chooses how much to produce. Firms two and three then observe firm one’s choice of output. In the second stage, firm two only selects its output. In the third and last stage, having observed also the output choice by firm two, firm three chooses output. Then all firms collect profits. How much will the three firms produce in this case if they maximize profits?

c) Which form of competition do the firms prefer? Which one would the consumers prefer? Explain your answer.
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4. This is PART (II) of the exam and it has TWO QUESTIONS.
   Answer both questions.

Good Luck!
II.1 Solow Model

NOTE: PLEASE DRAW A BOX AROUND YOUR FINAL ANSWERS IF YOU WANT TO GET FULL CREDIT

Consider the discrete-time Solow model. Production is given by

\[ Y_t = K_t^\alpha (A_tL_t)^{1-\alpha}, \]

where \( K_t \) is capital, \( A_t \) is technology, and \( L_t \) is labor. Technology and labor grow according to

\[ A_{t+1} = (1+g)A_t, \]
\[ L_{t+1} = (1+n)L_t. \]

Capital is accumulated through

\[ K_{t+1} = I_t + (1-\delta)K_t. \]

Households save a constant rate of output \( s \). However, there is an investment tax credit, i.e. the government taxes consumption at rate \( \tau \) and channels the taxes to investment. Hence, given the savings rate \( s \),

\[ C_t = (1-s)Y_t, \]
\[ I_t = sY_t. \]

(1) Find a formula for the evolution of capital per efficiency unit of labor, \( k_t = \frac{K_t}{A_tL_t} \), and steady-state capital per efficiency unit of labor \( k^* \).

(2) Explain how an increase in \( s \) affects the steady-state level of capital per efficiency unit of labor, \( k^* \), and output per efficiency unit of labor, \( y^* \), and consumption per efficient worker, \( c^* \). Give both the mathematical argument (you can use a graph for this) and the economic intuition.

(3) Suppose consumers prefer to set their savings rate, \( s \), to maximize \( c^* \). What would the savings rate be? Recent data shows that labor share in output, \( 1-\alpha \), is declining. Would the optimal savings rate (the one that maximizes \( c^* \)) be higher or lower? Give both the mathematical argument and the economic intuition.

(4) Suppose the government finances itself through taxation of consumption so that \( C_t = (1-s)(1-\tau)Y_t \), where \( \tau \) is the tax rate. Write tax revenue as a function of model parameters. What would the effect of the decline in labor share be on tax revenues if consumers prefer to set their savings rate, \( s \), to maximize \( c^* \)? (Assume \((1+g)(1+n)-(1-\delta)<1\), which is true for realistic parameter values.)

(5) How would the change in labor share of output and the change in tax rate affect long-run growth rate of output?
II.2 RBC Model and Fiscal Policy

NOTE: PLEASE DRAW A BOX AROUND YOUR FINAL ANSWERS IF YOU WANT TO GET FULL CREDIT

Consider the real business cycle model that we studied in class. Production is given by

\[ Y_t = A_t N_t^{1-\alpha}, \]

where \( A_t \) is technology, and \( N_t \) is labor. The household maximizes utility:

\[ \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\phi}}{1+\phi} + B \log G_t \right), \]

where the parameter \( B \) measures how much people value government spending. The budget constraint for the household is

\[ S_{t+1} = (1+r_p)S_t + \pi_t N_t + \tau_t (1+r_p)C_t, \]

where \( \pi_t \) are profits, \( \tau_t \) is a tax on consumption. The gov't uses the tax proceeds to finance the spending, and the budget is balanced each period:

\[ G_t = \tau_t C_t. \]

The resource constraint for the overall economy is:

\[ C_t + G_t = Y_t, \]

so that \( S_t = 0 \) in equilibrium (there is no capital). Both labor and goods markets are assumed to be competitive. Each period, the government decides on \( G_t \) and adjusts \( \tau_t \) in consequence.

1. Write the first-order conditions characterizing the optimal demand for labor by the firm, and write the first-order conditions characterizing the optimal choice of labor by the household.

2. Compute the equilibrium employment, consumption, output, and the real wage, as a function of \( A_t \) and \( \tau_t \).

3. How does an increase in \( \tau_t \) affect the economy (consumption, output, employment, and the real wage)? Provide some intuition for your result.

4. What is the amount of revenue raised by the government, as a function of \( \tau_t \)? What is the \( \tau_t \) that maximizes government spending?

5. What is the level of \( \tau_t \) that maximizes household utility? How is it different from the level of \( \tau_t \) that maximizes \( G_t \)? Explain intuitively.
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   Use of calculator is permitted and applicable statistical tables are provided.

4. This is PART (III) of the exam and it has TWO QUESTIONS:
   (Statistics) Answer one question: III-A.
   (Econometrics) Answer one question: III-B1 or III-B2 or III-B3.
   Do not submit answers to more than one question in each section. You will not receive any credit if the examiners cannot determine which question you intend to answer.

Good Luck!
In a grocery store, the waiting time in a self-service checkout lane, $X$, is a random variable distributed exponentially with parameter $\theta$ and the probability density function

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

a. Show that $E(X) = \theta$ and $Var(X) = \theta^2$.

Hint: You can use the following fact for the Gamma function:

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy = (\alpha - 1)!$$ for a positive integer $\alpha$.

d. In a sample of 100 people who used a self-service checkout lane, the average waiting times was 6.5 minutes.

i. What is the maximum likelihood estimate of the probability that the waiting time in a self-service checkout lane is no more than 5 minutes?

ii. Based on your answer in (c), find a 95% confidence interval for $\theta$ in this sample.
PART III-B: ECONOMETRICS (CHOOSE ONE QUESTION III-B1, III-B2 OR III-B3)

III-B1

Consider the following model:

\[
\begin{align*}
(1) \quad C_t &= \alpha_0 + \alpha_1 Y_t + \alpha_2 i_t + \varepsilon_t \\
(2) \quad Y_t &= C_t + G_t \\
(3) \quad i_t &= \mu_0 + \mu_1 Y_t + \mu_2 M_t + \theta_t
\end{align*}
\]

where \( C = \) national consumption, \( Y = GDP, \) \( i = \) an interest rate, \( G = \) government spending on goods and services, \( M = \) money supply.

You consider government spending and money supply to be exogenous.

a. Would an ordinary least squares estimate of equation (1) be unbiased? Why (not)?

b. Is the necessary condition satisfied for equation (3) to be identified? Explain.

c. If you estimate equation (3) by two-stage-least-squares, what would be the instrumental variable?

d. Write out the form of the first-stage equation for your estimate in (c).

e. In this first-stage equation, how would you test for the relevance of your instrument?

f. Is it possible to test for the exogeneity of this instrument in (c)? Why (not)?

III-B2

You want to investigate whether the number of points the basketball player Kobe Bryant scores in a game has any influence on the number of points he scores in the next game.

a. Give possible reasons why this influence may be positive or negative. Take into account that the game is played by many players.

b. You specify the following equation for estimation: \( Y_t = \alpha + \beta Y_{t-1} + \mathbf{x}_t' \gamma + \varepsilon_t, \) where \( Y_t \) is the number of points scored by Bryant in his \( t \)-th game of the season, \( x_t \) is a bunch of control variables (e.g. the strength of the team he is playing against etc.) and \( \varepsilon_t \) is an error term. What could possibly \( \varepsilon_t \) capture? What could be some reasons why this error could be autocorrelated?

c. Explain in the light of previous part, why OLS is not appropriate for estimating this equation, but IV is. Can you suggest some natural instrument(s)?

d. Is there any way of testing whether the error term is indeed autocorrelated?

e. Suppose you can’t reject the null that errors are iid. Assume that your software can run both OLS and IV procedures (and provide all outputs from them), but cannot directly test the null that \( \beta = 0. \) What exactly will you do to carry out the test?

III-B3 on next page
Let $Y_i$ be generated as

$$Y_i = 1 \{ \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + u_i > 0 \}; \ i = 1, ..., n,$$

where $u_i$ is independent of $(X_{i1}, X_{i2})$ and follows the (standard) Logistic distribution. Suppose further that $u_i; i = 1, ..., n$ are IID. Recall that the Logistic distribution has the following CDF:

$$F(u) = \frac{1}{1 + e^{-u}}$$

i) Express $P(Y_i = 1 \mid X_{i1}, X_{i2})$ as a function of parameters $(\beta_0, \beta_1, \beta_2)$.

ii) Derive the log-likelihood function for this model. Discuss how to estimate $(\beta_0, \beta_1, \beta_2)$ by a maximum likelihood estimator (MLE).

iii) Discuss how to estimate $(\beta_0, \beta_1, \beta_2)$ by a nonlinear least squares estimator (NLS). Which of the two estimators (MLE and NLS) should one prefer? Explain.

iv) Suppose that the maximum likelihood estimate is $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2) = (0.1, 0.3, -0.5)$. Calculate the change in the predicted probability when $X_{i1}$ changes from 1.0 to 0.8 while fixing $X_{i2}$ to 3.