The Shock Tube Problem

ME702 Final project

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The Shock Tube problem

- Studied by Gary A. Sod in 1978
- 1D problem
- Analytical solutions are known
- Used to test and validate computational fluid models
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\[ p = 100 \text{ kPa} \]
\[ u = 0 \text{ m/s} \]
\[ \rho = 1.000 \text{ kg/m}^3 \]

\[ p = 10 \text{ kPa} \]
\[ u = 0 \text{ m/s} \]
\[ \rho = 0.125 \text{ kg/m}^3 \]
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Governing Equations

**Conservation of mass**
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0
\]

**Conservation of momentum**
\[
\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot [\vec{u} (\rho \vec{u})] + \nabla p + \nabla \cdot \vec{T} = 0
\]

**Conservation of energy**
\[
\frac{\partial (\rho E)}{\partial t} + \nabla \cdot [E(\rho \vec{u})] + \nabla \cdot (\rho \vec{u}^2) + \nabla \cdot (\vec{T} \cdot \vec{u}) + \nabla \cdot \vec{j} = 0
\]
Governing Equations

Conservation of mass
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Conservation of momentum
\[ \frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot [\vec{u}(\rho \vec{u})] + \nabla p + \nabla \cdot \vec{T} = 0 \]

Conservation of energy
\[ \frac{\partial (\rho E)}{\partial t} + \nabla \cdot [E(\rho \vec{u})] + \nabla \cdot (\rho \vec{u}^2) + \nabla \cdot (\vec{T} \cdot \vec{u}) + \nabla \cdot \vec{j} = 0 \]

Assume:
- inviscid flows (no viscosity)
- no heat transfer
Analytical solutions
OpenFOAM 2D shock tube set-up

The diagram illustrates a rectangular shock tube with dimensions:
- Length: 10 m
- Width: 2 m
- Height: 0.2 m

The tube is set up in a coordinate system with the y-axis vertical, the x-axis horizontal, and the z-axis perpendicular to both.
OpenFOAM 2D shock tube set-up

2 m

0.2 m

10 m

grid refinement
OpenFOAM 2D shock tube set-up

![Diagram of the shock tube set-up with dimensions and empty walls.]
OpenFOAM 2D shock tube set-up

- $p = 100 \text{ kPa}$
- $u = 0 \text{ m/s}$
- $T = 348.432 \text{ K}$

- $p = 10 \text{ kPa}$
- $u = 0 \text{ m/s}$
- $T = 278.746 \text{ K}$

non-uniform internal flow
OpenFOAM 2D shock tube set-up

\[ \Delta t = 10^{-6} \text{ s} \]

\[ t = 10 \text{ ms} \]

sampling rate: 0.5 ms

resolution cases: 20x20x1
30x30x1
50x50x1
100x100x1
300x300x1
400x400x1
OpenFOAM rhoCentralFoam solver

- Greenshields et al., 2010
- compressible fluids $\nabla \cdot (\rho \overrightarrow{u}) \neq 0$
- finite volumes method
- values are provided at the centroid of the volume cell
- cells are contiguous polyhedral volumes
- volume integrals in divergence and gradient terms are converted to surface integrals via Gauss’s theorem
Results: time evolution
line cuts along x-axis
Results: time evolution

line cuts along x-axis

- pressure
- temperature
- speed
- density

20x20x1
Results: time evolution

line cuts along x-axis

20x20x1

50x50x1

pressure

temperature

speed

density
Results: time evolution

line cuts along x-axis

100x100x1

300x300x1

pressure

temperature

speed

density
Results: resolution
line cuts along x-axis
Results: resolution

line cuts along x-axis

- pressure
- temperature
- speed
- density
Results
line cuts along x-axis for 300x300x1 resolution grid

pressure

temperature

speed

density
Results

line cuts along x-axis for 300x300x1 resolution grid

- pressure
- temperature
- speed
- density
Results: 2D plots

xy plane cut
Results: 2D plots

xy plane cut

pressure

temperature

speed

density
Results: 2D plots

xy plane cut

- Pressure
- Temperature
- Speed
- Density

Shock wave
Rarefaction wave
Python solvers

Lax-Friedrichs (1-step)
Python solvers: Lax-Friedrichs (1-step)
Python solvers: MacCormack (2-step)
OpenFOAM vs. MacCormack

- Pressure
- Speed
- Density
Conclusions

- OpenFOAM can simulate the 2D shock tube problem
- Solutions match to the analytical ones for resolution of at least 300x300
- 2D solutions are the same for the 1D case
- Can extract 1D solutions to 2D and 3D case, if the diaphragm is along x only
Conclusions

- OpenFOAM can simulate the 2D shock tube problem
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- MacCormack 2-step scheme also approximates well the analytical solutions, but additional artificial viscosity is needed
- MacCormack 2-step scheme has more diffusion
Temperature is obtained given that $e = C_V T = (\gamma - 1)RT$:

$$\begin{align*}
T &= \frac{1}{C_V} \left( \frac{E}{\rho} - \frac{u^2}{2} \right)
\end{align*}$$