Further Investigations Into a Low-Order Model of Fan Broadband Noise

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Two aspects of a low-order method for predicting broadband interaction noise downstream of a fan stage in a turbofan engine are investigated. First, the ability to include real vane geometry effects by utilizing an asymptotic based unsteady gust response method is considered. The results indicate that the asymptotic method is too constrained by thin-airfoil parameter restrictions to be applicable to modern fan exit guide vanes. Also, the results demonstrate that any model for vane thickness that relies on rapid distortion theory in the gust interaction formulation will overpredict the sound especially at higher frequencies. A second investigation was made into the modification of the low-order method to directly use propagating acoustic modes calculated for a flat-plate cascade as opposed to just using the unsteady surface pressure in the model. The current implementation of the acoustic mode based prediction is not producing results similar to those shown in the literature previously. As such the method requires further development before conclusive statements regarding its prediction capability and computational requirements as compared to the original method can be made.

1. Introduction

Some extensions of a low-order method for simulating broadband interaction noise downstream of a fan stage in a turbofan engine are explored in this paper. What is termed the original low-order method in this paper has been described in detail previously. The formulation was based on the general method presented by Nallasamy and Envia and is similar in many respects to that described by Posson et al. The method first models the surface response of the fan exit guide vanes (FEGVs) to multiple individual gust disturbances and then uses a Green’s function method to compute the broadband sound power levels in the duct. The method models the exit guide vanes as strips of flat-plate cascades whose unsteady response to an incident gust is calculated semianalytically.

Last year, the preliminary exploration of including influence of vane shape in the formulation was described. Vane shape was included by utilizing the asymptotic method of Ayton and Peake to predict the unsteady response of a single real geometry, lifting airfoil, to a gust. This real airfoil asymptotic response model was used in place of the flat-plate cascade response model in the low-order method. A summary of the final conclusions from the study are now provided in Section 3.A. of this paper.

A related, alternative low-order method for predicting the broadband noise due to rotor wake interaction with the exit guide vanes has been presented in the literature. In the method, a strip-theory relying on a flat-plate cascade model is also utilized; however, the acoustic field in the duct is determined from the acoustic modes associated with the cascade. One does not use the Green’s method to determine the duct pressure from the vane unsteady pressure distribution. A comparison between the acoustic mode based method and the original method in terms of the accuracy of prediction and the computational requirements

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is sought when the basic formulations are clearly connected and the model assumptions and input values are identical. The formulation of the cascade acoustic mode based method is described in Section 2.C. Preliminary results from the comparison of the two methods are given in Section 3.B.

2. Method

A. The original method

The original low-order prediction of broadband FEGV noise begins with an assumption that the rotor wake turbulence is convected by the mean wake flow and can be viewed as a combination of Fourier components. Each component can be modeled as a three-dimensional gust disturbance with a specified frequency and wave number vector. The amplitude of each gust is specified by the turbulence spectrum. The full vane response to each gust is created using a strip theory in which each strip is considered independently. An integral equation is solved to obtain the unsteady surface pressure at each strip using the method described by Ventres.\(^\text{11}\) Once the unsteady surface pressure on each strip is known, it is appropriately weighted by the inflow turbulence spectrum and used to compute the acoustics downstream of the vane via the Green’s function for a cylindrical annulus. The inflow turbulence is modeled using a Liepmann spectrum that relies on passage averaged values of turbulence intensity and turbulence length scale taken either from experiment or computation. Other similar formulations include the variation of the turbulent intensity across a rotor wake passage in the inflow turbulence model.\(^2,3\) However, it has been shown that the final results for the exhaust duct broadband noise are only slightly lower at high frequency when one uses the average passage value of the turbulent intensity.\(^4\) Thus, the average passage value at each radial strip is used with this model currently.

B. The method for including vane shape effects

A method for including the effect of vane shape was investigated. The semianalytical flat-plate cascade response in the original model was replaced by an asymptotic formulation. A working asymptotic cascade formulation was not available, so the single airfoil response was utilized in order to make preliminary conclusions as to the effectiveness of the methodology. The asymptotic single airfoil response solution was derived by Ayton and Peake.\(^6\) They utilize a perturbation analysis that provides a leading edge solution with a trailing edge correction. They show that the nondimensional unsteady pressure distribution along the chord of a flat-plate airfoil at 0 angle of attack due to a unit amplitude gust with upwash of the form \(u_t \exp i(\omega t - k \cdot x)\) being carried by a flow with Mach number \(M\) has the form \(h_l - h_t\) where

\[
h_l = -ie^{-\pi/4} \frac{1}{\sqrt{\pi k_1 x (1 + M)}} e^{\frac{i k_1 M}{\beta^2} (1 - M) x}
\]

\[
h_t = -ie^{-\pi/4} \frac{1}{\sqrt{\pi k_1 (1 + M)}} \text{Erfc} \left[ e^{-\pi/4} \sqrt{-2 \frac{k_1 M}{\beta^2}} (x - 1) \right] e^{\frac{i k_1 M}{\beta^2} (1 - M) x}
\]

Results from this formulation were compared to the integral equation method for determining the unsteady surface pressure on a single airfoil due to a unit amplitude gust and the results compare perfectly even for rather low values of \(k_1 M\). The real geometry solution is too lengthy to include here and instead the readers are directed to the original paper.\(^6\)
C. Low-order method based on cascade acoustic modes

In order to provide a one-to-one comparison of the original method and one that bases the broadband prediction upon cascade acoustic propagating modes, acoustic mode formulation was derived in the exact manner that the original method was derived. The broadband acoustic power in the duct downstream of the fan exit guide vane due to rotor-wake interaction with the FEGV is computed assuming that downstream of the FEGV there is a constant area annulus. The power can be calculated based on the component of the intensity in the axial direction

\[ P = \int_0^{2\pi} \int_{R_{\text{hub}}}^{R_{\text{tip}}} (\mathbf{I} \cdot \hat{x}_{\text{axial}}) r dr d\theta \]  

(3)

The intensity will be calculated based on the acoustic pressure and acoustic velocity in the duct at a given axial location. The pressure and velocity are computed in the frequency-wave-number domain using a strip theory approach which will be described in this section.

The spectrum of the rotor wake turbulence which interacts with the exit guide vanes is defined by the velocity correlation

\[ Q_{2,2}(x, t, r) = \frac{1}{(2\pi)^3} \iiint E_{2,2}(k, t, r) e^{ik \cdot x} dk \]  

(4)

which is linked to the turbulence intensity via Eq. 1-42 in Hinze\(^2\)

\[ Q_{2,2}(x, t, r) = (u_2)_A(u_2)_B = u_{2_A}(\xi, t')u_{2_B}(\xi + x, t' + t) \]  

(5)

when \( A = B \). \( u'^2 \) is related to \( Q_{2,2} \). For this application, the spectrum is modeled using the Liepmann description

\[ E_{2,2}(k, t) = (2\pi)^3 \frac{2u'^2}{\pi^2} \frac{L_s^5(k_1^2 + k_3^2)}{(1 + (kL_s)^2)^3} \]  

(6)

with Hinze Eq. 1-36 giving the notation \( u'_n = \sqrt{u_n'^2} \) and \( k = \sqrt{k_2^2 + k_2^2 + k_3^2} \). The transform pair is defined in the opposite manner in Hinze, hence the extra \( 2\pi \) factors. \( L_s \) is understood to be the length scale in the flow or streamwise direction. It is noted that for the rotor wake problem, the energy spectrum is considered dependent upon radial location. The data are obtained at various radial stations and it is seen that both the length scale and the turbulence intensity vary in the radial direction.

With this understanding and notation, the derivation for the vane response to incoming turbulence can be formulated. The turbulence is convected with the wake in the rotor frame of reference, but the vane response depends upon the upwash velocity in its frame of reference. Hence, \( x \) is defined as the vane frame variables and \( X \) as the rotor frame variables. Here, the overbar notation from Hinze is replaced with brackets ( ) and the terms averaged and expected value are taken to be the same. In the rotor frame, the turbulent velocity is carried by the mean flow so we can write

\[ u_2(X, r, t) \approx u_2(X - W_t, r) \]

Here the vectors \( X, W, \) and \( \hat{n} \) are taken to be in the moving rotor frame with \( W \) being the mean flow velocity. The correlation function of the upwash then is

\[ \langle u_2(X - W_t, r)u_2^*(Y - W_T, r) \rangle \]  

(7)
In the stator frame the upwash correlation transform is defined as
\[
\langle \tilde{w}\tilde{w}^* \rangle = \langle \tilde{u}_2(k, \omega, r)\tilde{u}_2^*(K, \nu, r) \rangle = \iiint_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle u_2(x, t, r)u_2^*(y, \tau, r) \rangle e^{i\omega t - ikx} e^{-i\nu \tau + iKY} \, dx \, dy \, dt \, d\tau
\]
(8)

Here the integrals all range from $-\infty$ to $\infty$, and $\omega$ and $\nu$ are radial frequencies. The `\~` indicates transform in three spatial dimensions and time.

The coordinate transform between the stator and rotor frames of reference is given by
\[
X = x + D + \Omega x_3 \dot{t} e_2 \\
Y = y + D + \Omega y_3 \tau \dot{e}_2
\]
(9)

where $D$ has the form $(D_1, D_2, 0)$ and by choice of coordinates and the way the strips are defined $x_3 = X_3$ and $y_3 = Y_3$. Thus, Eq. 8 can be rewritten as follows.
\[
\langle \tilde{w}\tilde{w}^* \rangle = \iiint_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle u_2(X - Wt, r)u_2^*(Y - W\tau, r) \rangle e^{i\omega t - ik(X - D - \Omega X_3 \dot{t} e_2)} e^{-i\nu \tau + iK(Y - D - \Omega Y_3 \tau \dot{e}_2)} \, dX \, dY \, dt \, d\tau
\]
(10)

The change of variables
\[
\xi = X - Y + W(t - \tau)
\]
is introduced giving
\[
\langle \tilde{w}\tilde{w}^* \rangle = \iiint_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle u_2(X - Wt, r)u_2^*(X - Wt - \xi, r) \rangle e^{i\omega t - ik(X - D - \Omega X_3 \dot{t} e_2)} e^{-i\nu \tau + iK(X - W(t - \tau) - \xi - D - \Omega (X_3 - \xi_3) \tau \dot{e}_2)} \, dX \, d\xi \, dt \, d\tau
\]
(11)

The integrals with respect to $t$ and $\tau$ give rise to delta functions such that
\[
\langle \tilde{w}\tilde{w}^* \rangle = (2\pi)^2 \iiint_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle u_2(X - Wt, r)u_2^*(X - Wt - \xi, r) \rangle \delta(\omega - k \cdot W + k_2 \Omega X_3) \delta(\nu - K \cdot W + K_2 \Omega (X_3 - \xi_3)) e^{-ik(X - D)} e^{ik(K \cdot (X - \xi - D))} \, dX \, d\xi
\]
(12)

The appearance of the delta functions shapes the connection between the radial frequency and the relative mean flow. However, the focus is on a particular strip here and the $X_3$ radial direction appears because the turbulence may have an unsteady variation in this direction. However, at the strip, the quantity $K \cdot W + K_2 \Omega Y_3 = K \cdot U(Y_3)$ and thus the delta function will give rise to the expected relation $K \cdot U = \nu$ for the strip located at radial location $Y_3$. Because it is assumed that the strip is affected only by itself (the correlation length scale is smaller than or on the order of the strip size) and to formally make the connection between the span and radial directions, $X_3$ and $X_3 - \xi_3$ in the delta functions are replaced by $r$. The integral with respect to $\xi$ now provides the transform of the expected value of the upwash correlation given earlier as $E_{2,2}$, and
\[
\langle \tilde{w}\tilde{w}^* \rangle = (2\pi)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\omega - k \cdot W + k_2 \Omega r) \delta(\nu - K \cdot W + K_2 \Omega r) \, e^{iD \cdot (k - K)} e^{iX \cdot (k - K)} \, dX
\]
(13)
Finally, the integration with respect to \( X \) can be performed and the relation \( U_o = W - \Omega r \) inserted to give

\[
\langle \tilde{w} \tilde{w}^* \rangle = (2\pi)^5 \delta (\omega - k \cdot U_o) \delta (\nu - K \cdot U_o) E_{2,2}(K, r)e^{iD \cdot (k - K)} \delta (k - K)
\] (14)

This expression for the amplitude of the transformed velocity in the upwash direction will be utilized to find the final form of the intensity.

The exit guide vane response to the turbulence and the subsequent acoustic waves produced in the annulus downstream must now be modeled. The Ventres method\textsuperscript{11} for determining the unsteady pressure response of a cascade vane to a gust-like disturbance forms the basis for the model. However, instead of determining the unsteady vane pressure and then applying the Green’s method to obtain the acoustic pressure away from the vane, the rectilinear acoustic modes will be used to determine the pressure in the duct. We assume that \( \Delta \tilde{p} \) has been calculated in the frequency-wave-number domain such that at any radial strip noted by \( r \) on the \( j \)th vane

\[
\Delta p_j(r, X_1, \omega, k) = \rho_o U_o \tilde{u}_2(r, \omega, k) \tilde{g}'(X_1, k_1g, k_2, M_g(r)) e^{ik_3r} e^{ijk \cdot h} d^3k
\] (15)

where the subscript \( g \) indicates that the Graham similarity rules\textsuperscript{13} have been used to obtain the response to a three-dimensional gust. \( h \) is the vane gap vector and \( \tilde{g}' \) indicates that this is related to the function \( \tilde{g} \times \text{Factor} \) given by Graham. We will label

\[
\Delta \tilde{p}(r, X_1, \omega, k) = \rho_o U_o \tilde{u}_2 \tilde{g}' e^{ik_3r}
\] (16)

The method of solution for the unit response function \( \tilde{g}' \) is described both in Goldstein’s book\textsuperscript{14} and in the Ventres report\textsuperscript{11}. The flat-plate cascade response depends on the wave number vector \( k \), the Mach number of the mean flow which is assumed to be in the chordwise direction, the gap-to-chord spacing, and the cascade stagger angle.

The acoustic pressure far away from the cascade can be calculated in terms of the acoustic modes that would propagate in a rectilinear geometry. The method for computing these acoustic modes is given in Goldstein’s book\textsuperscript{14} and are described in more detail in the paper by Atassi and Hamad.\textsuperscript{15}

For a cascade of stators, it has been shown that acoustic modes will cut-on and propagate away from the cascade when the condition

\[
(kh_{1g})^2 > (\Gamma_n)^2
\]
is satisfied, with the subscript $pg$ noting a variable in the Prandtl-Glauert plane and

$$k_1 = \frac{\omega}{U_o} \quad (17)$$

$$h_1 = h \sin \chi \quad (18)$$

$$h_2 = h \cos \chi \quad (19)$$

$$h_{pg} = h \sqrt{(\sin \chi)^2 + (\cos \chi)^2 \beta^2} \quad (20)$$

$$\kappa = \sqrt{\left(\frac{k_1 M}{\beta}\right)^2 - \left(\frac{k_3}{\beta}\right)^2} \quad (21)$$

$$K = \frac{k_1 M}{\beta^2} = \frac{\omega}{c_o \beta^2} \quad (22)$$

$$\Gamma_n = -2\pi n + \sigma + hKM(\sin \chi) \quad (23)$$

$$\sigma = k_1 h_1 + k_1 h_2 \quad (24)$$

$$\chi_{pg} = = \arctan \left( \frac{1}{\beta \tan \chi} \right) \quad (25)$$

$$\delta_n = \arccos \left( \frac{\Gamma_n}{\kappa h_{pg}} \right) \quad (26)$$

$$\theta_n^\pm = -\chi_{pg} \pm \delta_n \quad (27)$$

$$\alpha_n^\pm = \kappa \sin(\theta_n^\pm) \quad (28)$$

The gap at any radial location is $h = \frac{2\pi r}{V}$ where $V$ is the number of vanes. The acoustic pressure for mode $n$ far from the cascade is then given as

$$\tilde{p}_n^\pm(\omega, \mathbf{k}) = -\rho_o \tilde{u}_2 U_o \frac{\pi \cos \theta_n^\pm}{h_{pg} \sin \delta_n} f_o e^{-i[\omega t + (\kappa \sin \theta_n^\pm + MK) x_1 - \kappa \cos \theta_n^\pm \beta x_2 + k_3 x_3]} \quad (29)$$

where $x$ is again the coordinates in the stator frame of reference. The $\pm$ indicates upstream and downstream modes. The $f_o$ function is obtained through the integration of the unsteady pressure jump

$$\rho_o \tilde{u}_2 U_o f_o = \frac{1}{2\pi c} \int_{-1/2}^{1/2} \Delta \tilde{p}(x_1, 0, x_3) \exp^{-i(KM + \alpha_n^\pm) \frac{x_1}{c}} \, d(\frac{x_1}{c})$$

Because one needs the component of the intensity in the axial direction, it is helpful to rewrite the acoustic wave in the machine frame of reference $x_{1m}, x_{2m}, x_{3m}$. There is a one-to-one correspondence between $x_3$ and $x_{3m}$ (this coordinate can be thought of as the radial coordinate $r$) and the cascade stagger angle gives the relation between the other directions such that

$$x_{1m} = x_1 \cos \chi - x_2 \sin \chi \quad (30)$$

$$x_{2m} = x_1 \sin \chi + x_2 \cos \chi \quad (31)$$

$$M_1 = M \cos \chi \quad (32)$$

$$M_2 = M \sin \chi \quad (33)$$

$$\beta_1^2 = 1 - M_1^2 \quad (34)$$

$$\cos \chi_{pg} = \frac{\beta}{h_{pg}} \cos \chi \quad (35)$$

$$\sin \chi_{pg} = \frac{h}{h_{pg}} \sin \chi \quad (36)$$
The exponential in Eq. (29) can then be rewritten using trigonometric relations as

\[-i[\omega t + M_1K x_{1m} - \kappa \frac{h}{h_{pg}} (M_1 M_2 \cos \delta_n + \beta \sin \delta_n) x_{1m} - \kappa \frac{h}{h_{pg}} \beta_1^2 \cos \delta_n x_{2m} + M_2 K x_{2m} + k_3 x_{3m}]\]  

(37)

The definition of \(\delta_n\) is used to obtain

\[\kappa \cos \delta_n = \frac{h}{h_{pg}} [k_1 \sin \chi + k_2 \cos \chi - \frac{2n\pi}{h} + M K \sin \chi]\]  

(38)

When one also notes \(\left(\frac{h}{h_{pg}}\right)^2 = \frac{1}{\beta_1^2}\), the terms multiplying \(x_{2m}\) become

\[k_{2ac} = [k_1 \sin \chi + k_2 \cos \chi - \frac{2\pi n}{h}]\]  

(39)

With the relationship for \(\kappa \cos \delta_n\), one can also show that

\[\kappa \sin \delta_n = \frac{h}{h_{pg}} \beta_1^2 \sqrt{\frac{\beta_1^2}{\beta_1^2} - (k_{2ac} + M K \sin \chi)^2}\]  

(40)

and then the term multiplying \(x_{1m}/c\) can be rewritten as

\[k_{1ac}^\pm = \left(\frac{1}{\beta_1^2} (-\frac{M_1 \omega}{c_o} + M_1 M_2 k_{2ac}) + \frac{1}{\beta_1^2} \sqrt{\frac{\omega}{c_o} - M_2 k_{2ac})^2 - \beta_1^2(k_{2ac}^2 + k_3^2)}\right)\]  

(41)

So the modal pressure is expressed in the machine axis frame as

\[\tilde{p}_n^\pm = -\rho_o \tilde{u}_{2o} U_o \frac{\pi c \cos \theta_n^\pm}{h_{pg} \sin \delta_n} f_o e^{i(-i\omega t + ik_{1ac} x_{1m} + ik_{2ac} x_{2m} - k_3 x_{3m})}\]  

(42)

Now that the acoustic pressure upstream and downstream of the cascade is known, the intensity can be computed. The linearized acoustic intensity is found using

\[I = \left(\frac{p}{\rho_o} + U_o \cdot \mathbf{u}_{ac}\right) (\rho_o \mathbf{u}_{ac} + \mathbf{U} \rho_{ac})\]  

(43)

Hence, in order to find the intensity, the acoustic velocity is needed. The linearized Euler equation gives

\[\frac{D \mathbf{u}_{ac}}{Dt} + \frac{1}{\rho_o} \nabla p = 0\]  

(44)

\[\rho_o (-\omega + U_o \cdot k) \tilde{u}_{ac} + k \tilde{p} = 0 \text{ in the stator frame}\]  

(45)

\[\rho_o (-\omega + U \cdot k_{ac}) \tilde{u}_{acm} + k_{ac} \tilde{p} = 0 \text{ in the machine frame}\]  

(46)

\[\tilde{u}_{acm} = -\frac{k_{ac}}{\rho_o (-\omega + U \cdot k_{ac})} \tilde{p}\]  

(47)

The tilde still denotes a variable transformed in time and three spatial in the stator frame of reference. \(U_o\) is the mean flow specified in the stator frame and \(U\) is specified in the machine frame and their amplitudes are equivalent \(U_o = U\).
The axial component of the intensity, $I \cdot \mathbf{x}_{\text{axial}}$ is aligned with $x_1$. The subscript 1 denotes this direction here

$$I \cdot \mathbf{x}_{\text{axial}} = \left( \frac{p}{\rho_o} + \mathbf{U} \cdot \mathbf{u}_{ao} \right) (\rho_o u_{ac1} + U_1 \rho_{ac})$$ \hspace{1cm} (48)

$$= \frac{1}{(2\pi)^8} \int \int \int \int \int \left( \frac{\bar{p}}{\rho_o} + \mathbf{U} \cdot \mathbf{u} \right) e^{-i\omega t + i\mathbf{k} \cdot \mathbf{x}}$$

$$\times (\rho_o \bar{u}_{ac1} + U_1 \rho_{ac}) \mathbf{e}^{i\omega t - i\mathbf{k} \cdot \mathbf{x}} d\omega d\mathbf{k}$$ \hspace{1cm} (49)

$$= \frac{1}{(2\pi)^8} \int \int \int \int \bar{p}(\omega, \mathbf{k}) \hat{p}^\dagger (\nu, \mathbf{K}) (1 - \frac{\mathbf{U} \cdot \mathbf{k}_{ac}}{\lambda_\nu})(-\frac{K_{1ac}}{\lambda_\nu} + \frac{U_1}{c_o^2}\nu, \mathbf{K})$$

$$\times e^{-i(\omega - \nu)t + i(\mathbf{k} - \mathbf{K}) \cdot \mathbf{x}} d\omega d\mathbf{k} \hspace{1cm} (50)$$

where we have used the relation, $\bar{p} = \hat{p}_{ac} c_o^2$ and

$$\lambda_\omega = -\omega + \mathbf{U} \cdot \mathbf{k}_{ac\text{dim}}$$ \hspace{1cm} (51)

$$\lambda_\nu = -\nu + \mathbf{U} \cdot \mathbf{K}_{ac\text{dim}}$$ \hspace{1cm} (52)

The Fourier transform of the pressure by modes is given in Eqs. (29) and (42).

$$\hat{p}_{n\pm}^\dagger (\omega, \mathbf{k}) \hat{p}_{n\pm}^\dagger (\nu, \mathbf{K}) = \rho_o^2 U_o^2 \bar{u}_2(\omega, \mathbf{k}) U_2(\nu, \mathbf{K})^* f_o(\omega, \mathbf{k}) f_o^*(\nu, \mathbf{K}) \left( \frac{\pi c \cos \theta_n^2}{h_{pg} \sin \delta_n} \right) \mathbf{k} \left( \frac{\pi c \cos \theta_n^2}{h_{pg} \sin \delta_n} \right) \mathbf{K}$$ \hspace{1cm} (53)

and the magnitude of the transformed upwash velocity is given in Eq. (14). These can be substituted into Eq. (50) then the integration with respect to $\mathbf{K}$ and $\nu$ can be completed. The delta functions require that $\mathbf{K} = \mathbf{k}$ and then that $\nu = \omega$. The expression can be simplified then as

$$I_1 = \sum_n \frac{1}{(2\pi)^3} \int \int \int \rho_o^2 U_o^2 |f_o|^2 \left( \frac{\pi c \cos \theta_n^2}{h_{pg} \sin \delta_n} \right)^2 \times$$

$$E_{2,2}(\mathbf{k}, r) \delta(\omega - \mathbf{k} \cdot \mathbf{U}_o)(\pm \frac{\omega}{\rho_o c_o^2}) \sqrt{\left( \frac{\omega}{c_o} - M_2 k_{2ac} \right)^2 - \beta_2^2 (k_{2ac}^2 + (k_3)^2)} d\omega d\mathbf{k}$$ \hspace{1cm} (54)

Integration with $k_1$ gives the relation $k_1 = (\omega - k_2 U_{o2})/U_{o1}$ which in the stator frame where $U_{o2} \sim 0$ gives $k_1 \sim \overrightarrow{\mathbf{K}}$. (To perform the integration with $k_1$ and utilize the delta function one must make the substitution $k_{new} = -k_1 U_{o1}$.) This relation has already been utilized in the derivation and does not add any additional constraint now.

$$I_1 = \sum_n \frac{1}{(2\pi)^3} \int \int \int \rho_o U_o^2 |f_o|^2 \left( \frac{\pi c \cos \theta_n^2}{h_{pg} \sin \delta_n} \right)^2 \times$$

$$E_{2,2}(\mathbf{k}, r)(\pm \frac{\omega}{U_{o1} c_o} \sqrt{\left( \frac{\omega}{c_o} - M_2 k_{2ac} \right)^2 - \beta_2^2 (k_{2ac}^2 + (k_3)^2)} d\omega dk_2 dk_3$$ \hspace{1cm} (55)

The power per Hertz contributed by the radial strip of interest is obtained by integrating the intensity over the annular area of interest (between radial locations $R_e$ and $R_{e+1}$), replacing $\omega = 2\pi f$ and dropping the integration with respect to $f$. We also use the fact that $U_o = U_{o1}$ because the flow is completely along the chordwise direction in the stator frame.

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The total Power/Hertz contributed by all the strips is nondimensionalized by $\rho_o c_o^2 R_{tip}^3$ and $E_{2,2}$ from Eq. (6) is substituted to arrive at

$$
\frac{\text{Power}}{\text{Hertz}}_{nd} = \sum_{i} \frac{1}{(2\pi)^2} \pi (R_{i+1}^2 - R_i^2) \int_{k_2} \int_{k_3} \rho_o U_o |f_o|^2 \left( \frac{\pi c \cos \theta_n^\pm}{h_{pg} \sin \delta_n} \right)^2 \times
$$

$$
E_{2,2}(k, r) \left( \pm \frac{\omega}{\lambda^2} \sqrt{\left( \frac{\omega}{c_o} - M_2 k_{2ac} \right)^2 - \beta_1^2 (k_{2ac}^2 + (k_3)^2)} \right) dk_2 dk_3
$$

(56)

with $N_R$ being the number of strips and $u_{nd}^2 = u^2 / U_o^2$.

It is noted that this is the exact form of the power given in the thesis by Logue except for an additional restriction on $k_3$ imposed in that thesis. The restriction that selects $k_3$ in the manner required to obtain allowable wave number vectors for the tonal problem is not imposed here just as it was not imposed in the original Green’s function method for modeling the broadband noise.

3. Results

A. Effect of vane shape

In order to consider the effect of vane shape, the flat-plate cascade unsteady response code that forms the core solver of the original method was replaced by a flat-plate airfoil unsteady solver. The single airfoil model cannot capture the vane surface pressure of a cascade of closely spaced vanes. As such, when one uses the method modified to run with the flat-plate airfoil model to predict the noise from the Source Diagnostic Test (SDT) baseline vane case at approach rotor speed, the prediction varies greatly from the one obtained utilizing the flat-plate cascade response model. This is shown in Figure 1(a). With this prediction difference in mind, one can still explore the effect of including vane thickness and camber and flow incidence (angle of attack) using the method based on the single airfoil response model. The camber and angle of attack results in Figures 1(c)-(d) were obtained using a 1% thick airfoil. Camber is shown to slightly increase the sound power level at higher frequency. Angle of attack is shown to slightly decrease sound power level at all frequencies. The amount of camber and angle of attack that can be included is limited by the thin airfoil approximation essential in the asymptotic model. For the 1% thick airfoil used in these tests, only camber up to 5% and angles of attack up to 4 degrees can be simulated. Therefore the true nature of the SDT vane which is 7% thick and has approximately 9% camber cannot be modeled with this approach.

In Figure 1(b), thickness is shown to reduce the sound power level at lower frequencies but at higher frequencies it is increased. This trend is due to the rapid distortion assumption that is used in the gust response calculation. When a blade-vortex interaction (BVI) type model is used added thickness reduces the lift response across the entire frequency range. Therefore, the trend at high frequency with thickness shown in Figure 1(b) is believed to be unphysical and an outcome of the modeling assumptions.
Figure 1. Effect of geometry on sound power level for the SDT baseline case at approach speed.
B. Predictions made using rectilinear cascade acoustic modes as basis for duct pressure

In this section, results from two low-order broadband prediction methods are compared:

1. the original method that uses the cascade vane unsteady pressure response on strips to build the 3D vane unsteady pressure response and then integrate the vane response together with the appropriate Green’s function to obtain the pressure in the annular duct.

2. the acoustic mode based method that determines the acoustic pressure in the annular duct based on the rectilinear propagating acoustic modes for each strip.

The formulation used for the second method is given in Section 2.C. Another similar approach was presented by Hanson and Horan\textsuperscript{7} with the main different being that an expression for the potential function describes the propagating acoustic mode instead of the pressure. The Hanson approach has been used to obtain predictions of the SDT cases\textsuperscript{17} and results were presented as part of the Broadband Benchmark Workshop in 2014. It was shown that the method provided reasonable predictions. The method based on the acoustic pressure modes was also previously shown to give reasonable results.\textsuperscript{8}

It was assumed then, that good comparisons would be obtained in this study as well. Currently, such positive results have not been realized. Future efforts may resolve issues in the implementation responsible for the lack of agreement across the two methods at this time. The validation work that has been undertaken and the current comparisons are given here.

In order to determine if the acoustic mode calculation is implemented properly, the 4th Category problem from the 3rd CAA Benchmark Workshop was considered. This category focused on harmonic excitation of a fan stator by the rotor wake. A narrow annulus was considered in order to allow cascade calculations to be validated. The stator vanes are flat-plates with no sweep or lean and chord-to-tip-radius ratio of 0.2618. The hub-to-tip ratio is 0.98. The flow mach number is 0.5. The gust amplitude is given as 10% of the mean flow. The problem models 16 rotors and 24 stators. The modal pressure amplitude of the acoustic waves in the annulus downstream and upstream of the stator for different rotor tip Mach numbers are provided for validation. Specifically, the pressure in the annulus is defined by

\[ p(r, \theta, x, t) = p_\infty \sum_{q=-\infty}^{\infty} \sum_{\mu=0}^{\infty} A_{q\mu} \Psi_{q\mu}(r) e^{i(q\theta - B\Omega t)} \]  

(58)

where $\Psi_{q\mu}$ are the normalized mode shape functions (combinations of Bessel functions), $B$ is the number of rotor blades, and $\Omega$ is the rotor rpm.

For the narrow annulus, a non dimensional lift value is also provided for validation. Four frequencies (rotor tip speeds) are given. Only for the higher two speeds are there propagating modes in the cascade analysis. So the lift and modal pressure amplitudes for just two of the speeds are given in the table below. The benchmark results give values calculated by both Namba and Schulten but here only Schulten’s values have been included. All of the comparisons are quite good.

A second narrow annulus problem was then created to consider the broadband calculation. Here the SDT low-count vane case at approach rotor speed provided input for the calculation. The rotor and stator counts were selected to match the SDT low-count case. While the outer radius of the stator is 11 inches as is the case for the SDT cases, the stator hub-to-tip ratio was chosen to be 0.98. The chord-to-tip-radius ratio was set based on the SDT low-count but the vanes were chosen to be flat plates at zero stagger. The mean flow, turbulence intensity, and length scale were chosen to be the SDT low-count values at mid-span.

The original broadband calculation based on the Green’s function method was compared to the method that uses the acoustic modes given in Equation (57). The same number of strips and the same input values for
Table 1. Narrow annulus results using several methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>( M_T, k_1 )</th>
<th>Up/down</th>
<th>2D Lift (complex, mag.)</th>
<th>Modal pressure amp. (complex, mag.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schulten</td>
<td>0.4763, 1.9949</td>
<td>Up</td>
<td>(0.01816, 0.1413), 0.1424</td>
<td>(-0.007538, 0.002055), 0.004813</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.01756, 0.1409), 0.1420</td>
<td>(-0.007303, 0.002328), 0.004665</td>
</tr>
<tr>
<td>Schulten</td>
<td></td>
<td></td>
<td>(0.01756, 0.1409), 0.1420</td>
<td>(-0.007705, 0.000980), 0.004767</td>
</tr>
<tr>
<td>Cascade mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green’s function</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Schulten</td>
<td>0.6495, 2.72</td>
<td>Down</td>
<td>same</td>
<td>(0.01061, 0.01556), 0.01881</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.01112, 0.01506), 0.01872</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.01127, 0.01503), 0.01848</td>
</tr>
<tr>
<td>Schulten</td>
<td>0.09709, 0.1586</td>
<td>Up</td>
<td>(0.09656, 0.1591), 0.1850</td>
<td>(0.007364, -0.002453), 0.004762</td>
</tr>
<tr>
<td></td>
<td>(0.09656, 0.1591), 0.1860</td>
<td></td>
<td></td>
<td>(0.007272, -0.002475), 0.004681</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.007369, -0.001835), 0.004615</td>
</tr>
<tr>
<td>Schulten</td>
<td>0.09656, 0.1591</td>
<td>Down</td>
<td>same</td>
<td>(-0.009946, 0.005870), 0.01155</td>
</tr>
<tr>
<td></td>
<td>0.09709, 0.1586</td>
<td></td>
<td></td>
<td>(-0.009699, 0.006215), 0.01152</td>
</tr>
<tr>
<td></td>
<td>0.09656, 0.1591</td>
<td></td>
<td></td>
<td>(-0.009909, 0.006209), 0.01168</td>
</tr>
<tr>
<td>Schulten</td>
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<tr>
<td>Cascade mode</td>
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<td>Green’s function</td>
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</tbody>
</table>

Figure 2. SDT low count at approach based narrow annulus problem. Comparison between solutions obtained using the two low-order methods.

Next, the radial extent was increased such that the hub-to-tip ratio was changed from 0.98 to 0.9, 0.8 and then 0.5. The stator geometry in each case is a flat-plate with the chordlength matching the SDT low-count set at zero stagger. The mean flow and turbulence parameters are constant across the span and identical to those used in the narrow annulus model problem. The results from the acoustic mode based method shown in Figure 3 are readily explained. At each strip the only difference in the cascade parameters is the spacing (gap-to-chord-ratio). For the vane geometry considered here, the gap-to-chord value varies from 0.48 at the tip to 0.83 at the hub. This slight change in the cascade parameters leads to only a small change in the set of propagating modes and their amplitudes. Thus, for each strip the integrals in Eq. (57) evaluate to almost the same number. The main difference then is the annular area that weighs that strips contribution to the total power. In Figure 3(a), the total power that would be obtained if indeed the only difference from strip to strip was the annular area is shown in gray. The actual computed results that account for the slight difference in the propagating modes due to the gap-to-chord differences are shown in black. However, if each result is scaled by the span-to-tip-radius ratio, the results collapse as shown in Figure 3(b).
Figure 3. Downstream broadband noise results for model problem using the acoustic mode method.

While the results in Figure 3(a) follow logically from the formulation, they do not match those obtained when using the original method shown in Figure 4. The Green’s function based method predicts that there is little difference in the noise produced by the differing span vanes at the lower frequency but at the higher frequency, the spectral shape changes and the larger span vane would produce more noise. Thus there is no collapse of the data with any scaling value.

This model problem with a span-to-tip-radius ratio of 0.5 is very close to the actual SDT low-count problem especially when one selects the trailing edge stagger angle for use in the original method. Therefore it is not surprising that the predictions from the two methods for the SDT low-count vane at approach rotor speed differ particularly at the higher frequencies as shown in Figure 5. As mentioned previously, others have used the acoustic mode based method to produce predictions that are much closer to those obtained using the original method. Further investigation is needed to identify issues with the current implementation of the acoustic mode based method.

4. Conclusions

In this paper, a method for including real vane geometry in a low-order prediction method for broadband fan noise was consid-
An asymptotic single airfoil response formulation was adapted and used to investigate the feasibility of utilizing such an approach to capture real vane geometry effects. It is shown that the limitations on the asymptotic method, namely thin-airfoil restrictions, limit the utility of the method for modeling real fan vane geometries. In addition, the use of a rapid distortion based unsteady response model may not be valid for simulating vanes with appreciable thickness. Alternate methods for obtaining the response of a cascade of real vanes to a gust will be considered in the future.

The difference between two low-order techniques for predicting broadband interaction noise from a fan stage that are built upon unsteady cascade theory and strip theory is explored. The original method developed over the past several years utilizes the cascade surface response to a gust for multiple strips from hub-to-tip and numerous gusts with different frequency-wave number combinations to estimate the total unsteady vane surface response to a turbulent inflow. A Green’s function approach is then used to determine the sound produced in the annulus downstream of the vanes. A second method, described in this paper, utilizes the propagating acoustic modes created by a cascade gust interaction. The rectilinear acoustic modes for each strip produced by numerous gusts with varying frequency-wave number combinations are “mapped” into the annular duct and appropriately weighted based on strip location. The results from the two methods do not agree. Because other researchers have previously shown that the acoustic mode based method can better predict fan interaction noise than what is shown in this paper, it is believed the current formulation must be revisited and updated.

Acknowledgments

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References


