Minimal Energy Routing with Latency
Quality-of-Service Guarantees

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Focus on low data rate SNETs serving time-critical applications (e.g., surveillance, environmental monitoring).

A common strategy to preserve energy is to have SNET nodes shut down their radios (go to “sleep”).

Hence, SNETs can operate in an event-based sensing mode.

This creates challenges for (timely) routing data to the gateway.

Trade-off: Long hops (lots of energy) vs. short hops (large delay).
Outline

- **System Model.**
- **Performance metrics and analysis.**
- **Routing algorithms.**
- **Numerical results.**
- **Conclusions.**
System Model

- Continuous time $t$.
- $N$ nodes, 1 gateway (node $N + 1$): $(\mathcal{V}, \mathcal{E})$.

\[ X^t_k: \text{channel state} \] between nodes $s(k)$ and $d(k)$ at time $t$
- Stochastic process, independent of all the other channels.
- Two-state Markov process: 1 (“good”) and 0 (“bad”).

\[ Y^t_i: \text{state of node } i \text{ at time } t; 1 \text{ (ON)} \text{ or } 0 \text{ (OFF)} \]
- Two-state Markov process.
- $Y^t_{N+1} = 1$, for all $t$.

Energy consumption model for each link $k \in \mathcal{E}$
- $c_k$: power consumption before connection is established (Watt).
- $g_k$: energy consumption after connection is established (Joule).
Comments on the System Model

- Channel model.
  - Finite state Markov model for Rayleigh fading channels.
  - Gilbert-Elliot model.

- Sleeping schedule model.
  - No time synchronization required.
  - Randomized transition between ON and OFF (good for security).

- Few data at a low rate.
  - Data is generated at a slow time scale.
  - Transmission, propagation and queueing delays negligible.
  - Only causes of latency
    - Bad channel conditions.
    - Sleeping nodes.
Objective

Let $p$ the path from node 1 to the gateway.

- Necessary and sufficient condition for successful delivery on link $k$ at time $t$: **channel is good** $(X_k^t = 1)$ and **downstream node is ON** $(Y_{d(k)}^t = 1)$.

- $L_k$: (random) time needed for node $s(k)$ to successfully deliver a packet to $d(k)$.

- Total time for packet delivery on path $p$: $\sum_{k \in p} L_k$.

- **Energy consumption** on path $p$: $T_p = \sum_{k \in p} (c_k L_k + g_k)$.

- **Latency probability** $P(L_p \geq d)$ for a constant $d$ on path $p$.

**Objective**

**Characterize** $E(T_p)$ and $P(L_p \geq d)$, and **optimize** by selecting routing strategies.
Performance Metrics

- **Expected energy consumption**

\[ E(T_p) = \sum_{k \in p} E(T_k) \]

(We can obtain a closed-form expression)

- **Latency probability** \( P(L_p \geq d) = P\left(\sum_{k \in p} L_k \geq d\right) \)

- Requires convolution of latencies on all links in path \( p \) ...
- Computationally *expensive* for long paths.
- **Significant** communication overhead.
- We have developed tight approximations of \( P(L_p \geq d) \) with 
  **much lower complexity**.
Bound on latency probability $P(L_p \geq d)$

Latency probability is hard to compute exactly. Consider bounds and approximations ...

**Chernoff bound:** For any $\theta \geq 0$

$$P(L_p \geq d) \leq \exp \left( \sum_{k \in p} \Lambda_k(\theta) - \theta d \right),$$

where $\Lambda_k(\theta)$ is the **logarithmic moment generating function** of $L_k$ (it depends on the transition probability matrices of channel and sleeping schedule Markov chains).
Large Deviations-type asymptotic for $P(L_p \geq d)$

Let $\lambda_k$ the maximum eigenvalue of $H_k$ (a matrix that depends on the transition probabilities of the Markov chains).

**Theorem**

For any $\mathbf{p} \in \mathcal{P}$, we have

$$\lim_{d \to \infty} \frac{1}{d} \log P(L_{\mathbf{p}} \geq d) = \max_{k \in \mathbf{p}} \lambda_k,$$

where $\lambda_k$ is a quantity that characterizes link $k$.

Interpretation: $P(L_{\mathbf{p}} \geq d) \approx e^{d \max_{k \in \mathbf{p}} \lambda_k}$, i.e., it decreases exponentially w.r.t. $d$.

- “Bottleneck link”.
- **Convenient** to characterize and update.
Energy vs. delay trade-off

- Generally, $E(T_p)$ and $P(L_p \geq d)$ cannot be minimized on the same path $p \in \mathcal{P}$.

- A possible formulation to capture the trade-off

  $\min_{p \in \mathcal{P}} \sum_{k \in p} E(T_k)$
  
  s.t. $P(L_p \geq d) \leq \epsilon$.

- Resource constrained shortest path problem. **NP-hard**.

- Alternative objective: weighted sum of $E(T_p)$ and (Chernoff bound) exponent of $P(L_p \geq d)$.

  $\min_{p \in \mathcal{P}} \left( E(T_p) + \beta \min_{\theta \geq 0} (\Lambda_p(\theta) - \theta d) \right)$

  where $\beta$ is a positive constant, $\Lambda_p(\theta) = \sum_{k \in p} \Lambda_k(\theta)$.

- Nontrivial **global optimization** problem.
Energy vs. delay trade-off (cont.)

- Exchange the order of minimization

\[
\min_{\theta \geq 0} \left( -\beta \theta d + \min_{p \in \mathcal{P}} (E(T_p) + \beta \Lambda_p(\theta)) \right)
\]

- The inner minimization is a **shortest path problem**.
- Objective function: **continuous, piecewise convex** w.r.t. \( \theta \).
Solution approaches

- **Approach I**: *Convex polynomial underestimation* approach (centralized).

![Typical shape of the objective function](image_url)
Solution Approaches (cont.)

- **Approach II:** Simulated annealing based approach.
  - Scoring function:
    \[
    \nu(p) = \mathbb{E}(T_p) + \beta \min_{\theta \geq 0} (\Lambda_p(\theta) - \theta d)
    \]
    The second term is a surrogate for \( \beta \log \mathbb{P}(L_p \geq d) \).
  - **Large deviations**-based scoring function:
    \[
    \log \mathbb{P}(L_p \geq d) \approx \log(-\lambda_p \mathbb{E}(L_p)) + d \lambda_p
    \]
    \[
    \nu(p) = \mathbb{E}(T_p) + \beta \log(-\lambda_p \mathbb{E}(L_p)) + \beta d \lambda_p
    \]
    where \( \lambda_p \triangleq \max\{\lambda_k \mid k \in p\} \).
  - Arbitrarily choose an initial path.
  - Change the path configuration locally.
  - Accept/reject the new path according to the Metropolis criterion.
  - **Distributed** algorithm.
Simulated annealing algorithm
Optimization over duty cycles

- Nodes have power consumption $q_i$ ($i = 1, \ldots, N$) when they are ON, but NOT transmitting any packets.
- What is the fraction of time nodes should stay ON?
- This adds a new dimension to the problem over which one can further optimize.
- Solution approach: simulated annealing algorithm combined with local minimization methods.
Numerical results: solution quality comparison

Setting:
- 300 networks with random topologies.
- Each network has 50 nodes and 1 gateway.

$f_{\text{min}}$: benchmark optimal value obtained by exhaustive search.
- Empirically, centralized method always finds the optimal solution.
- Simulated annealing based algorithm: objective value $f^*$.

<table>
<thead>
<tr>
<th>$\frac{f^* - f_{\text{min}}}{f_{\text{min}}}$</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of cases</td>
<td>87.0%</td>
<td>88.0%</td>
<td>92.7%</td>
<td>95.0%</td>
</tr>
</tbody>
</table>

- Mean = 2.2%, Std = 8.8%.
- Using our large deviations approximation: 1 order of magnitude faster with less communication overhead and simplified computation.
- Further optimizing over duty cycles: In 92/100 instances had improvement, on average by 16.5%.
Summary

- Considered the expected energy consumption and the latency probability in sensor networks in the presence of varying channel conditions and sleeping schedules.
- Energy vs. latency trade-off.
- **Large deviations asymptotic** for the latency probability.
- Centralized and distributed solution approaches:
  - There are significant gains from optimizing the duty cycle and the sleeping schedule.
  - We considered a randomized sleeping schedule (no time synchronization, security advantages).