An Optimal Control Approach for the Persistent Monitoring Problem
X. C. Lin, C. G. Cassandras, and X. C. Ding
Center for Information and Systems Engineering, Boston University

Abstract
We propose an optimal control framework for persistent monitoring problems where the objective is to control the movement of mobile agents to minimize an uncertainty metric in a given mission space. For a single agent in a one-dimensional space, we show that the optimal solution is obtained in terms of a sequence of switching locations, thus reducing it to a parametric optimization problem. Using Infinitesimal Perturbation Analysis (IPA) we obtain a complete solution through a gradient-based algorithm. We illustrate our approach with numerical examples.

Problem Formulation
Goal:
Control the movement of mobile nodes to minimize an uncertainty metric in a given mission space.

Probability of Detection:
We associate with every point \( x \in [0, L] \) a function \( p(x, s) \) captures the probability of detecting an event at this point.

Uncertainty Value:
\( R_i \) measures the uncertainty at selected sampling points \( \alpha_i \in [0, L] \).

Optimization
\[
\min_{s(t)} \int_0^T \sum_{i=1}^M R_i(t) \, dt
\]
subject to
\[
0 \leq s(t) \leq L, \quad -1 \leq u(t) \leq 1
\]

Numerical Experiment
Two numerical examples are presented in Fig. 3. The top two figures correspond to an example with \( L = 20, M = 21, T = 56 \); the bottom two figures correspond to an example with \( L = 100, M = 101, T = 980 \). Both experiments have \( A = 0.01, B = 3, r = 4, R_i(0) = 2 \).

Conclusions and Future work
We have formulated a persistent monitoring problem where we consider a dynamic environment with uncertainties at points changing depending on the proximity of the agent. We obtained an optimal control solution that minimizes the accumulated uncertainty over the environment, in the case of a single agent and 1-D mission space. The solution is characterized by a sequence of switching points, and we use an IPA-based gradient algorithm to compute the solution. Ongoing work aims at solving the problem with multiple agents and a richer dynamical model for each agent, as well as addressing the persistent monitoring problem in 2-D and 3-D mission spaces.

Fig. 1: A queueing system analog of the persistent monitoring problem
Fig. 2: Hybrid automaton for each \( x_i \). Red arrows represent events when the control switches between 1 and -1. Blue arrows represent events when \( R_i \) becomes 0. Black arrows represent all other events.
Fig. 3: Numerical results. Left figures correspond to optimal agent trajectory. Right figures are \( J \) versus iterations.

Hamiltonian Analysis
For system evolves in an interior arc, the Hamiltonian is
\[
H(x, \lambda, u) = \sum_{i=1}^M R_i(t) + \lambda_i(t) u(t) + \sum_{i=1}^M \lambda_i(t) \dot{R}_i(t)
\]

Lemma:
On an optimal trajectory, \( s^*(t) \neq 0 \) and \( s^*(t) \neq L \) for all \( t \in (0, T] \)

Infinitesimal Perturbation Analysis (IPA)
IPA Review:
\[ \partial \partial_{\theta} x \partial \partial_{\theta} t \] measures the uncertainty at selected sampling points \( \alpha_i \in [0, L] \).

Objective function gradient evaluation:
\[
\nabla J(\theta) = \frac{1}{T} \sum_{t=1}^T \nabla R_i(t) \theta
\]
\[ \theta^{\text{new}} = \theta - \eta \nabla J(\theta) \]
\( \nabla J(\theta) \) is the projection of the gradient \( \nabla J(\theta) \) onto the feasible set.