An Overcomplete Dictionary Approach to Imaging of Moving Targets with Multi-static SAR
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Introduction

- Conventional synthetic aperture radar (SAR): imaging of stationary scenes
  motion introduces ambiguities
- Multi-static SAR:
  suitable for imaging of dynamic scenes
  time, frequency and geometric diversity
- Conventional approach:
  matched filtering to each hypothesized velocity independently
- Our approach:
  - Reflectivity estimation over an overcomplete dictionary of velocity vectors
  - Sparse scene imaging with only a few measurements

Moving Target Model

- SAR image:
  - complex valued spatial reflectivity function at some reference time
- Scene model:
  - a sparse set of point reflectors with unknown constant velocities
- Received signal for far-field sensing and narrow-band transmission:

\[ r_k(t) = \int_{|x|<c} s(x; t_{ref}) \tau_k(t - t_{ref}(x; v_k; t_{ref})) \, dx \]

- The reflected waveform delay:
  \[ \tau_k(x; v_k) = t_{ref} - \frac{x^T v_k}{c} - \frac{t_{ref} - t_{ref}(x; v_k; t_{ref})}{c} \]

Transmitted Waveforms

- Overcomplete dictionary \( \Phi \) depends on transmitted waveforms and data collection geometry
- Seek waveforms to lead to an incoherent/RIP dictionary
- Ultra narrow band (UNB) and chirp waveforms lead to Fourier relationship between measurements and the spatial reflectivity

\[ r_k(t) \approx \sum_{p} s_p e^{-j2\pi f_0 (\Delta t + t_{ref}(x_p; v_p; t_{ref}))} \]

Overcomplete Dictionary Approach

- Overcomplete set of hypothesized scatterer velocities
  \( \mathbf{v}_p \in \mathcal{V} = \{ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K \} \)

- Conventional approach:
  matched filtering to each hypothesized velocity independently
- Linear forward model, with quantization noise \( q \)

\[ \mathbf{r} = \Phi \mathbf{s} + \mathbf{n} \]

- Image formation through sparsity preserving optimization

\[ \hat{\mathbf{s}} = \arg \min_{\mathbf{s}} \| \mathbf{s} \|_0 \]

subject to \( \| \mathbf{r} - \Phi \mathbf{s} \|_2^2 \leq \delta \)

\[ \| \mathbf{s}_p \|_0 \in \{0, 1\}, \quad \forall p \]

- Convex relaxation leads to SOCP program

Examples

- Sparse scene with two moving objects
- New overcomplete dictionary reconstruction

Conclusion

- Overcomplete dictionary approach provides:
  - Unified estimation of motion and scattering
  - Focused imaging for dynamic scenes
  - Potential measurement reduction with multifunction sensing
  - Exploits recent results in sparse representations and CS