1. Motivation

I. Why Stochastic Flow Model (SFM)?

- Aggregates many events into simple continuous dynamics, preserves only events that cause drastic change, thus making computation efficient (e.g., orders of magnitude faster simulation)
- Allows simple, unbiased performance sensitivity estimates, not possible in the underlying DES
- Intended for developing control schemes rather than for performance analysis

II. Why Infinitesimal Perturbation Analysis (IPA)?

- Controller (Parameterized by θ)
- MODEL ABSTRACTION
  - Unbiased estimators
  - General distributions
  - Simple implementation

2. Multiclass SFM

I. Problem Formulation

- PROBLEM: Determine (θ1, ..., θn) to optimize system performance

II. General Multiclass Dynamics

FCFS policy in SFM

Set of INFLOW CHANGE EVENTS

\[ \frac{dL_1}{dt} = \begin{cases} 0, & x(t) = 0 \text{ and } x(s) > 0 \text{ for } s < t \\ \frac{\lambda}{\mu} - \frac{\mu}{\lambda}, & x(t) = 1, \frac{\lambda}{\mu}, \frac{\mu}{\lambda} \text{ otherwise} \end{cases} \]

III. Event Classification

- NOTATION: \( t_1, t_2, ..., t_n \) event times in increasing order
  1. Exogenous event: Uncontrolled random change in external processes: \( C(t_1), \theta(t_2) \)
  2. Endogenous event: State \( \theta_\omega(t) \) satisfies \( \theta_\omega(t_1, t_2, t_3) = \theta \) for some continuously differentiable \( \theta_\omega(t) \)
  3. Induced event: Change in \( \theta_\omega(t) \) at time \( t_2 \) because of change in \( \theta_\omega(t) \) at time \( t_1 \) such that:

\[ \tau_1(t_1, t_2, t_3) = \theta_\omega(t_1, t_2, t_3) \]

Induced events (red dots) can be induced by:

1. Exogenous event, e.g., \( \tau(t_1, t_2, t_3) \)
2. Endogenous event, e.g., \( \tau(t_1, t_2, t_3) \)
3. Another induced event, e.g., \( \tau(t_1, t_2, t_3) \)

3. IPA and Performance Optimization

I. General IPA Derivation: Three Equations

IPA goal: for performance metric \( J(\theta, t, \omega, T) \): Obtain unbiased estimates of performance metric \( \frac{dJ}{d\theta} \)

\[ \frac{dJ}{d\theta} = \sum \left[ \frac{\partial J}{\partial \theta} \right] \]

STATE DERIVATIVE:

\[ \frac{dJ}{d\theta} = \sum \left[ \frac{\partial J}{\partial \theta} \right] \]

EVENT TIME DERIVATIVE:

1. Exogenous events: By definition, \( \frac{dJ}{d\theta} \)
2. Endogenous events: Recall that \( \frac{dJ}{d\theta} \)
3. Induced events: \( \sum \left[ \frac{\partial J}{\partial \theta} \right] \)

II. Apply General Equations on Multiclass SFM

Lemma 1: If either one of two holds, then \( \frac{dJ}{d\theta} \) only depends on information on event times

\[ L(\theta) = \text{inflow change event, for all } t \]

\[ \frac{dL}{dt} = \text{induced event, if } \theta(t_1, t_2, t_3) \]

Theorem: Under some (mild) technical conditions, the IPA gradient estimators derived above are UNBIASED

3. Numerical Results: A Resource Contention Game

- Algorithm effective: Convergence attained
- Gap between system-centric and user-centric optimization: Price of anarchy
- Gap may disappear in some situations (C. Yao and C.G. Cassandras, 2009)