MA 581 Qualifying Exam Problems **April 2016**

1. Let X be a random variable with a geometric distribution with parameter $0 < p < 1$.

   1) Find $P(X = 3)$.
   2) Find $P(X = 7 | X > 5)$.
   3) Find the moment generating function $M_X(t)$ of X.
   4) Compute the expected value $\mathbb{E}[X]$ using $M_X(t)$.
   5) Use the fact $\text{Var}[X] = (1-p)/p^2$ to compute $\mathbb{E}[X(1-X)]$.

   Suppose $X_1, X_2, \ldots$ are independent and identically distributed like X. Let $S_n = X_1 + X_2 + \ldots + X_n$.

   6) Find $\text{Var}[\bar{X}_4]$, where $\bar{X}_4 = S_4/4$.

   7). Find the exact distribution of S_4 and justify your answer.

2. Let X be a rv with pdf given by $f(x) = cx^3$, for $1 < x < 3$, where c is a constant.

   a. Determine the value of c.
   b. Find P(2 < X < 2.5).
   c. Find the cdf F of X.
   d. Compute P(X > 2.2 | X < 2.7).
   e. Compute E(X).
   f. Compute stdev(X).
   g. Compute $E[\cos(X) / X^3]$ exactly.
   h. Compute the pdf of X^2.
   i. Suppose that 10 students each independently generate their own value of X. Find the probability that exactly 7 of those students generate a value greater than 2.2.
   j. Suppose Y is a rv such that X and Y are iid (where X is the rv of this problem). Compute

      stdev (3X – 7Y).