

Qualifying Exam: CAS MA 576

Boston University, Spring 2012

Problem 1. Let Y_1, \dots, Y_n be iid random variables with probability density function

$$f_Y(y) = \frac{e^{y\theta} \cos(\theta)}{2 \cosh(\pi y/2)}, \quad y \in \mathbb{R}.$$

- (a) Show that this distribution belongs to the exponential family.
- (b) Find the natural parameter, cumulant function, and dispersion parameter for this distribution.
- (c) Compute the mean and variance of this distribution. Express the variance as a function of the mean.
- (d) Compute the canonical link for a GLM from this distribution.
- (e) Write down an expression for the deviance associated with this distribution.

Problem 2 Consider the data in the table below from a study of automobile accidents in Florida in 1988.

	Ejected?	Fatal incident	Nonfatal incident
Seat belt used	Yes	14	1105
	No	483	41111
Seat belt not used	Yes	497	4624
	No	1008	157342

A logistic binomial GLM was used to model the likelihood of a fatal incident as a function of whether a seat belt was used and whether the driver was ejected. The maximum likelihood values for the coefficients and the variance-covariance matrix are given below:

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Coefficients: (Intercept)          belt          eject
                -5.0436             -1.7173         2.7978
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var-cov:      (Intercept)          belt          eject
(Intercept)  0.0009735536 -0.0009350671 -0.0009322571
belt         -0.0009350671  0.0029176633  0.0008062108
eject       -0.0009322571  0.0008062108  0.0030532124
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(Dispersion parameter for binomial family taken to be 1)

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Null deviance: 3568 on 3 degrees of freedom
Residual deviance: 2.854 on 1 degrees of freedom
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- (a) Estimate the probability that an automobile accident where the driver did not use a seat belt and was not ejected resulted in a fatality?
- (b) Estimate a 95% confidence interval on the probability from part (a).
- (c) How much does wearing a seat belt affect the odds of having a fatal incident?
- (d) Is the effect in part (c) statistically significant?
- (e) Provide a point estimate and a 95% confidence interval for the probability that an automobile accident where the driver did not use a seat belt and was ejected resulted in a fatality.