Qualifying Exam: CAS MA 582

Boston University, Spring 2011

Problem 1. Let $X_1,\ldots,X_n$ be independent identically distributed random variables with probability density function $f_X(x) = \frac{x^3 e^{-x/\theta}}{6\theta^4}$. It is known that $E[X_i] = 4\theta$ and that $Var[X_i] = 4\theta^2$.

(a) Write down the log likelihood for the data as a function of θ.

(b) Compute the maximum likelihood estimator, $\hat{\theta}$, for θ.

(c) Compute the mean and variance of $\hat{\theta}$.

(d) Compute the Fisher information for θ. Is $\hat{\theta}$ efficient?

(e) Let $\lambda = 1/\theta$. Find the maximum likelihood estimator of λ and compute its asymptotic distribution as $n \to \infty$.

Problem 2.

We say the rv X has the W distribution with parameter $\theta > 0$ (written $X \sim W(\theta)$) if X has pdf $f(x)= 3x^2/\theta^3$, for $0 < x < \theta$, and $f(x) = 0$, elsewhere.

Consider the parameterized W family $\{W(\theta) : \theta > 0 \}$.

a. Let Y_n be the maximum of the random sample of size n. Show that Y_n is a consistent estimator of θ.

b. Find the pdf of Y_n. (*Hint:* Find the cdf first.)

c. Show that Y_n is NOT an unbiased estimator of θ.

d. Show that $n(\theta - Y_n)$ converges in distribution, and find its asymptotic distribution explicitly.

e. Find an unbiased estimator of θ, call it T_n. Show that T_n is a consistent estimator of θ.

f. Show that $n(\theta - T_n)$ converges in distribution, and find its asymptotic distribution explicitly.