Qualifying Exam: CAS MA 583

Boston University, Spring 2008

Problem 1. Let X_1 be the first arrival time of an Inhomogeneous Poisson process with rate function $\lambda(t)$.

(a) Write down the density function for X_1.

(b) Let $Z_1 = \int_0^{X_1} \lambda(t)dt$. Find the density function of Z_1.

(b) Let $\lambda(t) = \frac{1}{(t+1)}$. Compute $\Pr[X_1 > 1]$.

Problem 2. Assume we have $2n$ numbered balls distributed between 2 urns. Each minute, we select a number uniformly between $\{1,...,2n\}$, find which urn the ball with that number is in, and switch it to the other urn.

(a) Construct a Markov chain describing the number of balls in each urn after each step. How many states does this chain have? What are the transition probabilities? Is the chain irreducible? Classify the states as transient, null recurrent, or positive recurrent. What is the period of each state?

(b) How many stationary distributions will this Markov chain have? Write down a stationary distribution if at least one exists. If not, prove that no stationary distribution exists.

(c) Determine whether this chain is time reversible.