1. Let \( X_1, X_2, \ldots, X_n \) be a random sample from an exponential distribution with unknown parameter \( \theta > 0 \). That is, the pdf \( f(x; \theta) \) is given by
\[
f(x; \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \text{for } x > 0, \theta > 0 \\ 0, & \text{elsewhere,} \end{cases}
\]
where \( \theta > 0 \) is to be estimated.
   a. Find the maximum likelihood estimator (MLE) \( \hat{\theta}_{MLE} \) of \( \theta \).
   b. Show that \( \hat{\theta}_{MLE} \) is unbiased for \( \theta \).
   c. Show that \( \hat{\theta}_{MLE} \) is a consistent estimator for \( \theta \).
   d. Show that \( \hat{\theta}_{MLE} \) is asymptotically normal.
   e. Compute Fisher's information for \( \theta \).
   f. Is \( \hat{\theta}_{MLE} \) an efficient estimator of \( \theta \)?

2. Fix \( \lambda > 0 \), and let \( X \sim \text{Pois}(\lambda) \).
   a. Find the mgf of \( X \).
   b. If \( X \) and \( Y \) are iid \( \sim \text{Pois}(\lambda) \), prove that \( X+Y \sim \text{Pois}(2\lambda) \).
   c. If \( Y_n \sim \text{Pois}(n\lambda) \), show that \( \sqrt{n}(Y_n/n - \lambda) \) converges in distribution, and identify its asymptotic distribution completely and explicitly. (Hint: Use part (b))
   d. Show that \( \sqrt{n}(\sqrt{Y_n/n} - \sqrt{\lambda}) \) converges in distribution, and identify its asymptotic distribution completely and explicitly.

3. Let \( \overline{X} = (X_1, X_2, \ldots, X_n) \) be a random sample from a population with unknown mean \( \theta \) and known finite variance \( \sigma^2 \).
   a. Find a best linear unbiased estimator (BLUE) \( T = T(\overline{X}) \) of \( \theta \).
   b. Find the variance of the BLUE \( T = T(\overline{X}) \) of \( \theta \).
   c. Let \( T = T(\overline{X}) \) be the BLUE of \( \theta \) and let \( T_i = T_i(\overline{X}) \) be another linear unbiased estimator of \( \theta \). Show that \( \text{Cov}_{\theta}(T, T_i) = \text{Var}_{\theta}(T) \).